2 Electrostatic Potential and Capacitance

2.1 Introduction

In Chapter 1, we learned about the types of electric charge, the forces acting between the charges, the electric fields produced by a point charge and by different charge distributions and Gauss' theorem. The **force** acting on a given **charge q** can be found by knowing the electric field. Now, if the electric charge is able to move due to this force, it will start moving and in such a motion work will be done. So, now in this chapter we shall study in detail, the physical quantities like electrostatic energy, electrostatic potential that give information about the work done on the charge. Moreover electric potential and electric field, both the quantities can be obtained from each other. We will also know the relation between them.

A simple device which stores the electric charge and electrical energy is a **capacitor.** We shall also study about the capacitance of a capacitor, the series and parallel combinations of capacitors, the electrical energy stored in it, etc. The capacitors are used in different electrical and electronic circuits e.g. electric motor, flashgun of a camera, pulsed lasers, radio, TV etc. At the end of the chapter we shall see about a device–with the help of which we can get a very large potential difference–Van de Graaff generator.

2.2 Work done during the Motion of an Electric Charge in the Electric Field

We had seen in Chapter-1 that when an electric charge q is placed at a point in an electric field $\vec{E}$, a force $\vec{F} = q\vec{E}$, acts on it. Now, if this charge is able to move, it starts moving. To discuss the work done during such a motion, initially we will consider a unit positive charge.

As shown in the figure 2.1, we want to take a unit positive charge (q = +1 C charge) from point A to point B, in the electric field produced by a point charge (Q), and also want to find the **work done by the electric field** during this motion. Many different paths can be thought of to go from A to B. In the figure 2.1 ACB and ADB paths are shown as illustrations.

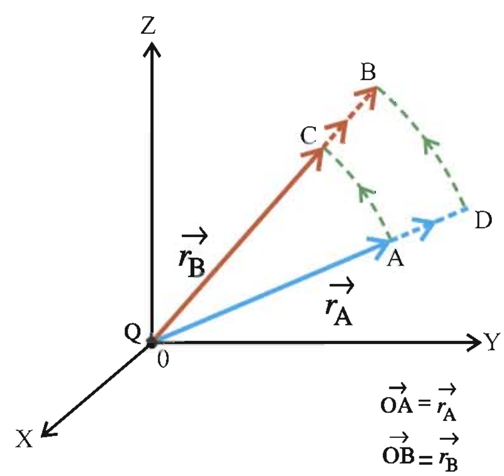


Figure 2.1 Work during the Motion of a Charge

According to the definition, the force on the unit positive charge at a given point, is the electric field $\vec{E}$ at that point. According to the formula $E = \frac{kQ(1)}{r^2}$ this force varies continuously with distance. Hence the work done by the electric field on unit positive charge in a small displacement is given by $dW = \vec{E} \cdot \vec{dr}$ and the work done during

A to B by $W_{AB} = \int_A^B \vec{E} \cdot \vec{dr}$ (2.2.1)

Here, $\int_A^B \vec{E} \cdot \vec{dr}$ is called the **line integral of electric field** between the points A and B.

ACB Path : (1) First, we go from A to C on the circular arc AC having radius OA and then we go from C to B in $\vec{OC}$ direction. The electric field produced by Q, is normal to the arc AC at every point on it (the angle between $\vec{E}$ and $\vec{dr}$ = 90°). Hence $W_{AC} = \int_A^C \vec{E} \cdot \vec{dr} = 0$.

The work done by the electric field on the path CB, is

$$W_{CB} = \int_C^B \vec{E} \cdot \vec{dr} \quad (2.2.2)$$

$$= \int_C^B \frac{kQ}{r^2} \hat{r}_B \cdot dr \hat{r}_B = kQ \int_C^B \frac{1}{r^2} dr = kQ \left[-\frac{1}{r} \right]_{r_C}^{r_B}$$

$$W_{CB} = kQ \left[\frac{1}{r_C} - \frac{1}{r_B} \right] \quad (2.2.3)$$

Thus, on the path ACB, the work done by the electric field

$$W_{ACB} = W_{AC} + W_{CB} = kQ \left[\frac{1}{r_C} - \frac{1}{r_B} \right] \quad (2.2.4)$$

Here, since $r_C < r_B$, it is self-evident that this work is positive.

(2) Path ADB : From A to D, just like the above, the work done by the electric field is obtained as $W_{AD} = kQ \left[\frac{1}{r_A} - \frac{1}{r_D} \right]$. Moreover, since the electric field is normal to the arc DB, the work done in this motion = 0.

Hence the work done by the electric field on ADB path is

$$W_{ADB} = W_{AD} + W_{DB} = kQ \left[\frac{1}{r_A} - \frac{1}{r_D} \right] \quad (2.2.5)$$

Here $|\vec{r_D}| = |\vec{r_B}|$ and $|\vec{r_A}| = |\vec{r_C}|$. Hence from equations 2.2.4 and 2.2.5,

$$W_{ACB} = W_{ADB} = W_{AB} = kQ \left[\frac{1}{r_A} - \frac{1}{r_B} \right] \quad (2.2.6)$$

Thus, in an electric field, the work done by the **electric field** in moving a unit positive charge from one point to the other, **depends only on the positions of those two points and does not depend on the path joining them.**

Now, if we move the unit positive charge from point B to A, **on any path,** the work done by the electric field, will be given by (according to equation 2.2.6)

$$W_{BA} = kQ\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad (2.2.7)$$

If a unit positive charge is taken from point A to B **on any path** and then is brought back to A on any path, a closed loop is formed (e.g. ACBDA or ADBCA) and on this closed loop the total work done by the electric field ($\oint \vec{E} \cdot \vec{dr}$); will be $W_{AB} + W_{BA} = 0$ (using equations 2.2.6 and 2.2.7). You are aware of the fact that a field with this property is known as a **conservative field.** Thus electric field is also a conservative field. [In Standard 11 you had also seen that the gravitational field is also a conservative field.]

Although we have considered the work done on unit positive charge, all these aspects are also applicable to the work done on any charge q, but for that, the right hand side of the above equations for the work, should be multiplied by q. e.g., Work for A to B will be $W_{AB} = \int_A^B q\vec{E}\cdot\vec{dr}$. Moreover, you will be able to understand that instead of the work done **by the electric field,** if we want to find the **work required to be done by the external force against the electric field** (for the motion without acceleration), then the **negative sign** will have to be put on the right hand side of the above equation (2.2.1) for the work. Hence for unit positive charge, such a work will be given by $W'_{AB} = -\int_A^B \vec{E}\cdot\vec{dr}$, which is the same in magnitude as work given by equation 2.2.1 but has the opposite sign to it. For charge q such a work will be given by $W''_{AB} = -\int_A^B q\vec{E}\cdot\vec{dr}$.

From this discussion we should remember that $\int_A^B \vec{E}\cdot\vec{dr}$, that is the line integral of electric field between A to B – is the work done by the electric field in moving a unit positive charge from A to B and it does not depend on the path. Moreover, $\oint \vec{E}\cdot\vec{dr} = 0$. $\vec{E}\cdot\vec{dr}$ is also sometimes written as $\vec{E}\cdot\vec{dl}$ where $\vec{dl}$ is also a small displacement vector

2.3 Electrostatic Potential

We know that the work done by the electric field in moving a unit positive (+1 C) charge from one point to the other, in the electric field, depends only on the positions of those two points and not on the path joining them.

If we take a reference point A, and take the unit positive charge from point A to B; A to C; A to D; ..... etc in the electric field, then the work done by the electric field is obtained as $W_{AB} = \int_A^B \vec{E}\cdot\vec{dr}$, $W_{AC} = \int_A^C \vec{E}\cdot\vec{dr}$, $W_{AD} = \int_A^D \vec{E}\cdot\vec{dr}$,... respectively. But the reference point A is already fixed, hence the above mentioned work depends on the position of the other points (B, C, D, ...) only. Conventionally the reference point is taken as a point at infinite distance from the source of electric field. Hence to bring a unit positive charge from that point to a point P in the field, the work done by the electric field is given by the formula $W_{\infty P} = \int_\infty^P \vec{E}\cdot\vec{dr}$

and it becomes the function only of the position of point P. But, if we want to find the work required to be done against the electric field; in order that the motion becomes "motion without acceleration,"

the formula $W'_{\infty P} = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr}$ has to be used.

An important characteristic of an electric field is called electrostatic potential and with reference to the work done on unit positive charge, it is defined as under :

"Work required to be done against the electric field in bringing a unit positive charge from infinite distance to the given point in the electric field is called the electrostatic potential (V) at that point."

Here the meaning of "against the electric field" is actually "against the force by the electric field". We will call the electrostatic potential as electric potential in short.

According to the above definition, the electric potential at a point P is given by the formula :

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr} \qquad (2.3.1)$$

In other words this formula represents the definition of electric potential.

From this formula the potential difference betwen points Q and P is given by

$$V_Q - V_P = \left(-\int_{\infty}^{Q} \vec{E} \cdot \vec{dr}\right) - \left(-\int_{\infty}^{P} \vec{E} \cdot \vec{dr}\right) \qquad (2.3.2)$$

$$= \int_{Q}^{\infty} \vec{E} \cdot \vec{dr} + \int_{\infty}^{P} \vec{E} \cdot \vec{dr} = \int_{Q}^{P} \vec{E} \cdot \vec{dr} \qquad (2.3.3)$$

$$= -\int_{P}^{Q} \vec{E} \cdot \vec{dr} \qquad (2.3.4)$$

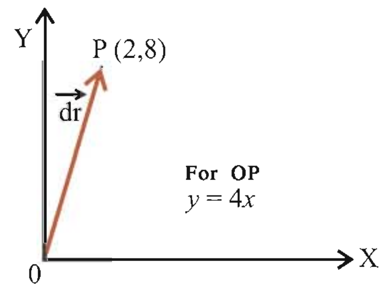
This potential difference shows the work required to be done to take a unit positive charge from P to Q, against the electric field and in that sense it also shows the potential of Q with respect to P. Very often the potential difference is in short written as p.d. also. The unit of electric potential (and hence that of the potential difference also) is joule / coulomb which is called volt (symbol V) in memory of the scientist Volta. i.e., volt $= \frac{\text{joule}}{\text{coulomb}}$ or $V = \frac{J}{C}$. It's dimensional formula is $M^1L^2T^{-3}A^{-1}$.

Electric potential is a scalar quantity. Moreover, we have obtained electric potential from the vector quantity-electric field $\vec{E}$ (See equation 2.3.1). In future we will also obtain electric field from the electric potential. In the calculations involving electric field $\vec{E}$, its three components E_x, E_y, E_z have to be considered and the calculations become longer, while in the calculations involving the electric potential, only one scalar appears and hence the calculations become shorter and easier. Hence the concept of electric potential is widely used. Absolute value of electric potential has no importance, only the difference in potential is important.

[For Information Only : Galvani (1737–1798) produced electricity by placing two different metallic electrodes in the tissue of frog. He called it Animal Electricity. Volta explained that the above process had nothing to do with the characteristics of the frog, but one can generate electricity by placing two dissimilar metallic electrodes on any wet body. He was the one who designed the electro chemical cell, which we studied earlier as voltaic cell.

The importance of electric potential in electricity is similar to the importance of temperature in thermodynamics and the height of fluid in hydrostatics. The electricity flows (i.e. the electric current flows) from an electrically charged material having higher electric potential to an electrically charged material having lower electric potential. Quite similar to water, which flows from a higher level to a lower level or like the flow of heat which flows from a region having higher temperature to a region having lower temperature. Thus, the direction of the flow of electric current between two materials depends on their electric potentials.]

Illustration 1 : Suppose an electric field due to a stationary charge distribution is given by $\vec{E} = ky\hat{i} + kx\hat{j}$, where k is a constant. (a) Find the line integral of electric field on the linear path joining the origin O with point P(2, 8), in the Figure. (b) Obtain the formula for the electric potential at any point on the line OP, with respect to (0, 0)



Solution : (a) The displacement vector $\vec{dr}$ on the line OP is $\vec{dr} = dx\hat{i} + dy\hat{j}$

$$\therefore \vec{E}\cdot\vec{dr} = (ky\hat{i} + kx\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= kydx + kxdy = k(ydx + xdy)$$

Moreover, on the entire OP line $y = 4x$ ($\because$ the slope of a straight line is constant)

$\therefore$ $dy = 4dx$

$\therefore$ The line integral of electric field from O to P, is

$$\int_O^P \vec{E}\cdot\vec{dl} = k\int_O^P (ydx + xdy) = k\int_{(0,0)}^{(2,8)} [4xdx + x(4dx)] = k\int_0^2 8x\,dx \qquad \text{(A)}$$

$$= 8k\left[\frac{x^2}{2}\right]_0^2 = 16k$$

(b) In order to obtain the potential at any point Q(x, y) on the line OP with respect to (0, 0) we can use $V(Q) = -\int_O^Q \vec{E}\cdot\vec{dl}$

$$\therefore V(Q) = -\int_{(0)}^{(x)} 8kx\,dx \text{ ... (from equation A)}$$

$$= -8k\left[\frac{x^2}{2}\right]_0^x = -4kx^2$$

Illustration 2 : The electric field at distance r perpendicularly from the length of an infinitely long wire is $E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density of the wire. Find the potential at a point having distance b from the wire with respect to a point having distance a from the wire ($a > b$). [**Hint :** $\int \frac{1}{r} dr = ln\ r$].

Solution : $V_b - V_a = -\int_a^b \vec{E} \cdot \vec{dr}$

$$= -\int_a^b \frac{\lambda}{2\pi\epsilon_0 r} dr \quad (\because \vec{E} \parallel \vec{dr})$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \int_a^b \frac{1}{r} dr = -\frac{\lambda}{2\pi\varepsilon_0} [ln\ r]_a^b = -\frac{\lambda}{2\pi\varepsilon_0 r} [ln\, b - ln\, a]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} [ln\, a - ln\, b]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} ln \left(\frac{a}{b}\right)$$

For reference point a, taking $V_a = 0$

$$\therefore V_b = \frac{\lambda}{2\pi\varepsilon_0} ln \left(\frac{a}{b}\right)$$

Illustration 3 : An electric field is represented by $\vec{E} = Ax\hat{i}$, where $A = 10 \frac{V}{m^2}$. Find the potential of the origin with respect to the point (10, 20)m.

Solution : $\vec{E} = Ax\hat{i} = 10x\hat{i}$

$$V(0, 0) - V(10, 20) = -\int_{(10,20)}^{(0,0)} \vec{E} \cdot \vec{dr}$$

$$= -\int_{(10,20)}^{(0,0)} (10x\hat{i}) \cdot (dx\hat{i} + dy\hat{j}) = -\int_{10}^{0} 10x\, dx$$

$$= -10\left[\frac{x^2}{2}\right]_{10}^{0} = [0 - (-500)] = 500 \text{ volt}$$

Since V(10, 20) is to be taken as zero,

V(0, 0) = 500 volt.

2.4 Electrostatic Potential Energy

In the previous article (2.2), we had discussed the work done by the electric field on a unit positive charge and then also on the charge q, during the motion in the electric field. Moreover we had also talked about the work required to be done by the external force against the electric field, in which the **motion of charge is without acceleration only.** Hence its velocity remains constant and its kinetic energy does not change. But the work done by this external force is stored in the form of potential energy of that charge. From this, the electric potential energy is defined as under :

"The work required to be done against the electric field in bringing a given charge (q), from infinite distance to the given point in the electric field is called the electric potential energy of that charge at that point." Here "motion without acceleration" is implied when we mentioned" "work required to be done."

From the definitions of electric potential energy and the electric potential, we can write the electric potential energy of charge q at point P, as

$$U_P = -\int_{\infty}^{P} q\vec{E}\cdot\vec{dr} = -q\int_{\infty}^{P} \vec{E}\cdot\vec{dr} \quad (2.4.1)$$

$$= qV_P \quad (2.4.2)$$

Moreover, we can also call the electric potential at point P as the electric potential energy of unit positive charge (q = +1 C) at that point. That is,

$$\begin{Bmatrix}\text{electric potential} \\ \text{at a given point}\end{Bmatrix} = \begin{Bmatrix}\text{electric potential energy of unit} \\ \text{positive charge at that point}\end{Bmatrix}$$

For more clarity in this discussion, we note a few important points as under :

(1) When we bring charge q (or a unit positive charge) from infinite distance to the given point or when we move it from one point to the other in the field, the **positions of the sources (charges) producing the field are not changed.** (We will imagine these sources as being clamped on their positions by some invisible force !!)

(2) The absolute value of the electric potential energy is not at all important, only the difference in its value is important. Here, in moving a charge q, from point P to Q, **without acceleration, the work required to be done by the external force,** shows the difference in the electric potential energies ($U_Q - U_P$) of this charge q, at those two points.

$$\therefore\ U_Q - U_P = -q\int_{P}^{Q} \vec{E}\cdot\vec{dr} \quad (2.4.3)$$

(3) Here, electric potential energy is **of the entire system** of the sources producing the field and the charge that is moved, for **some one configuration,** and when the configuration changes the electric potential energy of the system also changes. e.g., when the distance between them is r, it is one configuration and if distance r changes, the configuration is also said to be changed and hence the electric potential energy of the system is also said to be changed. But as the conditions of the sources producing the field are not changed, the entire change in the electric potential energy is **experienced by this charge q only which we have moved.** Hence we are able to write $U_Q - U_P$ as the difference in potential energy **of this charge q only.** Because of this reason we have mentioned "potential energy of charge q" for equation 2.4.1 and "potential energy of unit positive charge" in the discussion that followed it.

2.5 Electric Potential due to a Point Charge

We want to find the electric potential V(P), due to a point charge q, at some point P, at a distance r from it.

For this we will put the origin of co-ordinate axes 0, at the position of that charge. See figure 2.2. Here $\vec{OP} = \vec{r}$. According to the definition of electric potential we can use the equation.

$$V(P) = -\int_{\infty}^{P} \vec{E}\cdot\vec{dr} \quad (2.5.1)$$

Moreover, we can also write this equation in another form as

$$V(P) = \int_{P}^{\infty} \vec{E}\cdot\vec{dr} \quad (2.5.2)$$

because, $\int_{\infty}^{P} \vec{E}\cdot\vec{dr} = -\int_{P}^{\infty} \vec{E}\cdot\vec{dr}$.

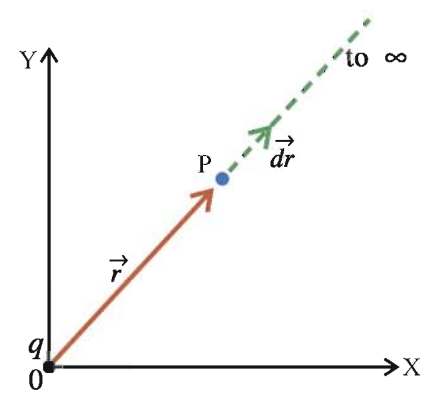


Figure 2.2 Potential due to Point Charge

At this point P, $\vec{E} = \frac{kq}{r^2}\hat{r} \quad (2.5.3)$

$\therefore$ From equation 2.5.2

$$V(P) = \int_P^{\infty} \frac{kq}{r^2} \hat{r} \cdot dr\hat{r} = \int_r^{\infty} \frac{kq}{r^2} dr$$

$$= kq \int_r^{\infty} \frac{1}{r^2} dr = kq \left[-\frac{1}{r}\right]_r^{\infty}$$

$$V(P) = \frac{kq}{r} \qquad (2.5.6)$$

$$\text{or} \quad V(P) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \qquad (2.5.7)$$

This equation is true for any charge, positive or negative. The potential due to a positive charge is positive and that due to a negative charge is negative (as q is to be put with negative sign in the above equation.)

It is self evident from equation 2.5.6 that as the distance r increases, the electric potential decreases as $\frac{1}{r}$. In case of potential also superposition principle is applicable. To find the electric potential due to many point charges we should find the potential due to every charge according to equation 2.5.7 and they should be added algebraically.

Illustration 4 : A point P is 20 m away from a 2 μC point charge and 40 m away from a 4 μC point charge. Find the electric potential at P.

(1) Find the work required to be done to bring 0.2 C charge from infinite distance to the point P.

(2) Find the work required to be done to bring –0.4 C charge from infinite distance to the point P. [$k = 9 \times 10^9$ N m^2 C^{-2}]

Solution : $V_P = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = k\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$$= 9 \times 10^9 \left[\frac{2 \times 10^{-6}}{20} + \frac{4 \times 10^{-6}}{40}\right] = 1800 \text{ volt}$$

(1) $W_1 = V_P q_1' = (1800)(0.2) = 360$ J.

(2) $W_2 = V_P q_2' = (1800)(-0.4) = -720$ J

2.6 Electric Potential due to an Electric Dipole

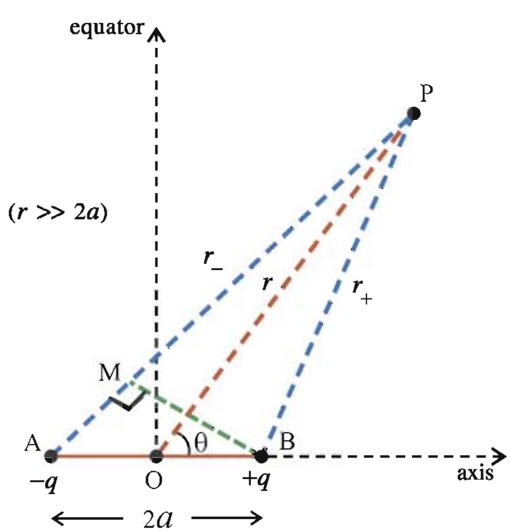


Figure 2.3 Potential due to an Electric Dipole

We have seen in Chapter-1 that two equal and opposite charges ($+q$ and $-q$) separated by a finite distance ($= 2a$) constitute an electric dipole.

Such a dipole is shown in the figure 2.3, with the origin of co-ordinate system O at its mid-point. The magnitude of the dipole moment of the dipole is $p = q(2a)$ and its direction is from negative to the positive charge that is, in AB direction.

We want to find the electric potential at point P far away from the mid-point O of dipole and in the direction making an angle θ with the axis of the dipole. Let OP = r, AP = r_-, and BP = r_+. At P, the electric potential is equal to the sum of the potentials produced by each of the charges.

$$\therefore\ V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-} \quad (2.6.1)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r_- - r_+}{r_- r_+}\right] \quad (2.6.2)$$

Since P is a far distant point, $r >> 2a$ and hence we can take AP || OP || BP. In this condition the figure 2.3 shows that

$$\left\{\begin{array}{l}\text{for numerator of equation (2.6.2), } r_- - r_+ = AM = 2a\ cos\theta \\ \text{and for denominator, } r_- \approx r_+ \approx r\end{array}\right\} \quad (2.6.3)$$

We have considered a very far distant point as compared to the length ($2a$) of the dipole. The molecular dipoles are very small and such an approximation is very well applicable to them. From equations (2.6.2) and (2.6.3), we get

$$V(r) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2a\,cos\theta}{r^2}\right) \quad (2.6.4)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\,cos\theta}{r^2} \quad (2.6.5)$$

Writing the unit vector in the direction $\vec{OP}$ as $\hat{r}$, we can wirte $\vec{p}\cdot\hat{r} = p\ cos\theta$.

$$\therefore\ V(\vec{r}\,) = \frac{1}{4\pi\varepsilon_0}\ \frac{\vec{p}\cdot\hat{r}}{r^2}\ (\text{for } r >> 2a) \quad (2.6.6)$$

Note : The dipole obtained in the limits $q \to \infty$ and $a \to 0$, is called the point dipole. For such a point dipole the above equation is more accurate, while for the physical dipole - found in practice - this equation gives an approximate value of the electric potential. Let us note a few points evident from equation (2.6.4), as under :

(1) Potential on the Axis : For a point on the axis of the dipole

$$\theta = 0 \text{ or } \pi.\ \therefore\ V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

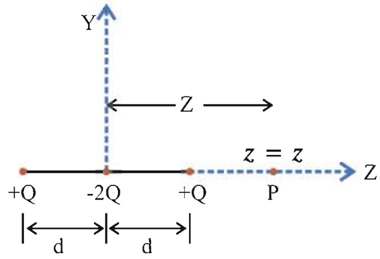
From the given point, if the nearer charge is $+q$, then we get V as positive and if it is $-q$, then we get V as negative.

(2) Potential on the Equator : For a point on the equator $\theta = \frac{\pi}{2}$ $\therefore$ V = 0

(3) The potential at any point depends on the angle between its position vector $\vec{r}$ and $\vec{p}$.

(4) The potential due to a dipole decreases as $\frac{1}{r^2}$ with distance (while the potential due to a point charge decreases as $\frac{1}{r}$ with distance). We have seen in Chapter 1 that the electric field due to a dipole decreases as $\frac{1}{r^3}$)

Ilustration 5 : When two dipoles are lined up in opposite direction, the arrangement is known as a quadruple (as shown in the Figure). (1) Calculate the electric potential at a point $z = z$ along the axis of the quadruple and (2) If $z >> d$, then show that,



$$V(z) = \frac{Q}{4\pi\varepsilon_0}\frac{2d^2}{z^3}$$

Note : $2|Q|d^2$ is called the quadruple moment.

Solution : (1) Let z be the Z co-ordinate of point P.

The electric potential at point P, due to +Q charge (which is at the left hand side of the origin) is,

$$V_1 = \frac{kQ}{z+d} \qquad (1)$$

The electric potential at point P due to the +Q charge which is at the right hand side of the origin is,

$$V_2 = \frac{kQ}{z-d} \qquad (2)$$

The electric potential at point P, due to –2Q charge present at the origin is,

$$V_3 = -\frac{k(2Q)}{z} \qquad (3)$$

∴ The total potential at point P,

$$V(z) = V_1 + V_2 + V_3$$

$$= kQ\left[\frac{1}{z+d}+\frac{1}{z-d}-\frac{2}{z}\right] = kQ\left[\frac{2z}{z^2-d^2}-\frac{2}{z}\right] = kQ\left[\frac{2d^2}{z(z^2-d^2)}\right]$$

(2) If $z >> d$, we can neglect d^2 in comparison with z^2 in the denominator of right hand side of the above equation.

$$\therefore\ V(z) = \frac{kQ(2d^2)}{z^3} = \frac{Q}{4\pi\varepsilon_0}\frac{2d^2}{z^3}$$

Illustration 6 : Charge Q is distributed uniformly over a non-conducting sphere of radius R. Find the electric potential at distance r from the centre of the sphere (r < R). The electric field at a distance r from the centre of the sphere is given as $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\, r\hat{r}$. Also find the potential at the centre of the sphere.

Solution : The electric potential on the surface of such a sphere is,

$$V(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

As a result, we can use the equation $V(r) - V(R) = -\int_R^r \vec{E}\cdot\vec{dr}$

$$\therefore\ V(r) - V(R) = -\int_R^r \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\, r dr\ \hat{r}\cdot\hat{r} \qquad (\because \vec{dr} = dr\,\hat{r})$$

$$= -\frac{Q}{4\pi\varepsilon_0 R^3}\int_R^r r dr = -\frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}\right]_R^r$$

$$= -\frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}-\frac{R^2}{2}\right]$$

$$\therefore\ V(r) = V(R) + \frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{R^2}{2} - \frac{r^2}{2}\right]$$

$$\therefore\ V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{Q}{8\pi\varepsilon_0 R^3}(R^2 - r^2)$$

$$\therefore\ V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right),\ r < R$$

At the centre of the sphere $r = 0$, $\therefore$ V (centre) $= \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{2R}\right)$.

2.7 Electric Potential due to a System of Charges

In a system of charges, point charges could have been distributed descretely (separated from each other) while in some system they could have been distributed continuously with each other. In some system of charges the distribution of charges could be a mixture of any type of these two distributions.

(a) Descrete Distribution of Charges :

In figure 2.4, point charges q_1, q_2, q_3 ...... q_n are shown as distributed descretely. The position vectors of these charges with respect to the origin of co-ordinate system are $\vec{r_1}$, $\vec{r_2}$,... $\vec{r_n}$ respectively. We want to find the electric potential due to this system, at point P with position vector $\vec{r}$. For this we will find the electric potential due to every point charge and then will make summation.

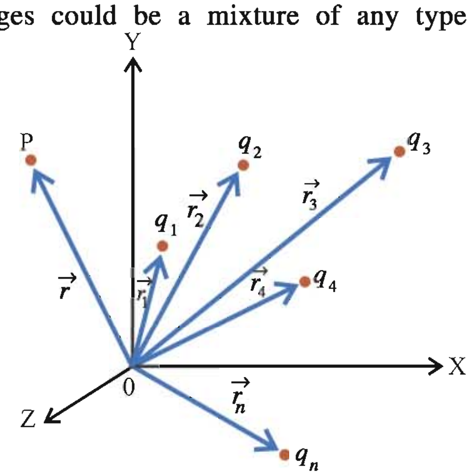


Figure 2.4 Potential Due to Descrete Charges

That is, $V = V_1 + V_2 + ... + V_n$ (2.7.1)

$$\therefore\ V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1p}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2p}} + ... + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{np}} \qquad (2.7.2)$$

Where r_{1p} = distance of P from $q_1 = |\vec{r} - \vec{r_1}|$.

Similarly r_{2p}, ..., r_{np} are the corresponding distances.

$$\therefore\ V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\vec{r} - \vec{r_1}|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{|\vec{r} - \vec{r_2}|} + ... + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{|\vec{r} - \vec{r_n}|} \qquad (2.7.3)$$

$$\therefore\ V = \sum_{i=1}^{n}\frac{kq_i}{|\vec{r} - \vec{r_i}|} \qquad (2.7.4)$$

(b) Electric Potential due to a Continuous Distribution of Charges :

Suppose in a certain region electric charge is distributed continuously. Imagine this region to be divided in a large number of volume-elements, each one with extremely small volume. If the volume of such an element having position vector $\vec{r}\,'$ is $d\tau'$ and at this position the volume-density of charge is $\rho(\vec{r}\,')$, then the charge in this element is $\rho(\vec{r}\,')\,d\tau'$, and it can be treated as a point charge. The electric potential due to this small, volume element at point P having the position vector $\vec{r}$, is

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{r}')d\tau'}{|\vec{r} - \vec{r}'|}. \quad (2.7.5)$$

By integrating this equation over the entire volume of this distribution, we get the total potential at point P, which can be written as under :

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{volume} \frac{\rho(\vec{r}')d\tau'}{|\vec{r} - \vec{r}'|} \quad (2.7.6)$$

If the charge distribution is uniform, $\rho(\vec{r}\,')$ can be taken as canstant (= ρ).

(c) A Spherical Shell with Uniform Charge Distribution :

In Chapter 1, we had seen that the electric field **at a point outside** and **at a point on the surface** of spherical shell with uniform charge distribution is equal to the electric field obtained by considering the entire charge of the shell as concentrated.at the centre of the shell. We have obtained the electric potential from the electric field ($V = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$). For electric potential also the entire charge can be considered as concentrated at the centre of the shell. Hence the potential at a point outside and at a point on the surface of the shell having charge q and radius R, is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \quad (\text{for } r \geq R) \quad (2.7.7)$$

where r = distance of the given point from centre of shell.

Moreover, we also know that the electric field inside the shell is zero. Hence during the **motion** of **unit positive charge inside the shell** no work is required to be done. Hence the potentials at **all points** inside the shell are equal having the value equal to the potential on the surface of that shell. i.e. $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$ (for $r \leq R$) (2.7.8)

(Note that here, only that work is accounted for which is done during the motion of unit positive charge from ∞ to the surface of the shell.)

2.8 Equipotential Surfaces

An equipotential surface is that surface **on which the electric potentials at all points are equal.**

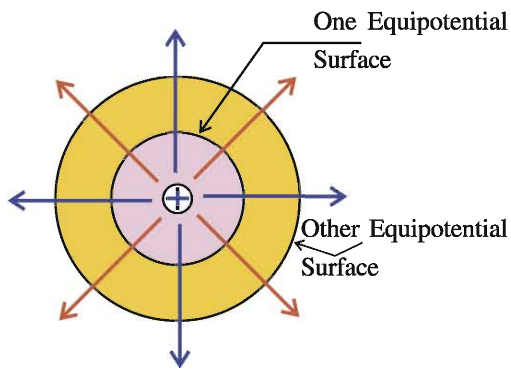


Figure 2.5 Equipotential Surfaces

The electric potential due to a point charge is given by $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$. Hence if r is constant, V also becomes constant. From this we can say that for a single point charge q, the equipotential surfaces are the surfaces of the spheres drawn by taking this charge as the centre. (See figure 2.5). The potentials on two such different surfaces are different but for all the points on the **same surface** the potentials **are equal.** The electric field produced by a point charge is along the radial directions drawn from it. [For $+q$ they are in radial directions going away from it and for $-q$ coming towards it.]. These radial lines are normal to those equipotential surfaces at every point. Hence at a given point the direction of electric field is normal to an equipotential surface passing through that point. We shall now prove that this is true not only for a point charge but in general for any charge configuration.

Suppose a unit positive charge is given a small displacement $d\vec{l}$ **on the** equipotential surface **(along this surface),** from a given point. In this process the work required to be done against the electric field (by the external force) is $dW = -\vec{E} \cdot d\vec{l}$ = potential difference between those two points.

But the potential difference on the equipotential surface = 0.

$\therefore \ \vec{E} \cdot d\vec{l} = 0 \Rightarrow E\, dl\, cos\theta = 0$, where θ = angle between $\vec{E}$ and $d\vec{l}$.

But $E \neq 0$ and $dl \neq 0 \ \therefore \ cos\theta = 0 \ \therefore \ \theta = \frac{\pi}{2} \ \therefore \ \vec{E} \perp d\vec{l}$.

But $d\vec{l}$ is along this surface. Hence the electric field $\vec{E}$ is normal to the equipotential surface at that point.

Like the field lines, the equipotential surface is also a useful concept to represent an electric field. For a uniform electric field prevailing in X-direction, the field lines are parallel to X-axis and equispaced, while the equipotential surfaces are normal to X-axis (i.e. parallel to YZ plane.) See figure 2.6.

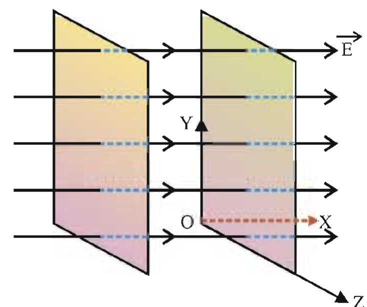


Figure 2.6 Equipotential Surface for a Uniform Electric Field

(a) Equipotential Surfaces of a Dipole (Only For Information)

(b) Equipotential Surfaces of a System of Two Positive and Equal Charge (Only for Information)

Figure 2.7

The equipotential surfaces of an electric dipole are shown in figure 2.7(a).

The equipotential surfaces of a system of two positive charges of equal magnitude are shown in figure 2.7(b).

2.9 Relation between the Electric Field and the Electric Potential

In article 2.3, we have obtained the electric potential $V = (-\int_{\infty}^{P} \vec{E} \cdot \vec{dr})$ from the electric field.

Now, if we know about the electric potential in a certain region, we can get the electric field from it as well.

We have seen in article 2.3, that from the line integral of electric field between points P and Q, we can get the potential difference between those two points. (Equation 2.3.4) as

$$V_Q - V_P = \Delta V = -\int_P^Q \vec{E}\cdot\vec{dr} \quad (2.9.1)$$

Now, if these points P and Q are very close to each other, then for such a small displacement $d\vec{l}$, integration is not required and only one term $\vec{E}\cdot d\vec{l}$ can be kept.

$$\therefore\ dV = -\vec{E}\cdot d\vec{l} \quad (2.9.2)$$

If $d\vec{l}$ is in the direction of $\vec{E}$, $\vec{E}\cdot d\vec{l} = E\ dl\ cos0° = E\ dl$

$$\therefore\ dV = -E\ dl$$

$$\therefore\ E = \frac{-dV}{dl} \quad (2.9.3)$$

This equation gives the magnitude of electric field in the direction of displacement $d\vec{l}$. Here $\frac{dV}{dl}$ = potential difference per unit distance. It is called the **potential gradient.** Its unit is $\frac{V}{m}$. From equation (2.9.3) the unit of electric field is also written as $\frac{V}{m}$, which is equivalent to $\frac{N}{C}$.

If we had taken the displacement $d\vec{l}$ **not** in the direction of electric field, but in some other direction, then $\frac{-dV}{dl}$ would give us the **component of electric field in the direction of that displacement.** e.g. If the electric field is in X-direction only and the displacement is in any direction (in three dimensions), then

$$\vec{E} = E_x\hat{i} \text{ and } d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ dV = -(E_x\hat{i})\ .\ (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= -E_x\ dx \quad (2.9.4)$$

$$\therefore\ E_x = \frac{-dV}{dx} \quad (2.9.5)$$

Similarly, if the electric field was only in Y and only in Z direction respectively, we would get,

$$E_y = \frac{-dV}{dy} \quad (2.9.6)$$

$$E_z = \frac{-dV}{dz} \quad (2.9.7)$$

Now, if the electric field also has all the three (x–, y–, z–) components then from equations (2.9.5) (2.9.6) and (2.9.7) we can write as under.

$$E_x = \frac{-\partial V}{\partial x},\ E_y = \frac{-\partial V}{\partial y},\ E_z = \frac{-\partial V}{\partial z} \quad (2.9.8)$$

$$\text{and } \vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) \quad (2.9.9)$$

Here $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$, $\frac{\partial V}{\partial z}$ show the partial differentiation of V(*x*, *y*, *z*) with respect to *x*, *y*, *z* respectively. Moreover, the partial differentiation of V(*x*, *y*, *z*) with respect to *x* means the differentiation of V with respect to **only *x*** (i.e. $\frac{\partial V}{\partial x}$) by taking *y* and *z* in the formula of V, as constants.

In equation (2.9.1), the values of $\vec{E}$ at all points between P and Q come in the calculation, while equations (2.9.3) and (2.9.8) give relation between the potential difference near a given point and the electric field at that point.

The direction of electric field is that in which the rate of decrease of electric potential with distance $\left(\frac{-dV}{dr}\right)$ is maximum and this direction is always normal to the equipotential surface.

This entire discussion is based on the property that electric field is a conservative field.

2.10 Potential Energy of a System of Point Charges

As shown in the figure 2.8, in a system of charges three point charges q_1, q_2 and q_3 are lying stationary at points A, B and C respectively. Their position vectors from the origin of a co-ordinate system are $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ respectively. We want to find the potential energy of this system.

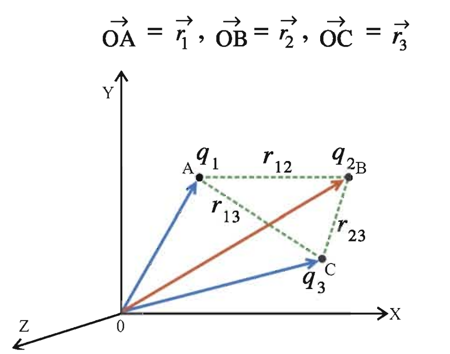


Figure 2.8 System of Point Charges

In the beginning we shall imagine that these charges are lying at infinite distances from the origin and also from each other. In this condition the electric force between them is zero, and their potential energy is also zero.

Moreover, the electric fields at A, B and C are also zero. From such a condition the work required to be done by the external forces (against the electric fields) to arrange them in the above mentioned configuration is stored in the form of potential energy of this system.

First, we bring the charge q_1 from infinite distance to point A. In this process since no electric field is present, the work done by the external force against the electric field is W_1 = zero. (You know that here the field produced by this charge itself is not to be considered.)

Now the charge set on q_1, produces an electric field and electric potential around it. The potential due to this charge q_1 at point B separated by distances r_{12} from it is (from equation 2.5.7) is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} \tag{2.10.1}$$

Where $r_{12} = |\vec{r_2} - \vec{r_1}|$

Hence the work required to be done by the external force to bring charge q_2 from infinite distance to point B, is $W_2 = q_2 V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$ (2.10.2) (from equation 2.4.2).

(If we want to consider a system of these **two charges only,** then the total work $W_1 + W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_2}{r_{12}}$ is the electric potential energy of this system.)

Now q_1 and q_2 both will produce electric fields and electric potentials around them. The electric potential produced due to them at point C is $V_C = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$ (2.10.3)

Therefore, the work required to be done to bring charge q_3 from infinite distance to point C is

$$W_3 = (V_C)q_3$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_{23}} \quad (2.10.4)$$

Hence the total work to be done to set these three charges in the above arrangement (= $W_1 + W_2 + W_3$) is the electric potential energy U of this system.

$$\therefore U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_{23}} \quad (2.10.5)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right] \quad (2.10.6)$$

$$= k\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right] \quad (2.10.7)$$

From this, in general, the potential energy of a system of n–charges can be wirtten as

$$U = \sum_{\substack{i=1 \\ i<j}}^{n} \frac{kq_iq_j}{q_{ij}} \quad (2.10.8)$$

As the electric field is conservative; it does not matter, which charge comes earlier or later. In that case the electric potential energy does not change (and given by equation 2.10.8 only)

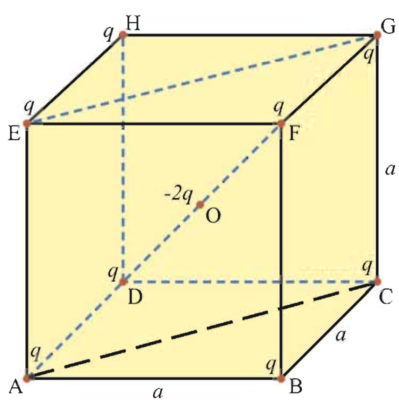


Illustration 7 : Calculate the potential energy of the system of charges, shown in the Figure.

Solution : The total potential energy of the system of charges is equal to the sum of the potential energy of all the pairs of charges.

(1) There are 12 pairs of charges like the AB pair. The distance between the electric charges in such pairs is equal to a.

The potential energy of all such pairs is

$$U_1 = \frac{kq^2}{a} \times 12 \quad (1)$$

(2) There are 12 pairs of charges like the AC pair. The distance between charges in such a pair is $a\sqrt{2}$. ($\because$ AC $= \sqrt{AB^2 + AC^2} = \sqrt{a^2 + a^2} = a\sqrt{2}$). Their potential energy is,

$$U_2 = \frac{kq^2}{a\sqrt{2}} \times 12 \quad (2)$$

(3) There are 4 pairs of charges like the AG pair. The distance between charges in these pairs is equal to $a\sqrt{3}$. ($\because$ AG = $\sqrt{AC^2 + CG^2}$ = $\sqrt{2a^2 + a^2} = a\sqrt{3}$)

Their potential energy is $U_3 = \frac{kq^2}{a\sqrt{3}} \times 4$

(4) There are eight pairs of electric charges similar to AO in which distance between charges is $\frac{a\sqrt{3}}{2}$. (AO = $\frac{AG}{2}$ = $\frac{a\sqrt{3}}{2}$)

Their potential energy is $U_4 = -\frac{kq\,2q}{\left(a\frac{\sqrt{3}}{2}\right)} \times 8$ (4)

$\therefore$ total potential energy $U = U_1 + U_2 + U_3 + U_4$

$$\therefore U = \frac{12kq^2}{a} + \frac{12kq^2}{a\sqrt{2}} + \frac{4kq^2}{a\sqrt{3}} - \frac{32kq^2}{a\sqrt{3}}$$

$$= \frac{kq^2}{a}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}} - \frac{32}{\sqrt{3}}\right] = \frac{kq^2}{a}\left[12(1 + \frac{1}{\sqrt{2}}) - \frac{28}{\sqrt{3}}\right]$$

2.11 The Potential Energy of an Electric Dipole in an External Electric Field

As shown in figure 2.9, an electric dipole AB is placed in a uniform electric field $\vec{E}$ in X–direction such that the axis of the dipole makes an angle θ with the field $\vec{E}$. Its dipole moment is $q(2a)$ in AB direction. The electric potential energy of this dipole means the algebraic sum of the electric potential energies of both of its charges ($+q$ and $-q$). We arbitrarily take the potential at the position of $-q$ charge as zero. Hence its potential energy becomes zero. Now we will find the potential enegy of $+q$ charge with respect to it and it will become the potential energy of the entire dipole.

AB = $2a$, AC = AB$cos\theta$

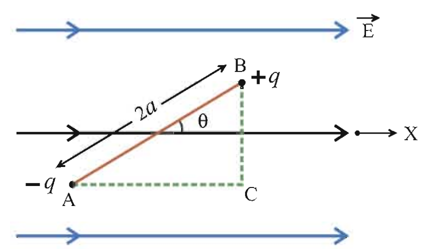


Figure 2.9 Potential Energy of Dipole

As the electric field is only in X-direction,

$$E = \frac{-\Delta V}{\Delta x} = \frac{-(V_B - V_A)}{AC}$$

$$= \frac{-V_B}{2a\cos\theta} \quad (\because V_A = 0) \qquad (2.11.1)$$

$$\therefore V_B = -E\,(2a\,\cos\theta) \qquad (2.11.2)$$

$\therefore$ Potential energy of $+q$ at B, is

$$U = qV_B = q[-E\;2a\;\cos\theta] \qquad (2.11.3)$$

$$= -E(q\;2a\;\cos\theta)$$

$$= -E\;p\;\cos\theta \;[\because q(2a) = p]$$

$$= -\vec{E}\cdot\vec{p} \qquad (2.11.4)$$

$\therefore$ The potential energy of the entire dipole $U = -\vec{E} \cdot \vec{p} = -\vec{P} \cdot \vec{p}$ (2.11.5)

We note a few points :

(i) If the axis of the dipole is normal to the electric field, then $\theta = \frac{\pi}{2}$ and

$$U = Ep\ cos\frac{\pi}{2} = 0$$

(ii) If the axis of the dipole is parallel to the field. ($\vec{AB} \parallel \vec{E}$)

Then $\theta = 0 \therefore U = -pE$. This is the **minimum value** of potential energy. Hence the dipole tries to arrange its axis parallel to the electric field, so that $\vec{p}$ becomes parallel to $\vec{E}$. In this condition dipole remains in stable equilibrium. (A system always tries to remain in such a state that its potential energy becomes minimum.) (For $\theta = \pi$, the dipole is in an unstable equilibrium.)

2.12 Electrostatics of Conductors

It is interesting to know the effects produced when metallic conductors are placed in the electric field or when electric charges are placed on such conductors.

(a) Effect of External Electric Field on Conductors :

In a metallic conductor there are positive ions situated at the lattice points and the free electrons are moving randomly between these ions. They are free to move within the metal but not free to come out of the metal. When such a conductor is placed in an external electric field $\vec{E}'$, the free electrons move under the effect of the force in the direction opposite to the field and get deposited on the surface of one end of conductor. And an equal amount of **positive charge** can be considered as deposited on the other end. Thus electric charges are **induced.** These induced charges produce an electric field $\vec{E}''$ inside the conductor, in the direction **opposite** to the external electric field $\vec{E}'$. When these two electric fields become equal in magnitude, the resultant (net) electric field ($\vec{E}$) inside the conductor becomes zero. (See figure 2.10). Now the motion of charges in the conductor stops, and the charges become steady (stationary) on the end-surfaces.

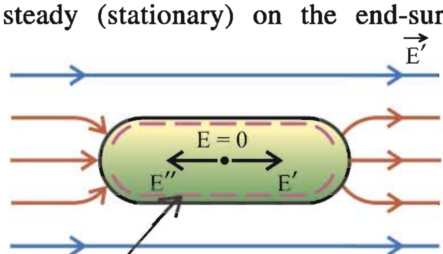


Figure 2.10 Conductor in Electric Field

Now let us consider a Gaussian Surface shown by dotted line, inside the conductor and close to the surface, as shown in figure 2.10. Every point on this surface is a point inside the conductor; the electric field $\vec{E}$ on this entire surface is zero. Hence the electric charge enclosed by it is also zero. ($\because \int \vec{E} \cdot \vec{dr} = \frac{q}{\varepsilon_0}$).

Thus in the case of a metallic conductor, placed in an external electric field,

(1) A steady electric charge distribution is induced on the surface of the conductor.

(2) The net electric field inside the conductor is zero.

(3) The net electric charge inside the conductor is zero.

(4) On the outer surface of the conductor, the electric field at every point is locally normal (perpendicular) to the surface. If the electric field were not normal (perpendicular) a component of electric field parallel to the surface would exist and due to it the charge would move on the surface. But now the motion is stoppd and the charges have become steady. Thus the component of electric field parallel to the surface would be zero, and hence the electric field would be normal to the surface.

(5) Since $\vec{E} = 0$ at every point inside the conductor, the electric potential everywhere inside the conductor is constant and equal to the value of potential on the surface.

(6) If there is a cavity inside the conductor then even when the conductor is placed in an external electric field ($\vec{E}'$), the net electric field inside the coductor is zero and also inside the cavity it is zero. Consider a Gaussian Surface around the cavity as shown in the figure 2.11. Since every point on this surface is a point inside the conductor, the electric field on this entire surface is zero.

Hence the total charge on the surface of the cavity is zero, ($\int \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$). And there is no charge inside the cavity. Hence the electric field everywhere inside the cavity is zero.)

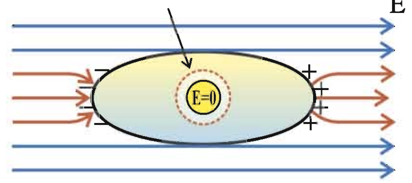


Figure 2.11
Cavity in a Conductor

This fact is called electrostatic shielding. If we are sitting in a car and suppose lightning strickes, we should close the doors of the car. (we suppose the car is fully made of metal !) By doing so, we happen to be in the cavity of car and we are protected due to electrostatic shielding.

(b) Effects Produced by Putting Charge on the Conductor :

In the above discussion we considered the effects produced when a metallic conductor is placed in an external electric field. Now we note the effects produced when a charge is placed on a metallic body, in the absence of an external electric field.

(1) Whether a metallic conductor is put in an external electric field or not and whether a charge is put or not, on it, in all such (but stable) conditions the **electric field everywhere inside** the conductor is always **zero.** This is a very important and a general fact. (This can be taken as a property to define a conductor).

(2) The charge placed on a coductor is always **distributed only on the outer surface** of the conductor. We can understand this by the fact that the electric field inside a coductor is zero. Consider a Gaussian Surface shown by the dots inside the surface and very close to it, (figure 2.12). Every point on it is inside the surface and not on the surface of conductor Hence the electric field at every point on this surface is zero. Hence according to Gauss's theorem the charge enclosed by that surface is also zero.

(3) In a stable condition these charges are steady on the surface. This shows that the electric field is locally normal to the surface. (See figure 2.12).

(4) The electric field at any point on the charged conductor is $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, where $\hat{n}$ = unit vector coming out from the surface normally. To prove this, we consider a Gaussian surface of a pill-box (a cylinder) of extremely small length and extremely small cross-section ds. A fraction of it is inside the surface and the remaining part is outside the surface. The total charge enclosed by this pill-box is $q = \sigma ds$; where σ = surface density of charge on the conductor. At every point on the surface of the conductor $\vec{E}$ is perpendicular to the local surface element. Hence it is parallel to surface vector ($\vec{E} \parallel \vec{ds}$).

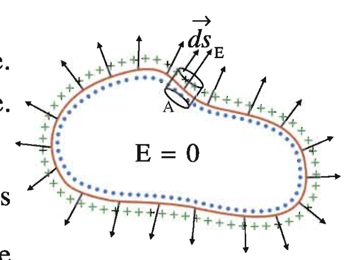


Figure 2.12

But inside the surface $\vec{E} = 0$. Hence the flux coming out from the cross-section of pill-box inside the surface = 0. For its side the area vector (surface vector) is normal to $\vec{E}$. Hence flux through it is zero. The flux coming out from the cross-section of pill-box outside the surface is $\vec{E} \cdot \vec{ds} = E ds$.

$\therefore$ Total flux = E ds

According to Gauss's theorem, $E\, ds = \frac{q}{\varepsilon_0} = \frac{\sigma\, ds}{\varepsilon_0}$ (2.12.1)

$\therefore\ E = \frac{\sigma}{\varepsilon_0}$ (2.12.2)

In the vector from $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ (2.12.3)

If σ is positive, $\vec{E}$ is in the direction of normal coming out from the surface. If σ is negative $\vec{E}$ is in the direction of normal entering into the surface.

(5) If some charge is placed inside a cavity in the conductor, then the charges are so induced on the surface of the cavity and on the outer surface of conductor that the electric field in the region which is inside the conductor but outside the cavity becomes zero. The electric field inside the cavity is non-zero and the electric field outside the conductor due to that charge is also non-zero.

[**Note (For information only) :** In the above discussion we have considered the coductors to be insulated.

The sharp ends of the conductor have a large electric charge density. The electric field near such a region is very strong. This strong electric field can strip the electrons from the surface of the metal. This event is known as Corona discharge. In general, this event is called dielectric breakdown.

The electrons escaping the surface of a metal perform an accelerated motion, colliding with the air particles coming in their way. The excited atoms of the energetic particles emit electromagnetic waves and a greenish glow is observed. Apart from the above process, the ionization of the air molecules also takes palce, during collision

Sailors long ago saw these glows at the pointed tops of their masts and spars and dubbed the phenomenon St. Elmo's fire.]

2.13 Capacitors and Capacitance

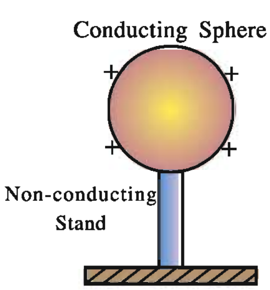


Figure 2.13

Consider an insulated conducting sphere as shown in the figure 2.13. Suppose we go on gradually adding positive charge on this sphere. As the charge on the sphere is gradually increased, the potential (V) on the surface of the sphere and the electric field around the sphere also go on gradually increasing. In this process at some one stage the electric field becomes sufficiently strong to ionize the air particles around the sphere. Hence the charge on the sphere is conducted through air and insulating property of air gets distroyed (i.e. it is not sustained.). This effect is called dielectric breckdown.

Thus the charge on the sphere is leaked and now the sphere is not able to store any additional charge. During this entire process the ratio of the charge (Q) on the sphere and the potential (V) on the sphere remains constant. This ratio is called the capacitance of the sphere. [$C = \frac{Q}{V}$]

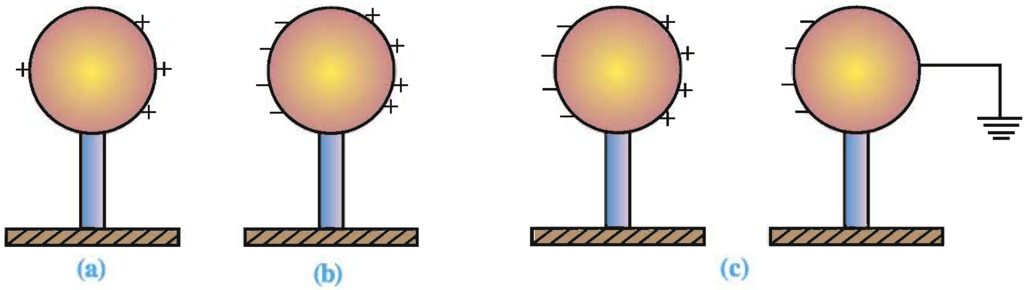


Figure 2.14

The maximum electric field upto which an insulating (non-conducting) medium can maintain its insulating property is called the **dielectric strength** of that medium (or the minimum electric field which starts ionization in a given non-conducting-medium is called its **dielectric strength.**). For air the dielectric strength is nearly 3000 $\frac{V}{mm}$.

Now, if we want to increase the capacity of the above mentioned sphere to store charge (capacitance C), then place another, insulated conducting sphere near the first one. So, electric charge is induced in this second sphere. See figure 2.14(b). If the second sphere is connected to Earth, as in figure 2.14(c) electrons from Earth will flow to it and neutralize the positive charge in it. Now due to negative charge on the second sphere the potential on the surface of the first sphere and the electric field near it are decreased. Now the capacity to store charge on the first sphere increases, as compared to earlier. In this condition also the ratio of the electric charge Q and the p.d. (V) between two spheres at every stage is found to be constant. This ratio is called the capacitance C of this system of two spheres. The value of this capacitance depends on the dimensions of the spheres, their relative arrangement and the medium between them.

"A device formed by two conductors insulated from each other is called a capacitor." These conductors are called the plates of the capacitor. The conductor with positive charge is called the positive plate and the one with negative charge is called the negative plate. The charge on the positive plate is called the charge on the capacitor and the potential difference between the two conductors is called the potential difference (V) between the two plates of the capacitor. Here the capacitance of the capacitor is $C = \frac{Q}{V}$.

The SI unit of capacitance is coulomb / volt and in memory of the great sceintist Michael Faraday it is known as Farad. Its symbol is F. Farad is a large unit for practical purposes and hence smaller units microfarad ($1\ \mu F = 10^{-6} F$) nanofarad ($1\ nF = 10^{-9} F$) and picofarad ($1\ pF = 10^{-12} F$) are used in practice.

A capacitor having a definite capacitance is shown by the symbol ⊣⊢ and the one having a variable capacitance is shown by the symbol ⊣⊢ (with arrow).

Moreover, a single conducting sphere of radius R and having charge Q can also be considered as a capacitor, because it also has 'some' capacity to store charge. For such a capacitor other conductor (with $-Q$ charge) is considered to be at infinite distance (separation). Taking the potential at infinite distance from the sphere as zero, the potential on the surface of this sphere is $V = \frac{kQ}{R}$. Hence the potential difference between this sphere and the other one imagined at infinite distance is also $V = \frac{kQ}{R}$.

∴ The capacitance of this sphere is $C = \frac{Q}{V} = \frac{QR}{kQ} = \frac{R}{k} = 4\pi\varepsilon_0 R$ ($\because$ $K = \frac{1}{4\pi\varepsilon_0}$). Earth can also be considered as a capacitor. You may calculate its capacitance.

2.14 Parallel Plate Capacitor

In such a capacitor, two conducting parallel plates of equal area (A) are insulated from each other and kept at a separation of (*d*). (See figure 2.15)

Considering vacuum (or air) as the non-conducting medium between them, we shall obtain the formula for its capacitance.

Suppose, the electric charge on this capacitor is Q. Therefore, the value of the surface density of charge on its plates is $\sigma = \frac{Q}{A}$. The value of d is kept very small as compared to the dimension of each plate. Due to this, the non-uniformity of the electric field near the ends of the plates can be neglected and in the entire region between the plates the electric field $\vec{E}$ can be taken as constant.

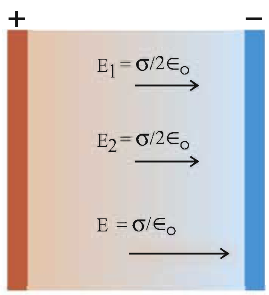


Figure 2.15 Parallel Plate Capacitor

The uniform electric field in the region between two plates due to the positive plate is $E_1 = \frac{\sigma}{2\varepsilon_0}$ in the direction from positive to negative plate. (2.14.1)

Similarly the uniform electric field in the same region due to the negative plate, is $E_2 = \frac{\sigma}{2\varepsilon_0}$ (2.14.2)

(Also in the direction from positive to negative plate.)

Since these two fields are in the same direction, the resultant uniform electric field is

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \qquad (2.14.3)$$

It is in the direction from positive to negative plate.

$$\therefore\ E = \frac{Q}{\varepsilon_0 A} \qquad (2.14.4)$$

In the regions on the other sides of the plates, E_1 and E_2 being equal but in opposite direction, the resultant electric field becomes zero.

If the potential difference between these two plates is V, then $V = Ed$ (2.14.5)

∴ From equations (2.14.4) and (2.14.5),

$$V = \frac{Q}{\varepsilon_0 A} d \qquad (2.14.6)$$

∴ From the formula $C = \frac{Q}{V}$, we get the capacitance of parallel plate capacitor as

$$C = \frac{\varepsilon_0 A}{d} \qquad (2.14.7)$$

From equation (2.4.7), it is clear that if the distance between two plates each of 1 m × 1 m is 1 mm, its capacitance is $C = \frac{(8.85\times10^{-12})(1)}{10^{-3}} = 8.85 \times 10^{-9}$F.

If we want 1F capacitance, then the area of each plate kept at a separation of 1 mm should be $A = \frac{Cd}{\varepsilon_0} = \frac{(1\times10^{-3})}{8.85\times10^{-12}} = 1.13 \times 10^8\ m^2$. Thus each of the length and the breadth of each plate should be nearly 1×10^4 m = 10 km.

2.15 Combinations of Capacitors

The system, formed by the combination of capacitors having capacitances C_1, C_2, ..., C_n has some equivalent (effective) capacitance C. We shall discuss two types of combinations.

(a) Series Combination of Capacitors

The arrangement formed by joining the capacitors having capacitances C_1, C_2, C_3, ..., C_n by conducting wires as shown in figure 2.16 is called the series combination of capacitors.

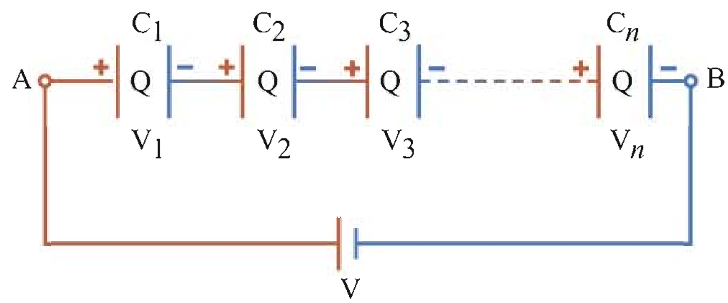


Figure 2.16 Series Combination of Capacitors

In such a condition the charge on every capacitor has the same value Q. As (−Q) charge is deposited by the battery on one plate, it induces (+Q) charge on the other plate. For this (−Q) charge from the second plate will be deposited on the near plate of the next capacitor. This induces +Q charge on the other plate. This continues further. Thus all capacitors have equal charge. but the potential difference between the two plates of different capacitors is different. From the figrue it is clear that

$$V = V_1 + V_2 + V_3 + ... + V_n \tag{2.15.1}$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ... + \frac{Q}{C_n} \tag{2.15.2}$$

$(\because C_1 = \frac{Q}{V_1}, \text{ ...... etc.})$

$$\therefore \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \tag{2.15.3}$$

If the effective capacitance of this combination is C,

$$\frac{V}{Q} = \frac{1}{C} \tag{2.15.4}$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \tag{2.15.5}$$

Thus the value of effective capacitance is even smaller than the smallest value of capacitance in the combination.

[Note that here the formula obtained for series combination is similar to the formula for effective (equivalent) resistance obtained for the parallel combination of the resistances.]

(b) Parallel Combination of Capacitors

The arrangement formed by joining the capacitors having capacitances C_1, C_2, C_3 by conducting wires as shown in figure 2.17 is called the parallel combination of capacitors.

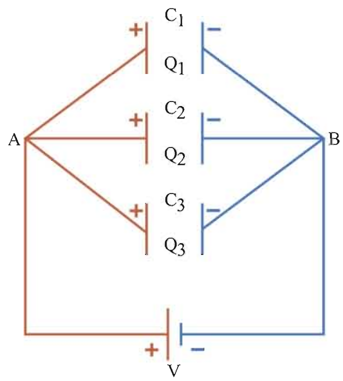


Figure 2.17 Parallel Combination of Capacitors

In such a combination the potential difference (V) between the plates of every capacitor is the same and is equal to the potential difference between their common points A and B. But the charge Q on every capacitor is different.

Here,
$$\left.\begin{aligned} Q_1 &= C_1 V \\ Q_2 &= C_2 V \\ Q_3 &= C_3 V \end{aligned}\right\} \quad (2.15.6)$$

And the total electric charge

$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$
$$= (C_1 + C_2 + C_3)V \quad (2.15.7)$$

If the effective capacitance of this parallel combination is C, then

$$C = \frac{Q}{V} = C_1 + C_2 + C_3 \quad (2.15.8)$$

If such n-capacitors are joined in parallel connection, the effective capacitance is

$$C = C_1 + C_2 + C_3 + ... + C_n \quad (2.15.9)$$

Here, as the values of capacitances are added the value of effective capacitance is even greater than the largest value of capacitance in the connection.

[Note that the formula obtained here for parallel combination is similar to the formula for effective (equivalent) resistance obtained for the series combination of resistances.]

Illustration 8 : Prove that the force acting on one plate due to the other in a parallel plate capacitor is $F = \frac{1}{2}\frac{CV^2}{d}$.

Solution : The electric field due to one plate is $E_1 = \frac{\sigma}{2\varepsilon_0}$ (1)

A second plate having charge σA is present in the above electric field.

∴ The force acting on the second plate is

$F = (\sigma A)E_1$

Substituting the value of E_1 from (1), we have,

$$F = \frac{\sigma^2 A}{2\varepsilon_0}$$

But $\sigma = \frac{Q}{A}$

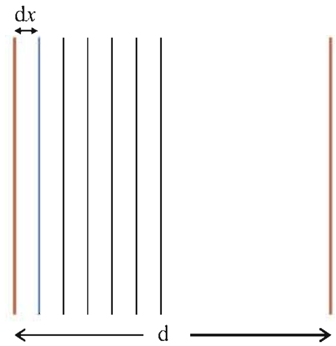
$$\therefore F = \frac{\frac{Q^2}{A^2}.A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{Q^2/d}{2\varepsilon_0 A/d} = \frac{Q^2}{2dC} \quad \left(\because \frac{\varepsilon_0 A}{d} = C\right)$$

$$\therefore F = \frac{1}{2}\frac{CV^2}{d} \quad (\because Q = CV)$$

Illustration 9 : Figure shows an infinite number of conducting plates of infinitesimal thickness such that consecutive plates are sparated by a small distance dx spread over a distance d to form a capacitor. Calculate the value of the capacitance of such an arrangement.

Solution : The capacitance of each of the capacitors in the above arrangement, $dC = \frac{\varepsilon_0 A}{dx}$

All these capacitors are in series combination with each other.
Therefore the total capacitance C is obtained from



$$\frac{1}{C} = \frac{1}{dC} + \frac{1}{dC} + \ldots$$

$$= \frac{dx}{\varepsilon_0 A} + \frac{dx}{\varepsilon_0 A} + \ldots$$

$$= \frac{1}{\varepsilon_0 A}(dx + dx + \ldots\ldots + dx)$$

$$\therefore \frac{1}{C} = \frac{d}{\varepsilon_0 A}$$

$$\therefore C = \frac{\varepsilon_0 A}{d}$$

This is equivalent to the capacitance of the capacitor formed by the first and the last plate of the above arrangement.

2.16 Energy Stored in a Charged Capacitor

In order to establish a charge on the capacitor, work has to be done on the charge. This work is stored in the form of the potential energy of the charge. Such a potential energy is called the energy of capacitor.

Suppose the charge on a parallel plate capacitor is Q. In this condition each plate of the capacitor is said to be lying in the electric field of the other plate.

The magnitude of the uniform electric field produced by one plate of capacitor is

$$= \frac{\sigma}{2\varepsilon_0} \ldots. \tag{2.16.1}$$

where $\sigma = \frac{Q}{A}$ and A = area of each plate.

Hence by taking arbitrarily the potential on this plate as zero, that of the other plate at distance d from it will be $= \left(\frac{\sigma}{2\varepsilon_0}\right)d$ (2.16.2)

From this, the potential energy of the first plate is zero and that of the second plate will be = (potential) (charge Q on it)

$$= \left(\frac{\sigma d}{2\varepsilon_0}\right)Q \tag{2.16.3}$$

$\therefore$ Energy stored in the capacitor

$$U_E = \frac{\sigma dQ}{2\varepsilon_0} = \left(\frac{Q}{A}\right)\frac{dQ}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A/d} \tag{2.16.4}$$

$$= \frac{Q^2}{2C} \tag{2.16.5}$$

where, $C = \frac{\varepsilon_0 A}{d}$ = capacitance of capacitor.

Moreover, $C = \frac{Q}{V}$. From equation (2.16.5) and this formula we can write

$$U_E = \frac{VQ}{2} \tag{2.16.6}$$

and $U_E = \frac{1}{2}CV^2$ (2.16.7)

We have derived these equations (2.16.5), (2.16.6) and (2.16.7) for the parallel plate capacitor, but in general they are true for all types of capacitor.

To show energy stored in the capacitor in the form of energy density :

The energy stored in the capacitor is $U_E = \frac{1}{2}CV^2$. This energy is stored in the region between the two plates, that is, in the volume Ad, where A = area of each plate and d = separation between them. Hence, if we write the energy stored **per unit volume** in the region between the plates – that is energy density – as ρ_E, then

$$\rho_E = \frac{U_E}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} \quad (2.16.8)$$

$$= \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\frac{V^2}{Ad} \quad (2.16.9)$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)\left(\frac{V}{d}\right) \quad (2.16.10)$$

$$= \frac{1}{2}\varepsilon_0 E^2 \qquad (\because \frac{V}{d} = E) \quad (2.16.11)$$

Where $\frac{V}{d}$ = E = electric field between the two plates. Thus the energy stored in the capacitor can be considered as the energy stored in the electric field between its plates.

We have obtained this equation for a parallel plate capacitor but it is a result in general and can be used for the electric field of any arbitrary charge distribution.

Illustration 10 : A capacitor of 4 μF value is charged to 50 V. The above capacitor is then connected in parallel to a 2 μF capacitor. Calculate the total energy of the above system. The second capacitor is not charged prior to its connection with the 4 μF capacitor.

Solution : The energy stored in the capacitor of 4 μF will be

$$W_1 = \frac{1}{2}C_1V^2$$

$$= \frac{1}{2} \times 4 \times (50)^2 = 2 \times 2500 = 5000 \ \mu J$$

The two capacitors are connected in parallel. Let q_1 and q_2 be the electrical charges on capacitors C_1 and C_2 respectively after connection. If V' is their common potential difference across the capacitors. $(V' = \frac{q_1}{C_1} = \frac{q_2}{C_2})$

$$\frac{q_1}{q_2} = \frac{C_1}{C_2}$$

$$\therefore \frac{q_1+q_2}{q_2} = \frac{C_1+C_2}{C_2} \quad (1)$$

By the law of conservation of charge.

$$q_1 + q_2 = Q \quad (2)$$

Where Q is the initial charge

Now, $Q = C_1V = (4)(50)$

$= 200 \ \mu C$

Putting equation (2) in equation (1) and substituting the value of Q, we have,

$$\frac{200}{q_2} = \frac{(4+2)}{2}$$

$$\therefore\ q_2 = \frac{200 \times 2}{6} = \frac{200}{3}\ \mu C$$

From Equation (2)

$$q_1 = 200 - \frac{200}{3}$$

$$= \frac{400}{3}\ \mu C$$

Calculation of energy : The energy of the first capacitor

$$\frac{q_1^2}{2C_1} = \left(\frac{400}{3}\right)^2 \times \frac{1}{2 \times 4} = 2222\ \mu J$$

The energy of the second capacitor

$$\frac{q_2^2}{2C_2} = \left(\frac{200}{3}\right)^2 \times \frac{1}{2 \times 2} = 1111\ \mu J$$

The total energy of the system, after combination = 2222 + 1111 = 3333 = μJ

Thus the energy decreased by 5000 − 3333 = 1667 μJ. This energy is dissipated in the from of heat.

2.17 Dielectric Substances and their Polarisation

Non-conducting materials are called **dielectric**. Faraday found that when a dielectric is introduced between the plates of a capacitor, the capacitance of the capacitor is increased. In order to understand how does this happen, we should know about the effects produced when a dielectric is placed in an electric field. Dielectric materials are of two types (1) polar and (2) non-polar.

A dielectric is called a polar dielectric if its molecules possess a permanent dipole moment (e.g. HCl, H_2O, ... etc.) If the molecules of the dielectric do not possess a permanent dipole moment, then that dielectric is called a non-polar dielectric (e.g. H_2, O_2, CO_2, ... etc.)

(a) Non-polar Molecule : In a non-polar molecule, the centre of the positive charge and the centre of the negative charge coincide with each other. Hence they do not possess a permanent dipole moment. Now, when it is placed in a uniform electric field ($\vec{E}_0$), these centres are displaced in mutually opposite directions. Hence they now, possess a dipole moment $p = qd$, where d = the distance between centres of positive and negative charges after being displaced, q = the value of positive or negative charge (See figure 2.18).

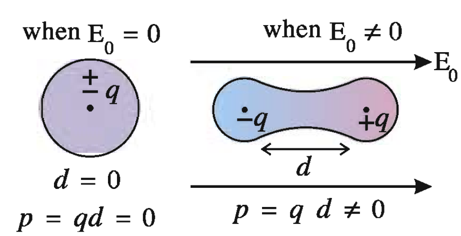


Figure 2.18 Polarisation of a Non-polar Molecule

Thus an electric dipole is induced in it. In other words due to an external electric field a dielectric made of such molecules is said to be polarised. If the external electric field ($\vec{E}_0$) is not very strong, it is found that this dipole moment of molecule is proportional to $\vec{E}_0$.

$$\therefore\ \vec{p} = \alpha \vec{E}_0 \qquad (2.17.1)$$

where α is called the polarisability of the molecule.

From units of $\vec{p}$ and $\vec{E}_0$, the unit of α is $C^2\ m\ N^{-1}$

(b) Polar Molecule : A polar molecule possesses a permanent dipole moment $\vec{p}$, but such dipole moments of different molecules of the substance are randomly oriented in all possible directions and hence the resultant dipole moment of the substance becomes zero.

Now, on applying an external electric field a torque acts on every molecular dipole. Therefore, it rotates and tries to become parallel to the electric field. Thus a resultant dipole moment is produced. In this way the dielectric made up of such molecules is said to be polarised. Moreover, due to thermal oscillations the dipole moment also gets deviated from being parallel to electric field. If the temperature is T, the dipoles will be arranged in such an equilibrium condition that the average thermal energy per molecule ($\frac{3}{2}k_B T$) balances the potential energy of dipole ($U = -\vec{p} \cdot \vec{E}_0$) in the electric field. At 0 K temperature since the thermal energy is zero, the dipoles become parallel to the electric field. We shall only discuss such an ideal situation.

(c) When there is air (or vacuum) between the charged plates of a capacitor, the electric field between the plates is $E_0 = \frac{\sigma_f}{\varepsilon_0}$. (2.17.2)

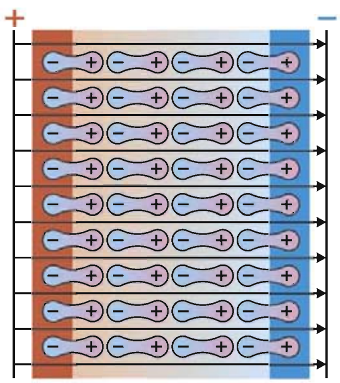


Figure 2.19 Polarisation in Dielectric

where σ_f = value of surface charge density on each plate.

The charge on these plates is called the free charge, because its value can be adjusted at our will (by joining proper battery).

Here, the area of each plate is = A. Now on placing a slab of dielectric material (polar or non-polar) in the region between the plates, the polarisation produced by the electric field $\vec{E}_0$ is shown in the figure 2.19. We want to find the electric field inside the dielectric.

It is clear from the Figure that the opposite charges in the successive dipoles inside the slab cancel the effect of each other, as they are very close to each other and a net (resultant) charge resides only on the faces of the slab, close to the plates. These charges are called **induced charges** or the **bound charges** or the **polarisation charges.** The charge induced on the surface of the slab close to the positive plate is $-\sigma_b A$ and that on the surface close to the negative plate is $+\sigma_b A$, where $-\sigma_b$ and $+\sigma_b$ are, the surface densities of the **bound charges** on **the** respective **surfaces.** This induced charges form a dipole. Its dipole moment is $P_{total} = (\sigma_b A)d$ (2.17.3)

where, d = thickness of the slab = distance between two plates. (if sides of slab touch the plates)

Here, Ad = volume of slab = V (2.17.4)

The dipole moment produced per unit volume is called the **intensity of polarisation** or in short **polarisation (P).**

$$\therefore P = \frac{P_{total}}{\text{volume}} = \frac{(\sigma_b A)d}{Ad} = \sigma_b \quad (2.17.5)$$

Thus the magnitude of polarisation(P) in a dielectric is equal to the **surface density** of **bound charges (σ_b),** induced on its surface. The electric field produced by these induced charges is in the direction opposite to the external electric field $\vec{E}_0$. Hence, now the resultant electric field E inside the dielectric can be considered as produced due to ($\sigma_f - \sigma_b$).

$$\therefore\ E = \frac{\sigma_f - \sigma_b}{\varepsilon_0} \qquad (2.17.6)$$

Thus the net electric field inside the dielectric is less than the applied (external) electric field. (But recall that net electric field in the conductor was zero.)

Moreover, it is found that if the external electric field (E_0) is not very strong, then the polarisation (P), is proportional to the net electric field (E) inside the dielectric.

i.e. $P \propto E$

$$\therefore\ P = \varepsilon_0 x_e E \qquad (2.17.7)$$

where x_e = constant, which is called the electric susceptibility of the dielectric medium. It depends on the nature of dielectric and the temperature. The dielectric obeying $P \propto E$ is called a linear dielectric.

From equation (2.17.7) $x_e = \dfrac{P}{\varepsilon_0 E}$ (2.17.8)

Using $E_0 = \dfrac{\sigma_f}{\varepsilon_0}$ and $P = \sigma_b$ in equation (2.17.6), we get,

$$E = \frac{\varepsilon_0 E_0 - P}{\varepsilon_0} \qquad (2.17.9)$$

$\therefore\ \varepsilon_0 E = \varepsilon_0 E_0 - \varepsilon_0 x_e E$ ($\because$ From equation 2.17.8 $P = \varepsilon_0 x_e E$) (2.17.10)

$$\therefore\ \varepsilon_0 E + \varepsilon_0 x_e E = \varepsilon_0 E_0 \qquad (2.17.11)$$

$$E\varepsilon_0(1 + x_e) = E_0\varepsilon_0 \qquad (2.17.12)$$

$\varepsilon_0(1 + x_e)$ is called the permittivity(ε) of the dielectric medium; i.e. $\varepsilon = \varepsilon_0(1 + x_e)$ (2.17.13)

$$\therefore\ E\varepsilon = E_0\varepsilon_0 \qquad (2.17.14)$$

$$\therefore\ E = \frac{E_0}{\varepsilon / \varepsilon_0} \qquad (2.17.15)$$

Here, $\dfrac{\varepsilon}{\varepsilon_0}$ is called the relative permittivity ε_r of the medium and is also called the dielectric constant of the medium K. Value of K is always greater than 1.

$$\text{Thus, } \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K \qquad (2.17.16)$$

From equations (2.17.13) and (2.17.16), $\dfrac{\varepsilon_0(1+x_e)}{\varepsilon_0} = K$

$$\therefore\ K = 1 + x_e \qquad (2.17.17)$$

This equation shows the relation between two electrical properties x_e and K of the dielectric.

Now, equation (2.17.15) can be written as

$$E = \frac{E_0}{K} \qquad (2.17.18)$$

Thus if the electric field in certain region in the free space is E_0, when a dielecrtic is placed in that region, the electric field in the dielectric becomes K^{th} part (i.e., $\frac{1}{K}$ times), the value in free space.

Electric Displacement : When a dielectric is placed between the plates of a capacitor, the net electric field produced in the dielectric is given by $E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$, where σ_f = value of surface density of free charges, σ_b = Value of surface density of bound charges.

Since $\sigma_b = P$, we get $E = \frac{\sigma_f - P}{\varepsilon_0}$ (2.17.19)

$\therefore \varepsilon_0 E + P = \sigma_f$ (2.17.20)

The directions of $\vec{E}$ and $\vec{p}$ are the same. $\varepsilon_0 \vec{E} + \vec{P}$ is called electric displacement $\vec{D}$.

$\therefore \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ (2.17.20)

It is a vector field. Using the definition of $\vec{D}$, many equations regarding electric field become simpler in form. Gauss' theorem in the presence of a dielectric is written as

$\oint \vec{D} \cdot d\vec{s} = q$ (2.17.22)

where q is only the free charge (it does not include the bound charge). Thus in case of dielectric the field related to the free charges is not $\vec{E}$, but it is $\vec{D}$, that is $\varepsilon_0 \vec{E} + \vec{P}$.

2.18 Capacitor with a Dielectric

When there is air (or vacuum) between the plates of the parallel plate capacitor, its capacitance is given by $C = \frac{\varepsilon_0 A}{d}$. (2.18.1)

where ε_0 = permittivity of vacuum, A = area of each plate and d = separation between two plates. Now if the entire region between these plates is filled with a dielectric medium having permittivity ε, then to obtain the formula for its capacitance C', ε should be placed in place of ε_0 in the above formula.

$\therefore C' = \frac{\varepsilon A}{d}$ (2.18.2)

$\therefore \frac{C'}{C} = \frac{\varepsilon}{\varepsilon_0} = K$ (2.18.3)

where K = dielectric constant of that medium.

$\therefore C' = KC$ (2.18.4)

Thus, putting a medium of dielectric constant K between the plates of the capacitor, its capacitance becomes K times and thus its capacity to store electric charge also becomes K times.

Illustration 11 : A capacitor consists of three parallel plates of equal area A. The distance between them is d_1 and d_2. Dielectric material having pemittivity ε_1 and ε_2 is present between the plates. (i) Calculate the capacitance of such a system. (ii) Express this capacitance in terms of K_1 and K_2.

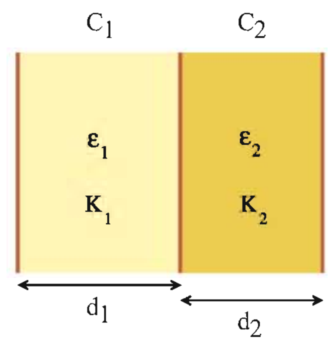
Solution : As shown in the figure, the two capacitors are connected in series. If C is the total capacitance, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{But } C_1 = \frac{\varepsilon_1 A}{d_1} \text{ and } C_2 = \frac{\varepsilon_2 A}{d_2}$$

$$\therefore \frac{1}{C} = \frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A}$$

$$= \frac{d_1\varepsilon_2 A + \varepsilon_1 A d_2}{\varepsilon_1\varepsilon_2 A^2} = \frac{d_1\varepsilon_2 + d_2\varepsilon_1}{\varepsilon_1\varepsilon_2 A}$$

$$\therefore C = \frac{\varepsilon_1\varepsilon_2 A}{d_1\varepsilon_2 + d_2\varepsilon_1} \text{ or } C = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}}$$



From $K_1 = \frac{\varepsilon_1}{\varepsilon_0}$, we get $\varepsilon_1 = \varepsilon_0 K_1$. Similarly $\varepsilon_2 = \varepsilon_0 K_2$, where ε_0 = permittivity of vacuum.

$$\therefore C = \frac{A}{\frac{d_1}{\varepsilon_0 K_1} + \frac{d_2}{\varepsilon_0 K_2}} = \frac{A\varepsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

2.19 Van-De-Graaff Generator

With the help of this machine, a potential difference of a few million (1 million = 10^6 = ten lac) volt can be established. By suitably passing a charged particle through such a high potential difference it is accelerated (to very high velocity) and hence acquires a very high energy ($\frac{1}{2}mv^2$). Because of such a high energy they are able to penetrate deeper into the matter. Therefore, fine structure of the matter can be studied with the help of them. The principle of this machine is as under.

Suppose there is a positive charge Q, on an insulated conducting spherical shell of radius R, as shown in the figure 2.20. At the centre of this shell, there is a conducting sphere of radius $r (r < R)$, having a charge q.

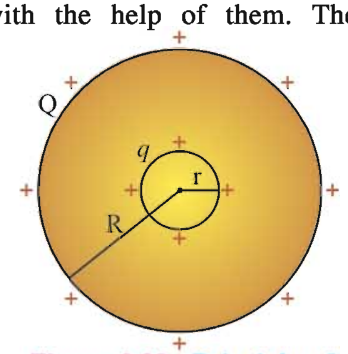


Figure 2.20 Principle of Van–de–Graaff Generator

Here the electric potential on the shell of radius R, is

$$V_R = \frac{kQ}{R} + \frac{kq}{R} \qquad (2.19.1)$$

and the potential on the surface of the sphere of radius r, is

$$V_r = \frac{kQ}{R} + \frac{kq}{r} \qquad (2.19.2)$$

It is clear from these two equations that the potential on the smaller sphere is more and the potential difference (p.d.) between them is

$$V_r - V_R = \left(\frac{kQ}{R} + \frac{kq}{r}\right) - \left(\frac{kQ}{R} + \frac{kq}{R}\right)$$

$$= kq\left[\frac{1}{r} - \frac{1}{R}\right] \qquad (2.19.3)$$

Hence, if the smaller sphere is brought in electrical contact with the bigger sphere then the charge goes from smaller on to the bigger sphere. Thus charge can be accumulated to a very large amount on the bigger sphere and thereby its potential can be largely increased.

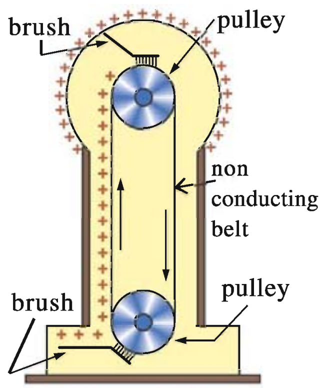


Figure 2.21
Van-de-Graaff Generator

The machine based on this principle made by Van-De-Graaff, is called the Van-De-Graaff generator.

As shown in the figure 2.21 a spherical shell of a few meter radius, is kept on an insulated support, at a height of a few meters from the ground.

A pully is kept at the centre of the big sphere and another pully is kept on the ground. An arrangement is made such that a non-conducting belt moves across two pulleys. Positive charges are obtained from a discharge tube and are continuously sprayed on the belt using a metallic brush (with sharp edges) near the lower pulley. This positive charge goes with the belt towards the upper pulley.

There it is removed from the belt with the help of another brush and is deposited on the shell (because the potential on the shell is less than that of the belt on the pulley.) Thus a large potential difference (nearly 6 to 8 million volt) is obtained on the big spherical shell.

Illustration 12 : Q amount of electric charge is residing on a conducting sphere having radius equal to R_1. This sphere is connected to another conducting sphere of radius R_2 by a conducting wire. Calculate the electric charge on each of the spheres. The two spheres are separated by a large distance.

Solution : Let q_1 and q_2 be the electric charge present on the two conducting spheres after being connected with each other.

$$\therefore Q = q_1 + q_2 \qquad (1)$$

The electric potentials of the two spheres have to be the same, since they are connected by a conducting wire

$$\therefore \frac{kq_1}{R_1} = \frac{kq_2}{R_2} \quad \therefore \frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad (2)$$

$$\therefore \frac{q_2 + q_1}{q_2} = \frac{R_1 + R_2}{R_2}$$

$$\therefore \frac{Q}{q_2} = \frac{R_1 + R_2}{R_2}$$

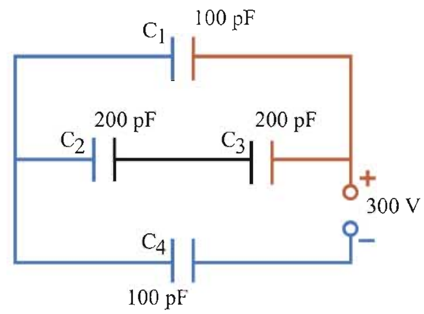
$$\therefore q_2 = \frac{R_2}{R_1 + R_2} Q \qquad (3)$$

Substituting the value of q_2 in the equation (1), we have

$$Q = q_1 + \frac{R_2}{R_1 + R_2} Q$$

This gives $q_1 = \frac{R_1}{R_1 + R_2} Q$.

Illustration 13 : Find the effective capacitance of the network shown in the figure and find the charge on each capacitor.



Solution : Here, C_2 and C_3 are in series. Their equivalent (effective) capacitance is $C' = \frac{C_2C_3}{C_2+C_3} = \frac{200\times200}{200+200}$ = 100*p*F.

This C' and C_1 are in parallel connection. Their equivalent capacitance is C'' = C' + C_1 = 100 + 100 = 200 *p*F.

This C'' and C_4 are in series. Their Equivalent capacitance is $C''' = \frac{C''C_4}{C''+C_4} = \frac{200\times100}{200+100}$ $= \frac{200}{3}pF$.

Now the charge supplied by the battery is $Q = C'''V = \left(\frac{200\times10^{-12}}{3}\right)(300) = 2 \times 10^{-8}C$.

∴ charge on C_4 and C'' are equal and each is equal to 2×10^{-8}C.

∴ Charge on C_4 is $Q_4 = 2 \times 10^{-8}C = Q''$ (on C'')

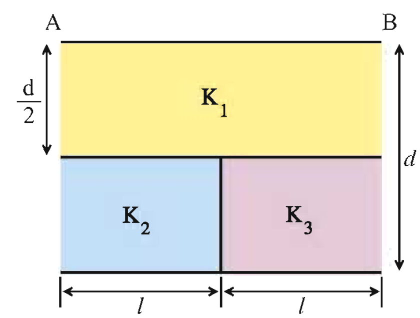
Charge on C'' is divided on C' and C_1. Since C' and C_1 are of equal values, that charge is divided equally on them.

∴ Charge on C_1 is $Q_1 = \frac{1}{2}Q_4 = 1 \times 10^{-8}C = Q'$... (on C')

The charge on C' has the same value as on C_2 and C_3.

∴ $Q_2 = Q_3 = 1 \times 10^{-8}$C.

Illustration 14 : Find the capacitance of the capacitor shown in the figure. Area of AB is A. K_1, K_2, K_3 are dielectric constants of respective materials.



Solution : We shall use the formula $C = \frac{\varepsilon A}{d} = \frac{K\varepsilon_0 A}{d}$ and also use the formulae for series and parallel connections. Here, the capacitors formed by K_2 and K_3 are in parallel and hence their equivalent capacitance is

$$C_{23} = C_2 + C_3 = \frac{K_2\varepsilon_0(A/2)}{(d/2)} + \frac{K_3\varepsilon_0(A/2)}{(d/2)}$$

$$= \frac{\varepsilon_0 A}{d}(K_2 + K_3)$$

The capacitor formed by k_1 is in series connection with this C_{23}. ∴ The equivalent capacitance of the entire system is

$$C = \frac{C_1C_{23}}{C_1+C_{23}} = \frac{\left(\frac{K_1\varepsilon_0 A}{d/2}\right)\left[\frac{\varepsilon_0 A}{d}(K_2+K_3)\right]}{\left(\frac{K_1\varepsilon_0 A}{d/2}\right)+\left[\frac{\varepsilon_0 A}{d}(K_2+K_3)\right]} = \frac{2\varepsilon_0 A}{d}\cdot\frac{K_1(K_2+K_3)}{(2K_1+K_2+K_3)}$$

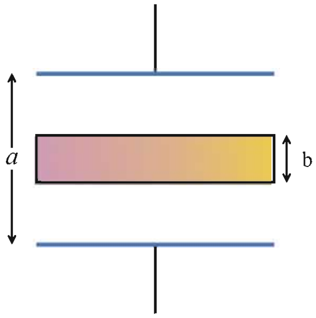
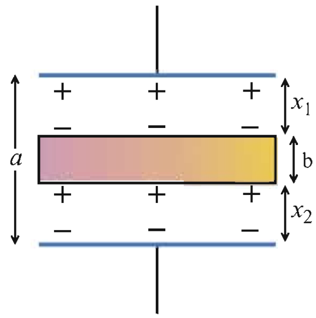


Illustration 15 : A capacitor has air between its plates having separation a. Now a metallic slab of thickness b is placed between its plates as shown in the figure. Show that now its capacitance is $C = \frac{\varepsilon_0 A}{a-b}$. Does this value of capacitance depend on the position of the metallic slab between the plates ?



Solution : One capacitor is formed in the upper region with thickness x_1. Let its capacitance = C_1. Other capacitor is formed in the lower region with thickness x_2. Let its capacitance = C_2. In the thickness b, there is metallic slab, hence no capacitor is formed (because its two surfaces cannot be considered as isolated from each other.) Here, C_1 and C_2 are in series. If their equivalent capacitance is C; $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{x_1}{\varepsilon_0 A} + \frac{x_2}{\varepsilon_0 A} = \frac{x_1 + x_2}{\varepsilon_0 A}$

$$\therefore\ C = \frac{\varepsilon_0 A}{x_1 + x_2}.$$

But Figure shows that $x_1 + x_2 = a - b$ $\therefore$ $C = \frac{\varepsilon_0 A}{a-b}$. This value does not depend on the position of the metallic slab. Put it anywhere but $(x_1 + x_2)$ remains constant and only in this much region capacitor is formed.

Illustration 16 : The area of each plate of a parallel plate capacitor is 100 cm^2 and the separation between the plates is 1.0 cm. When there is air between the plates, the capacitor is charged with a battery of 100 V. Now the battery is removed and a dielectric slab of thickness 0.4 cm and dielectric constant 4.0 is placed between the plates. (a) Find the capacitance before the dielectric is introduced. (b) Find the free charge on the plate and the surface charge density on it. (c) What is the electric field E_0 in the region between the plate and the dielectric ? (d) What is the electric field in the dielectric ? (e) What is the potential difference between two plates after the dielectric is introduced ?

Solution : $A = 100 \times 10^{-4} m^2$; $d = 1 \times 10^{-2} m$, $V_0 = 100$ V

$d' = 0.4 \times 10^{-2} m$, $K = 4.0$

(a) When there is air between the plates, the capacitance

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(100 \times 10^{-4})}{1 \times 10^{-2}} = 8.85 \times 10^{-12}\ F = 8.85\ pF.$$

(b) $q_0 = C_0 V_0 = (8.85 \times 10^{-12})(100) = 8.85 \times 10^{-10}$ C

This is free charge. The surface density of charge is

$$\sigma = \frac{q_0}{A} = \frac{8.85 \times 10^{-10}}{100 \times 10^{-4}} = 8.85 \times 10^{-8}\ C/m^2$$

(c) The electric field between the plate and the dielectric is produced by the charge on the plate, that is by the free charge.

$$\therefore\ E_0 = \frac{\sigma}{\varepsilon_0} = \frac{8.85 \times 10^{-8}}{8.85 \times 10^{-12}} = 10000\ V/m$$

(d) In the absence of dielectric the electric field at that place would be E_0. Now on putting the dielectric there the electric field is $E = \frac{E_0}{K} = \frac{10000}{4} = 2500$ V/m.

(e) Now, the potential difference (p.d.) between the plates (from $V = Ed$) is

$$V' = E_0(1 - 0.4) \times 10^{-2} + E(0.4 \times 10^{-2})$$

$$= 10000\ (0.6 \times 10^{-2}) + 2500 \times 0.4 \times 10^{-2}$$

$$= 60 + 10 = 70 \text{ V.}$$

Illustration 17 : A substance has a dielectric constant 2.0 and its dielectric strength is 20×10^6 V/m. It is taken as a dielectric material in a parallel plate copacitor. What should be the minimum area of its each plate such that its capacitance becomes 8.85×10^{-2} μF and it can withstand a potential difference of 2000 V ?

Solution : $K = 2$; $E = 20 \times 10^6$ V/m , $C = (8.85 \times 10^{-2}) \times 10^{-6}$ F

$V = 2000$ V, $A = ?$

Charge on capacitor $Q = CV = (8.85 \times 10^{-8})\ (2000) = 17.7 \times 10^{-5}$ C

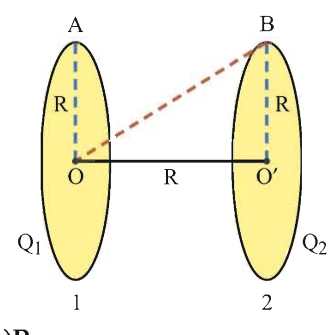
surface density of charge on the plate $\sigma = \frac{Q}{A} = \frac{17.7 \times 10^{-5}}{A}$ C/m^2.

If there were air between the plates, the electric field would be $E_0 = \frac{\sigma}{\varepsilon_0}$, but here a dielectric is placed. Hence the electric field is $E = \frac{E_0}{K} = \frac{\sigma}{K\varepsilon_0}$.

$$\therefore\ 20 \times 10^6 = \frac{17.7 \times 10^{-5}}{(A)(2)(8.85 \times 10^{-12})} \Rightarrow A = 0.5 \text{ m}^2$$

If the value of A is smaller than this, E becomes greater than 20×10^6 and dielectric breackdown occurs.

Illustration 18 : Two identical thin rings each of radius R m are kept on the same axis at a distance of R m apart. If charges on them are Q_1 C and Q_2 C respectively, calculate the work done in moving a charge Q C from the centre of one ring to that of the other.



Solution :

It is clear from the figure that $AO' = BO = \sqrt{R^2 + R^2} = (\sqrt{2})R$

Centre of each ring is at equal distance $= (\sqrt{2})R$ from the circumference of the other ring.

$$\therefore \text{ Potential at O is } V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R\sqrt{2}}$$

$$\text{and potential at O' is } V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R}$$

$\therefore$ Potential difference $\Delta V = V_1 - V_2 = \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2) + \frac{1}{4\pi\varepsilon_0 R\sqrt{2}}[Q_2 - Q_1]$

$$= \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2) - \frac{1}{4\pi\varepsilon_0 R\sqrt{2}}[Q_1 - Q_2]$$

$$= \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2)\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2)\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right] \text{ V}$$

$\therefore$ Work $W = q(\Delta V) = \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$ J

SUMMARY

1. The information about the work done in taking an electric charge from one point to the other in the electric field is obtained from the quantities called electric potential and electric potential energy.
2. $\int_A^B \vec{E}\cdot\vec{dr}$ is the line-integral of electric field between points A to B and it shows the work done by the electric field in taking a unit positive charge from A and B. Moreover, it does not depend on the path and $\oint \vec{E}\cdot\vec{dr} = 0$.
3. "The work required to be done against the electric field to bring a unit positive charge from infinite distance to the given point in the electric field, is called the electric potential (V) at that point".

   Electric potential at point P is $V_p = -\int_\infty^P \vec{E}\cdot\vec{dr}$

   Its unit is $\frac{\text{joule}}{\text{coulomb}}$ = volt. Symbolically $V = \frac{J}{C}$

   Its dimensional formula is $M^1L^2T^{-3}A^{-1}$

   Absolute value of electric potential has no importance but only the change in it is important.
4. "The work required to be done against the electric field to bring a given change (q) from infinite distance to the given point in the electric field is called the electric potential energy of that charge at that point."

   $U_p = -q\int_\infty^P \vec{E}\cdot\vec{dr} = qV_p$

   The absolute value of electric potential energy has no importance, only the change in it is important.
5. Electric potential at point P lying at a distance r from a point charge q is $V_p = \frac{kq}{r}$
6. The electric potential at a point at distance r from an electric dipole is $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$

   ... (for $r >> 2a$)

   Potential on its axis is $V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$. Potential on its equator is $V = 0$

7. Electric potential at a point $\vec{r}$ due to a system of point charges q_1, q_2, ... q_n situated at positions r_1, r_2, ... q_n is $V = \sum_{i=1}^{n} \frac{kq_i}{|\vec{r} - \vec{r}_i|}$.

The electric potential at point $\vec{r}$, due to a continuous charge distribution is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{volume} \frac{\rho(\vec{r}')d\tau'}{|\vec{r} - \vec{r}'|}$$

The electric potential due to a spherical shell is

$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$ ... (for $r \geq R$) and $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$ ... (for $r \leq R$)

8. A surface on which electric potential is equal at all points is called an equipotential surface. The direction of electric field is normal to the equipotential surface.

9. $E = \frac{-dV}{dl}$ gives the magnitude of electric field in the direction of $\vec{dl}$. To find E from V, in general, we can use the equation

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

The direction of electric field is that in which the rate of decrease of electric potential with distance $\left(\frac{-dV}{dl}\right)$ is maximum, and this direction is always normal to the equipotential surface.

10. The electrostatic potential energy of a system of point charges q_1, q_2, ... q_n situated respectively at r_1, r_2, ... r_n is

$$U = \sum_{\substack{i=1 \\ i<j}}^{n} \frac{kq_iq_j}{r_{ij}} \quad \text{where } r_{ij} = r_j - r_i$$

11. The electrostatic potential energy of an electric dipole in an external electric field $\vec{E}$, is $U = -\vec{E} \cdot \vec{p} = -E\, p \cos\theta$.

12. When a metallic conductor is placed in an external electric field,

(i) A stationary charge distribution is induced on the surface of the conductor.
(ii) The resultant electric field inside the conductor is zero.
(iii) The net electric charge inside the conductor is zero.
(iv) The electric field at every point on the outer surface of conductor is locally normal to the surface.
(v) The electric potential everywhere inside the conductor is the same constant.
(vi) If there is a cavity in the conductor then, even when the conductor is placed in an external electric field, the electric field inside the conductor and also inside the cavity is always zero. This fact is called the electrostatic shielding.

When electric charge is placed on the metallic conductor :

(i) The electric field everywhere inside the conductor is zero.
(ii) That charge is distributed only on the outer surface of the conductor.

(iii) The electric field on the surface is locally normal, and is given by $\vec{E} = \left(\frac{\sigma}{\varepsilon_0}\right)\hat{n}$.

(iv) If a charge is placed inside the cavity in the conductor, the electric field in region which is outside the cavity but in the conductor remains zero.

13. "A device formed by two conductors insulated from each other is called a capacitor". Its capacitance is $C = \frac{Q}{V}$ = constant. The unit of C is coulomb/volt, which is also called **farad.**

$1\ \mu F = 10^{-6}\ F;\ 1\ pF = 10^{-12}\ F$

14. The capacitance of the parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d}$.

15. If the effective capacitance in the series combination of capacitors is C,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

If the effective capacitance in the parallel combination of capacitors is C,

$$C = C_1 + C_2 + C_3 + ...$$

16. The energy stored in the capacitor is $U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{VQ}{2}$ and the energy density = energy per unit volume = $\frac{1}{2}\varepsilon_0 E^2$, where E = electric field.

17. When a dielectric is placed in an external electric field E_0, polarisation of dielectric occurs due to electical induction. The electric field produced by these induced charges is in the direction opposite to the direction of external electric field. Hence the resultant electric field E, inside the dielectric is less than the external electric field E_0. The dipole moment produced per unit volume is called the intensity of polarisation or in short polarisation P.P = σ_b.

Since $P \propto E$, $P = \varepsilon_0 x_e E$. x_e is called the electric susceptibility of the dielectric medium. $\varepsilon_0(1 + x_e)$ is called the permittivity ε of the dielectric medium. $\frac{\varepsilon}{\varepsilon_0}$ is called the relative permittivity of that medium and it is also called the dielectric constant K.

i.e. $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$.

$K = 1 + x_e$, $E = \frac{E_0}{K}$. Thus in the dielectric the electric field reduces to the K^{th} part.

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ is called the electric displacement. Gauss Law in the presence of dielectric is written as $\oint \vec{D} \cdot \vec{ds} = q$, where q is only the net free charge.

18. When there is air (or vacuum) between the plates of a parallel plate capacitor, the capacitance is $C = \frac{\varepsilon_0 A}{d}$.On placing a medium of dielectric constant K, the capacitance is $C' = KC$. Thus the capacitance becomes K times, due to the presence of the dielectric.

19. With the help of Van-De-Graf generator a p.d. of a few million volt can be established.

EXERCISE

For the following statements choose the correct option from the given options :

1. For a uniform electric field $\vec{E} = E_0(\hat{i})$, if the electric potential at $x = 0$ is zero, then the value of electric potential at $x = +x$ will be ....

(A) $x\ E_0$ (B) $-x\ E_0$ (C) $x^2\ E_0$ (D) $-x^2\ E_0$

2. The line integral of an electric field along the circumference of a circle of radius r, drawn with a point charge Q at the centre will be ....

(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (B) $\frac{Q}{2\varepsilon_0 r}$ (C) zero (D) $2\pi Qr$

3. A particle having mass 1 g and electric charge 10^{-8} C travels from a point A having electric potential 600 V to the point B having zero potential. What would be the change in its kinetic energy ?

(A) -6×10^{-6} erg (B) -6×10^{-6} J

(C) 6×10^{-6} J (D) 6×10^{-6} erg

4. The area of every plate shown in the Figure is A and the separation between the successive plates is d. What is the capacitance between points a and b ?

(A) $\varepsilon_0\ A/d$ (B) $2\varepsilon_0\ A/d$

(C) $3\varepsilon_0\ A/d$ (D) $4\varepsilon_0\ A/d$

5. A particle having mass m and charge q is at rest. On applying a uniform electric field E on it, it starts moving. What is its kinetic energy when it travels a distance y in the direction of force ?

(A) qE^2y (B) qEy^2 (C) qEy (D) q^2Ey

6. A parallel plate capacitor is charged and then isolated. Now a dielectric slab is introduced in it. Which of the following quantities will remain constant ?

(A) Electric charge Q (B) Potential difference V

(C) Capacitance C (D) Energy U.

7. A moving electron approaches another electron. What will happen to the potential energy of this system ?

(A) will remain constant (B) will increase

(C) will decrease (D) may increase or decrease

8. Energy of a charged capacitor is U. Now it is removed from a battery and then is connected to another identical uncharged capacitor in parallel. What will be the energy of each capacitor now ?

(A) $\frac{3U}{2}$ (B) U (C) $\frac{U}{4}$ (D) $\frac{U}{2}$

9. A uniform electric field is prevailing in Y-direction in a certain region. The co-ordinates of points A, B and C are (0, 0), (2, 0) and (0, 2) respectively. Which of the following alternatives is true for the potentials at these points ?

(A) $V_A = V_B$, $V_A > V_C$ (B) $V_A > V_B$, $V_A = V_C$

(C) $V_A < V_C$, $V_B = V_C$ (D) $V_A = V_B$, $V_A < V_C$

10. The capacitance of a parallel plate capacitor formed by the circular plates of diameter 4.0 cm is equal to the capacitance of a sphere of diameter 200 cm. Find the distance between two plates.

(A) 2×10^{-4} m (B) 1×10^{-4} m (C) 3×10^{-4} m (D) 4×10^{-4} m

11. The capacitance of a variable capacitor joined with a battery of 100 V is changed from 2 μF to 10 μF. What is the change in the energy stored in it ?

(A) 2×10^{-2} J (B) 2.5×10^{-2} J (C) 6.5×10^{-2} J (D) 4×10^{-2} J

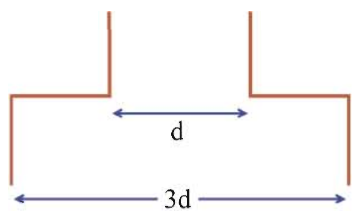
12. A parallel plate capacitor is charged with a battery, and then separated from it. Now if the distance between its two plates is increased, what will be the changes in electric charge, potential difference and capacitance respectively ?

(A) remains constant, decreases, decreases
(B) increases, decreases, decreases
(C) remains constant, decreases, increases
(D) remains constant, increases, decreases

13. 6 identical capacitors are joined in parallel and are charged with a battery of 10 V. Now the battery is removed and they are joined in series with each other. In this condition what would be the potential difference between the free plates in the combination ?

(A) 10 V (B) 30 V (C) 60 V (D) $\frac{10}{6}$ V

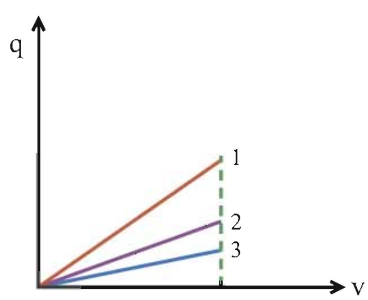
14. Six identical square metallic plates are arranged as in figure. Length of each plate is l. The capacitance of this arrangement would be ....



(A) $\frac{3\varepsilon_0 l^2}{d}$ (B) $\frac{4}{3}\frac{\varepsilon_0 l^2}{d}$

(C) $\frac{3}{2}\frac{\varepsilon_0 l^2}{d}$ (D) $\frac{4\varepsilon_0 l^2}{d}$

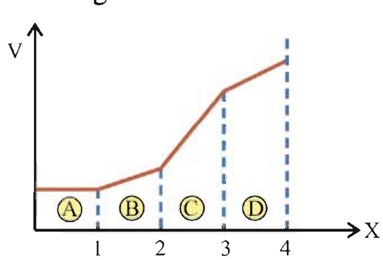
15. In the following table, the area of plates and separation between the plates are given. In the nearby Figure, $q - V$ graphs for them are shown. Determine which graph is for which capacitor.



Capacitor	area	separation
C_1	A	d
C_2	2A	d
C_3	A	$2d$

(A) $1 \rightarrow C_2$ $2 \rightarrow C_3$ $3 \rightarrow C_1$
(B) $1 \rightarrow C_1$ $2 \rightarrow C_2$ $3 \rightarrow C_3$
(C) $1 \rightarrow C_2$ $2 \rightarrow C_1$ $3 \rightarrow C_3$
(D) $1 \rightarrow C_3$ $2 \rightarrow C_1$ $3 \rightarrow C_2$

16. A $V-x$ graph for an electric field on X-axis is shown in the figure. In which region is the magnitude of electric field maximum ?



(A) A (B) B

(C) C (D) D

17. The distance between electric charges Q C and 9Q C is 4 m. What is the electric potential at a point on the line joining them where the electric field is zero ?
(A) 4 kQ V (B) 10 kQ V (C) 2 kQ V (D) 2.5 kQ V

18. If a capacitor having capacitance of 600 μF is charged at a uniform rate of 50 μC/s, what is the time required to increase its potential by 10 volts.
(A) 500 s (B) 6000 s (C) 12 s (D) 120 s

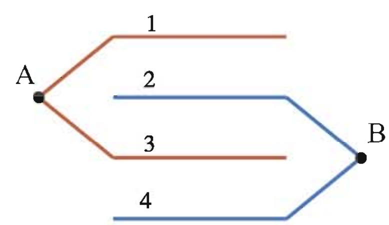
19. Two metallic spheres of radii R_1 and R_2 are charged. Now they are brought into contact with each other with a conducting wire and then are separated. If the electric fields on their surfaces are E_1 and E_2 respectively, E_1 / E_2 = ...
(A) R_2 / R_1 (B) R_1 / R_2 (C) R_2^2 / R_1^2 (D) R_1^2 / R_2^2

20. For a capacitor the distance between two plates is $5x$ and the electric field between them is E_0. Now a dielectric slab having dielectric constant 3 and thickness x is placed between them in contact with one plate. In this condition what is the p.d. between its two plates ?
(A) $\frac{13E_0x}{3}$ (B) $15\ E_0\ x$ (C) $7\ E_0\ x$ (D) $\frac{9E_0x}{2}$

21. In the figure area of each plate is A and the distance between consecutive plates is d. What is the effective capacitance between points A and B ?

(A) $\varepsilon_0 A/d$ (B) $2\varepsilon_0 A/d$

(C) $3\varepsilon_0 A/d$ (D) $4\varepsilon_0 A/d$



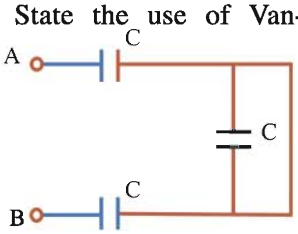
ANSWERS

1. (B)	2. (C)	3. (C)	4. (B)	5. (C)	6. (A)
7. (B)	8. (C)	9. (A)	10. (B)	11. (D)	12. (D)
13. (C)	14. (B)	15. (C)	16. (C)	17. (A)	18. (D)
19. (A)	20. (A)	21. (C)			

Answer the following questions in brief :

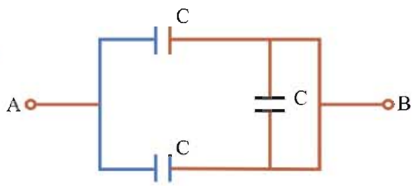
1. What is line integral of electric field ? What does it indicate ?
2. If the electric potential at point P is V_P, what is the electric potential energy of charge q at this point ?
3. What is the electric potential at a point on the equator of an electric dipole ?
4. What is electic potential gradient ? Give its unit.
5. Electric field is always .... to the equipotential surface and in a direction in which the rate of decrease of potential is ....
6. Give the formulae for the equivalent (effective) capacitance of capacitors in series and parallel combinations.
7. How does the energy density associated with an electric field depend on the value of electric field ?
8. What is meant by a non-polar molecule ?

9. Define intensity of polarisation (or in short polarisation) P.
10. Write the formula showing the relation between x_e and P.
11. Electric field at a point in free space is 100 N/C. What would be the electric field in a medium with dielectric constant 5, placed at that place ?
12. What is the meaning of the relative permittivity ε_r of a dielectric medium ?
13. State the use of Van-de-Graf generator.

14. 

What is the equivalent capacitance between points A and B in the figure ? (**Hint :** The last capcitor on right side is short circuited. ∴ it is not effective)

[**Ans :** C/2]

15. 

What is the equivalent capacitance between points A and B shown in the figure ? (**Hint :** The last capcitor on right side is short circuited. ∴ it is not effective)

[**Ans :** 2C]

Answer the following questions :

1. Show that the work done by the electric field in moving a unit positive charge from one point to the other point in an electric field depends only on the positions of those two points and not on the path joining them.
2. Define electric potential and give the formula corresponding to it. Write its units and dimensions.
3. Define electric potential and obtain the formula for the electric potential due to a point charge.
4. Derive the formula for the electric potential due to an electric dipole at a far distant point from it.
5. What is an equipotential surface ? Show that the direction of the electric field at a given point is normal to the equipotential surface passing through that point.
6. Obtain the formula which can give electric field from the electric potential.
7. Derive the formula for the electric potential energy of an electric dipole in a uniform electric field.
8. Explain in short the effects produced inside a metallic conductor placed in an external electric field.
9. What is a capacitor ? Give the definition, and units of capacitance. On which factors does the value of capacitance depend ? Give the symbol of capacitor.
10. Obtain the formula for the equivalent (effective) capacitance in the series / parallel combination of capacitors.
11. Obtain the formula for the capacitance of a parallel plate capacitor.
12. Obtain the formula for the energy, stored in the capacitor and also for the energy density.
13. Explain the polarisation produced in the dielectric placed between the two plates of a parallel plate capacitor and obtain the formula $P = \sigma_b$.
14. The resultant electric field inside a dielectric placed between two plates of a capacitor is $E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$. Hence obtain $E = \frac{E_0}{K}$, where E_0 = external electric field on the dielectric.
15. Using $E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$ obtain the formula for the electric displacement $\vec{D}$. State the importance of $\vec{D}$.
16. Obtain the formula showing the principle of Van-De-Graaff generator.
17. Only draw the Figure and explain the working of Van-de-Graf generator.

Solve the following examples :

1. q_1 = 2C and q_2 = −3 C charges are placed at (0, 0) and (100, 0)m points respectively. At which point(s) on the X-axis is the electrical potential zero ?

[Ans. : 40m, −200 m]

2. Two metallic spheres having radii a and b, are placed very far from each other and are joined by a conducting wire. The total charge on them is Q. Find (i) the charge on each sphere and (ii) potential on each sphere.

[Ans. : $Q_a = \frac{aQ}{a+b}$, $Q_b = \frac{bQ}{a+b}$, $V_a = V_b = \frac{kQ}{a+b}$]

3. In a certain region the electric potential is given by the formula $V(x, y, z) = 2x^2y + 3y^3z - 4z^4x$. Find the components of electric field and the vector electric field at point (1, 1, 1) in this field.

[Ans. : $E_x = 0$, $E_y = -11$ unit, $E_z = 13$ units, $\vec{E} = -11\hat{j} + 13\hat{k}$ unit]

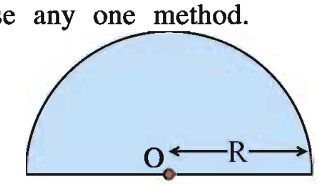
4. A spherical drop of water has 3×10^{-10} C amount of charge residing on it. 500 V electric potential exists on its surface. Calculate the radius of this drop. If eight such drops (Having identical charge and radii) combine to form a single drop, calculate the electric potential on the surface of the new drop. $k = 9 \times 10^9$ SI.

[Ans. : Radius of the first drop = 0.54 cm,
Electric Potential on the new drop = 2000 V]

5. Q amount of electric charge is present on the surface of a sphere having radius R. Calculate the total energy of the above system. [Ans. : $\frac{1}{2}\frac{kQ^2}{R}$]

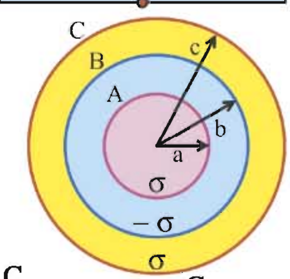
Note : The above example can be calculated in three different ways, (1) By multiplying the electric charge with the average value of the initial and the final electric potential, (2) By considering the above system to be a capacitor and calculating the energy of the capacitor and (3) By considering an electric charge q and taking the integration of the work done to increase the above charge by an amount dq. Use any one method.

6. σ is the uniform charge density present on the surface of a semi-sphere of radius R. Derive the formula for the electric potential at the centre.



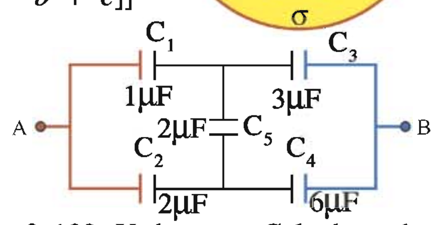
[Ans. : $\frac{R\sigma}{2\varepsilon_0}$]

7. Consider A, B and C to be the co-centric shells of metal. Their radii are a, b and c respectively ($a < b < c$). Their surface charge densities are σ, −σ and σ respectively. Calculate the electric potential on the surface of shell A.

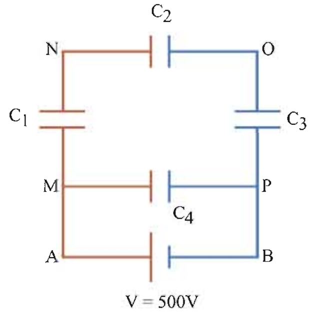


[Ans. : $\frac{\sigma}{\varepsilon_0}[a - b + c]$]

8. Calculate the equivalent capacitance between points A and B of the connections of capacitors shown in the figure. [Ans. : $\frac{9}{4}$ μF]

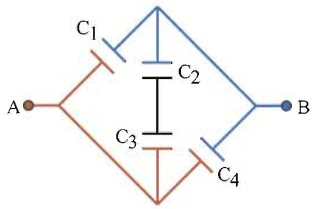


9. (1) A capacitor of 900 pF is charged with the help of 100 V battery. Calculate the electric potential energy of this capacitor. (2) The above capacitor is disconnected from the battery and is connected to another identical uncharged capacitor. What will be the total energy of the system ? [Ans. : (1) 4.5×10^{-6} J (2) 2.25×10^{-6} J]

10. 

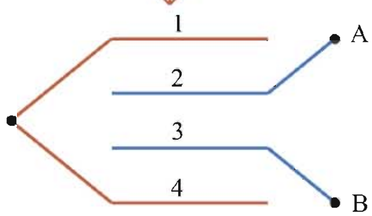
Calculate the equivalent capacitance for the connection of capacitors shown in the figure and the electric charge present on each of the capacitors. The value of each capacitance is 10 μF.

[**Ans. :** Equivalent capacitance = 13.3 μF $Q_1 = Q_2 = Q_3 = 1.7 \times 10^{-3}$ C, $Q_4 = 5.0 \times 10^{-3}$ C]

11. 

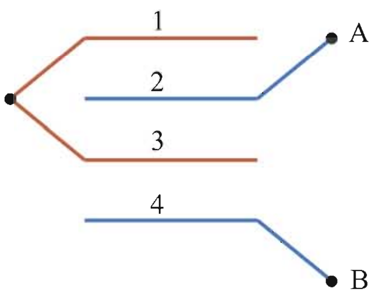
Find the equivalent capacitance between A and B in the circuit shown in the Figure. $C_1 = C_4 = 1$ μF; $C_2 = C_3 = 2$ μF.

[**Ans. :** 3 μF]

12. 

The area of each plate shown in the figure is A and the distance between consecutive plates is *d*. What is the equivalent capacitance between points A and B ?

[**Ans. :** $\frac{3}{2}\frac{\varepsilon_0 A}{d}$]

13. 

The area of each plate shown in the figure is A and the distance between consective plates is *d*. What is the equivalent capacitance between points A and B ?

[**Ans. :** $\frac{2}{3}\frac{\varepsilon_0 A}{d}$]