3

CURRENT ELECTRICITY

3.1 Introduction

In the previous two chapters, all electric charges (whether free or bound) were considered to be stationary (at rest) and we mainly studied the interaction between them. The study of this branch of electricity is called electrostatics.

In the present chapter, we will bring the charges in motion by providing energy to them. Such charges in motion constitute an electric current.

Such currents occur naturally in many situations. When there is lightning in the sky, charges flow from the clouds to the earth through the atmosphere. Flow of charges in lightning is of short duration, resulting in current called transient current. This flow of charges in lightning is not steady.

In everyday life, we see many devices where charges flow in a steady manner, like water flowing smoothly in a river. A cell-driven clock, torch and a transistor radio are examples of such devices.

In the present chapter, we shall study some of the basic laws concerning steady electric current and the quantities associated with flow of charges like electric current density, drift velocity and mobility. Moreover, we shall study about resistors, cells and their different connections, Kirchhoff's rules for the analysis of network and conversion of electrical energy into heat energy during conduction of electricity through conductors. Further, we shall get the information about potentiometer for the measurement of emf of a cell and wheatstone bridge which is used for the measurement of resistor.

The study of this branch of electricity is called **current electricity.**

3.2 Electric Current

The flow of electric charges constitutes an electric current.

If net amout of charge Q is flowing through a cross-sectional area of the conductor in time t, then for a steady flow of charge,

$$I = \frac{Q}{t} \qquad (3.2.1)$$

is defined as the current flowing through that cross-sectional area.

The amount of charge flowing per unit time across any cross-section of a conductor held perpendicular to the direction of flow of charge is called current (I).

Electric current (I) is considered as fundamental quantity in SI unit system. The SI unit of electric current is (a) which is equal to $\frac{\text{coulomb}}{\text{second}}$.

In the above equation (3.2.1), if we take,

$t = 1$ second

$Q = 1$ coulomb

then $I = 1$ ampere

If 1 coulomb of charge crosses any cross-section of a conductor. Perpendicular to the direction of current in 1 second, then the current through that cross-section is said to be 1 ampere.

For small currents, milliampere ($mA = 10^{-3}$ A) and microampere ($\mu A = 10^{-6} A$) units are used.

In metallic conductors the current is due to the motion of negatively charged electrons. In electrolytes, the current is due to the motion of both positive and negative ions moving in opposite directions. While in semi-conductors, partly the electrons and partly the holes (hole is the deficiency of electron in the covalent bond) are responsible for the flow of the current.

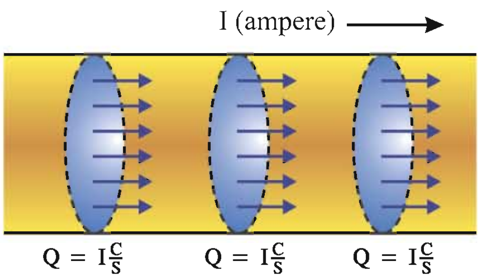


Figure 3.1 Conservation of Charge

Let I ampere current be flowing through any conductor as shown in figure 3.1. Hence, I coulomb electric charge is flowing through every cross-sectional area of the conductor per second.

In other words, the amount of electric charge entering any cross-section of the conductor from one side in a given time interval is equal to the amount of electric charge leaving that cross-section from the other side in the same interval of time. As a result of this **Electric charge is never accumulated at any point in the conductor. The electric charge is neither created nor destroyed at any point in the conductor. This means that electric charge is conserved.**

By convention, the direction of motion of positive charges is taken as the direction of electric current. It is called conventional current. However, in conductors the current is due to the motion of negatively charged electrons, so the direction of current is opposite to the electron current.

In some cases, current (rate of flow of charge) varies with time means the flow of charge is not steady. In this circumstances, let ΔQ be the net amount of electric charge flowing across any cross-sectional area of a conductor during the time interval Δt between times t and $(t + \Delta t)$, then the average electric current flowing during time interval Δt is given by,

$$\langle I \rangle = \frac{\Delta Q}{\Delta t}$$

The electric current at time t will be,

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad (3.2.2)$$

Illustration 1 : The current through a wire varies with time as $I = I_0 + \alpha t$, where $I_0 = 10$ A and $\alpha = 4$ As^{-1}. Find the charge that flows across a cross-section of the wire in first 10 seconds.

Solution : Current $I = \frac{dq}{dt} = I_0 + \alpha t$

$\therefore\ dq = (I_0 + \alpha t)dt$

Integrating on both sides,

$$\int dq = \int_{t=0}^{t=10} (I_0 + \alpha t)dt$$

$$\therefore\ q = \left[I_0 t + \frac{\alpha t^2}{2} \right]_{t=0}^{t=10} = 10\ I_0 + 50\ \alpha$$

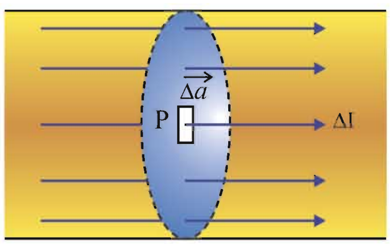
Substituting $I_0 = 10$ and $\alpha = 4$,

$q = 10(10) + 50(4) = 300$ C.

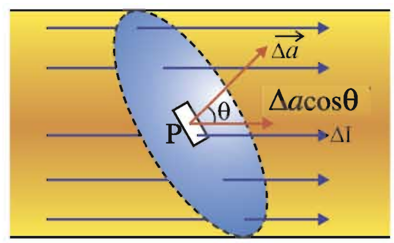
3.3 Electric Current Density

It is possible that the rate of flow of the electric charge through different cross-sectional areas of the conductor may not be same. Apart from this, the flow of the electric charge may not be perpendicular to the cross-sectional area of the conductor. In such circumstances, to study the flow of charge through a cross-section of the conductor at a particular point, a vector quantity known as electric current density $\vec{J}$ is defined.

To define the current density at a point P, imagine a small cross-section of area Δa through P perpendicular to the flow of charges as shown in figure 3.2 (a).



(a)



(b)

Figure 3.2 Cross–section of a Current Carrying Conductor

If ΔI be the current through the area Δa, the average current density is,

$$\langle J \rangle = \frac{\Delta I}{\Delta a}$$

The current density at the point P is,

$$J = \lim_{\Delta a \to 0} \frac{\Delta I}{\Delta a} = \frac{dI}{da} \tag{3.3.1}$$

The direction of the current density is the same as the direction of the current.

If a current I is uniformly distributed over an area A and is perpendicular to it,

$$J = \frac{I}{A} \tag{3.3.2}$$

Thus, **The electric current density at any point is defined as the amount of electric current flowing per unit cross-section perpendicular to the current at that point. (amount of electric charge flowing per unit time)**

The SI unit of the current density is Am^{-2}.

Now let us consider a cross-section Δa which is not perpendicular to the current, (Figure 3.2 (b)) then the component of cross-section in the direction of current $\Delta a \cos\theta$ should be considered.

Average current density at point P,

$$\langle J \rangle = \frac{\Delta I}{\Delta a \cos\theta} \tag{3.3.3}$$

Where, ΔI = Current flowing through small area Δa near point P.

and θ = angle made by the normal to the cross-section with the direction of the current.

For very small area $\overrightarrow{\Delta a}$,

current density $J = \lim_{\Delta a \to 0} \frac{\Delta I}{\Delta a \cos\theta} = \frac{dI}{da\cos\theta}$ (3.3.4)

$\therefore\ dI = J da \cos\theta$

$\therefore\ dI = \vec{J} \cdot \vec{da}$ (3.3.5)

Taking the surface integration of equation (3.3.5) over the entire cross–sectional area, we have,

$$\int dI = \int \vec{J} \cdot \vec{da}$$

$$I = \int_a \vec{J} \cdot \vec{da} \qquad (3.3.6)$$

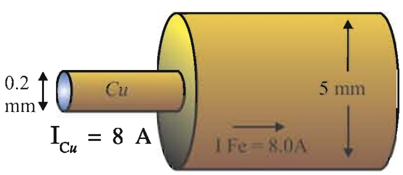
If the cross-sectional area is perpendicular to the current and if J is constant over the entire cross-section then,

$$I = \int \vec{J} \cdot \vec{da} = J \int da$$

$\therefore\ I = JA$ (3.3.7)

The concept of electric current density is very useful in the discussion of the flow of electric charges.

Illustration 2 : 0.2 mm diameter copper wire is connected to 5.0 mm diameter iron wire. The current flows through both the wires. If 8.0 A current flows through the copper wire, then calculate the following quantities.



(1) The current flowing through the iron wire and current density in it.

(2) The current density in the copper wire.

Solution : As per the conservation law of charges, equal amount of time is taken for a given quantity of charge to enter the copper wire and leave the iron wire.

(1) $\therefore\ I_{Cu} = 8.0 = I_{Fe}$

Let A_{Fe} be cross-sectional area of the iron wire and let d_{Fe} and r_{Fe} be the diameter and radius.

$$\therefore\ J_{Fe} = \frac{I_{Fe}}{A_{Fe}} = \frac{8.0}{\pi r_{Fe}^2} = \frac{8.0}{\pi\left(\frac{d_{Fe}}{2} \times 10^{-3}\right)^2} = \frac{8.0 \times 4}{(3.14)(5 \times 10^{-3})^2}$$

$$\therefore\ J_{Fe} = 407\ kA/m^2$$

(2) The current density in the copper wire $J_{Cu} = \frac{8.0}{(3.14)(0.1 \times 10^{-3})^2} = 2.5 \times 10^8\ A/m^2$

Illustration 3 : The current density along the axis of a cylindrical conductor having radius equal to R is given by $J = J_0 (1 - \frac{r^2}{R^2})$. Find the current along the length of the conductor. The distance from the axis is given by *r*.

Solution : Consider a ring of thickness *dr* at a distance *r* from the axis on the cross-section perpendicular to the axis of a cylinder.

The current flowing through the ring,

$$dI = \vec{J}\cdot\vec{da} = Jda \quad (\because \cos\theta = 1)$$

$$\therefore dI = J_0 \left(1 - \frac{r^2}{R^2}\right) (2\pi r dr)$$

The current along the length of the conductor,

$$I = \int dI = \int_{r=0}^{r=R} J_0 \left(1 - \frac{r^2}{R^2}\right)(2\pi r dr) = 2\pi J_0 \int_{r=0}^{r=R} \left(1 - \frac{r^2}{R^2}\right)(r)\ dr$$

$$I = 2\pi J_0 \int_0^R \left(r - \frac{r^3}{R^2}\right) dr = 2\pi J_0 \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$I = 2\pi J_0 \left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right] = 2\pi J0 \left[\frac{R^2}{4}\right]$$

$$I = \pi J_0 \frac{R^2}{2}$$

3.4 Ohm's Law

Why do we not experience a fatal shock on touching a 6V supply while on touching a 230 V source, one experiences a fatal shock ?

In these examples for different voltages, the electric current flowing through the body is different.

In 1828, a German physicist, George Simon Ohm was the first person to give a mathematical relationship between voltage and current, famously known as Ohm's law. Ohm experimentally proved that **"Under a definite physical condition, (e.g. constant temperature) the current (I) flowing through the conductor is directly proportional to the potential difference (V) applied across its ends."** This statement is called Ohm's law.

According to Ohm's law, $I \propto V$

$$\therefore \frac{V}{I} = \text{constant}$$

This constant ratio $\frac{V}{I}$ is called the resistance (R) of the conductor.

$$\therefore \frac{V}{I} = R \qquad (3.4.1)$$

$$\text{OR } V = IR \qquad (3.4.2)$$

The SI unit of resistance is $\frac{\text{volt}}{\text{ampere}}$ which is known as ohm, and is denoted by the symbol Ω. At a given temperature, the resistance R not only depends on the material of the conductor but also on the dimensions of the conductor.

The reciprocal of a resistance i.e. $\frac{1}{R}$ is called the conductance of the material of the given conductor. Its unit is Ω^{-1} or mho and is symbolised as ℧.

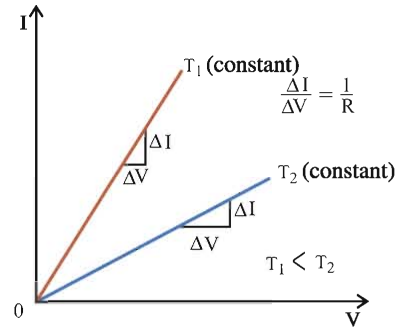


Figure 3.3 I – V Characteristics for a Conductor

Ohm's law is not a fundamental law of nature, like the gravitational law of Newton or the Coulomb's law for electrical charges. Ohm's law gives us the relationship between the potential difference across the conductor and the current flowing through it, under a given situation.

All the metals, some of the insulators and some of the electrical devices obey the Ohm's law. Such devices are called Ohmic devices.

The I – V graph for a conductor obeying Ohm's law at a constant temperature will be a straight line. i.e. such relation is linear.

3.4.1 Limitations of Ohm's Law

There do exist some materials and devices used in electric circuits where the proportionality of V and I does not hold. In such devices,

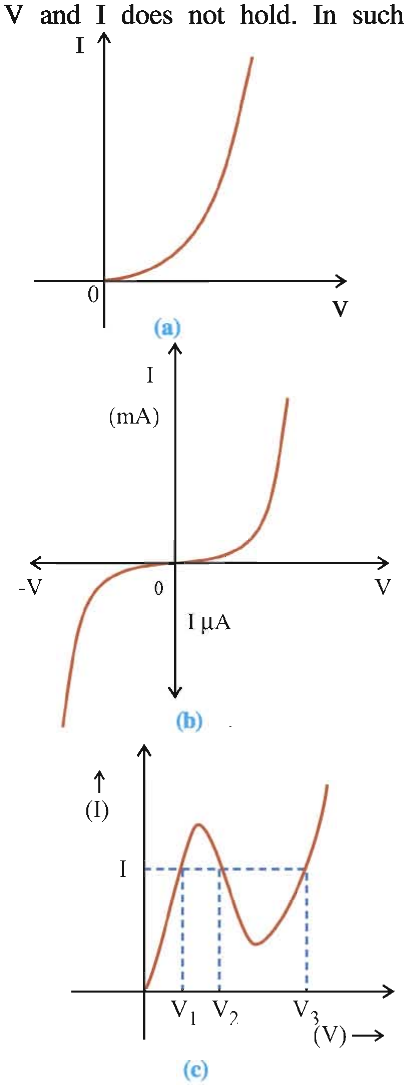


Figure 3.4 I-V Characteristics of Difference Devices

(1) V–I relations are non-linear. e.g. semi-conductor devices like diode and transistor. (figure 3.4 (a))

(2) The relation between V and I depends on the sign of V. In other words, if I is the current for a certain voltage V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction. This happens in a semi-conductor diode which we will study in future. (See figure 3.4 (b))

(3) The relation between V and I is not unique. i.e. there is more than one value of V for the same current I. A graph of device exhibiting such behaviour (e.g. tunnel diode) is shown in figure 3.4 (c).

Materials and devices not obeying Ohm's law are called non-ohmic devices. Such devices are widely used in electronic circuits.

3.5 Electrical Resistivity and Conductivity

Let us try to understand the dependence of the resistance (R) of a conductor on the dimensions of the conductor. Consider a conductor having cross–sectional area A and length l. Experimentally it is found that, at a given temperature, resistance (R) of a conductor is proportional to length of the conductor (l) and inversely proportional to the cross-sectional area (A).

$$R \propto l \text{ and } R \propto \frac{1}{A}$$

$$\therefore R \propto \frac{l}{A}$$

$$\therefore R = \rho.\frac{l}{A} \qquad (3.5.1)$$

Here the constant ρ is called the resistivity of the material. It depends on the material of the conductor, temperature and the pressure existing on the given conductor. It does not depend on the dimensions of the conductor.

The unit of resistivity ρ is ohm meter (Ωm).

(**Information :** The resistivity of a material changes at higher pressure, due to the changes in the composition of the crystals.)

Using equation (3.5.1), Ohm's law can be written as,

$$V = IR$$

$$V = \frac{I\rho l}{A} \qquad (3.5.2)$$

$$V = J\rho l \qquad (3.5.3)$$

where, $\frac{I}{A} = J$ is the current density.

Further, if E is the magnitude of uniform electric field in the conductor whose length is l, then the potential difference V across its ends is $V = El$

$$\therefore El = J\rho l$$

$$\therefore E = J\rho \qquad (3.5.4)$$

The current density $\vec{J}$ is a vector quantity and is directed along $\vec{E}$. Thus, the above equation can be written in the vector form as,

$$\vec{E} = \vec{J}\rho$$

$$\text{OR } \vec{J} = \frac{\vec{E}}{\rho} = \sigma\vec{E} \qquad (3.5.5)$$

where, $\sigma = \frac{1}{\rho}$ (reciprocal of resistivity) is called the conductivity of that material.

The unit of σ is $(\Omega\ m)^{-1}$ or mho m^{-1} ($℧\ m^{-1}$) or siemen m^{-1} (Sm^{-1}).

Note that equation (3.5.5) is the vector form of Ohm's law.

3.6 Drift Velocity, Mobility and its Relations with Current

In atoms and molecules, the negatively charged electrons and the positively charged nuclei are bound to each other due to Coulombian electric force. Bulk matter is made up of many molecules.

We will focus only on solid conductors in which current is carried by the negatively charged free electrons.

In metallic conductors, the electrons in the outershells are less bound with the nucleus. Due to thermal energy at room temperature, such valence electrons are liberated from the atom leaving behind positively charged ions. These ions are arranged in a regular geometric arrangement on the lattice points. The electrons liberated from the atom are called free electrons and ions are oscillating about their mean position.

In the absence of electric field, free electrons in a solid conductor move like the molecules in a gas due to their thermal velocities. During their motion they collide with the ions. The directions of their velocities after the collision are completely random. At a given time, there is no preferential direction for the velocities of the electrons. Such random motion of an electron is shown by continuous line AB in figure 3.5.

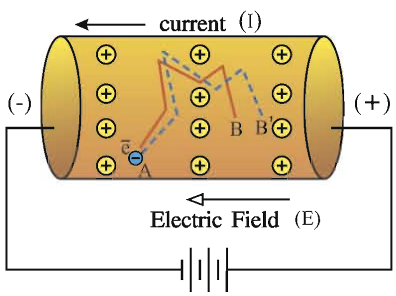


Figure 3.5 Drift Velocity

Thus on the average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction in the absence of an external electric field. Therefore the net charge passing through any cross-section of the conductor is zero hence there will be no flow of electric current in the conductor.

Now, an electric field (E) is applied across the conductor by connecting a battery between two ends of a conductor as shown in figure 3.5. Due to electric field in the conductor, the electron will experience a force F = Ee in the direction opposite to the electric field (towards the positive terminal of battery). The path of the electron will become AB' as shown by the dotted lines. This is because of the electron, executing the motion under the oscillatory electric field of the ions, constantly gets scattered from its path. This gives rise to the resistance in a conductor.

In the presence of electric field (E), the acceleration of the electron in the direction opposite to the electric field is $a = \frac{E \cdot e}{m}$. This acceleration of the electron is momentary, since the electrons are continuously colliding with the ions. (In the real sense, the electrons are scattered in the oscillating electric field of the ions.) As a result, the electrons are dragged in the direction opposite to the electric field. The velocity of the electron becomes zero after every such collision with the ions and after each collision the electron is accelerated once again due to electric field and collide with the ions. The above process keeps on repeating.

Thus, electron travels from A to B$'$ in the presence of electric field rather than travelling from A to B in the absence of electric field. The effective displacement of the electron is equal to BB$'$ in the presence of electric field. The velocity of electron corresponding to this displacement is known as the **drift velocity** (v_d). In this situation the average number of electrons passing through any corss-sectional area of the conductor is not zero in the presence of electric field. As a result, there will be a net flow of charge of current through the conductor.

The average time between two successive collisions of the electron with the ions is called relaxation time (τ).

The drift velocity achieved by the electron during the relaxation time (τ) is,

$$v_d = a\tau$$

$$v_d = \left(\frac{E \cdot e}{m}\right)\tau \quad (3.6.1)$$

Relation between the Drift Velocity and Current Density :

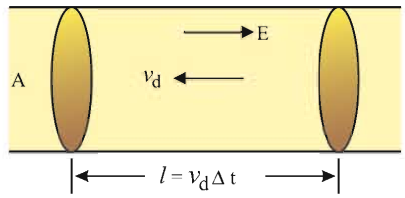


Figure 3.6

To find the relation between the current density and the drift velocity, let us consider a cylindrical conductor of uniform cross-sectional area A. An electric field E exists in the conductor when its ends are connected to the battery.

If the drift velocity of the electron is v_d, then distance travelled by the electron during time Δt is $l = v_d \Delta t$.

The volume of the portion of the conductor whose length is $v_d \Delta t = Al = Av_d \Delta t$.

If there are n free electrons per unit volume (number density) of the conductor, the number of free electrons in this portion is $= nAv_d \Delta t$.

All these electrons cross the area A in time Δt.

Thus, the charge crossing this area in time Δt is, $\Delta Q = nAv_d \Delta te$ (3.6.2)

$$\therefore \text{ Current } I = \frac{\Delta Q}{\Delta t} = nAv_d e \quad (3.6.3)$$

$$\text{and current density } J = \frac{I}{A} = nev_d \quad (3.6.4)$$

In general, equation (3.6.4) can be written in the vector from as,

$$\vec{J} = nq\vec{v}_d$$

For negative charge q, $\vec{J}$ and $\vec{v}_d$ will be in opposite direction.

Comparing two equations (3.5.5) and (3.6.4) of current density,

$$\sigma E = nev_d$$

Substituting the value of v_d from equations (3.6.1),

$$\sigma E = ne\left(\frac{Ee}{m}\tau\right)$$

$$\therefore \sigma = \frac{ne^2\tau}{m} \quad (3.6.5)$$

$$\text{since, } \sigma = \frac{1}{\rho}$$

$$\rho = \frac{1}{\sigma}$$

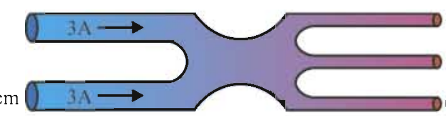
$$\therefore \rho = \frac{m}{ne^2\tau} \quad (3.6.6)$$

In a metal, number density n is not dependent on temperature to any appreciable extent. The oscillations of the ions increases with the temperature and become more erratic. As a result, the relaxation time (τ) decreases. Thus, the resistivity of the conductor increases with temperature according to above formula.

For insulators and semi-conductors, the relaxation time τ and number density n of charge carriers varies with the temperature.

The number density (n) of charge-carriers increases with temperature in semi-conductors. Therefore, the conductivity of semiconductor increases with temperature i.e. its resistivity (ρ) decreases.

Illustration 4 : 3A current is flowing through two identical conducting wires having diameter equal to 0.2 cm. These conducting wires are then split into three identical conducting wires, each having 0.1 cm diameter (as shown in the Figure). Calculate the drift velocities in the thicker and the thinner conductors.



The electron density = 7×10^{28} m^{-3}. All the conductors are made of the same material. The electric charge on electron is equal to = 1.6×10^{-19} C.

Solution : The current density in the thicker wire $J = \frac{I}{A} = \frac{3}{\pi r^2} = \frac{3}{\pi(0.1 \times 10^{-2})^2}$

Current density $J = nev_d$

$$\therefore v_d = \frac{J}{ne} = \frac{3}{\pi(0.1 \times 10^{-2})^2 \times 7 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$\therefore v_d = 8.5 \times 10^{-5}$ m s^{-1}

6A total current is flowing through the three indentical conductors (As per Kirchoff's First Law).

$\therefore$ The current flowing through each of the wires = 2 A

$$\therefore v_d' = \frac{J'}{ne} = \frac{2}{\pi(\frac{0.1}{2} \times 10^{-2})^2 \times 7 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= 2.3 \times 10^{-4} \text{ m s}^{-1}$$

Illustration 5 : A copper wire is stretched to make it 0.1% longer. What is the percentage change in its resistance ? [Assume that the volume of the wire remains constant.]

Solution : Suppose the length of the wire is l and area of cross-section is A.

The resistance of a wire, $R = \rho . \frac{l}{A}$

$$\therefore R = \frac{\rho l^2}{Al} = \frac{\rho l^2}{V} \qquad (1)$$

$$\frac{dR}{dl} = \frac{\rho}{V}.2l$$

$$\therefore dR = \frac{\rho}{V} 2l \cdot dl \qquad (2)$$

Taking ratio of equations (2) and (1),

$$\frac{dR}{R} = \frac{\frac{\rho}{V}.2ldl}{\frac{\rho}{V}.l^2}$$

$$\therefore \frac{dR}{R} = 2 \cdot \frac{dl}{l}$$

Percentage Change $\frac{dR}{R} \times 100\% = 2\left(\frac{dl}{l}\right) \times 100\% = 2\ (0.1\%)$

$= 0.2\ \%$

Thus, the resistance of the wire increases by 0.2%.

Note : If the change in the length of the wire is infinitesimally small, then the above method of differentiation can be used to calculate the change in the resistance. But if the change is very large, then we have to find the change in resistance according to the change in the length.

3.6.1 Mobility

We have seen that the conductivity of any material is due to the mobile charge-carriers. Mobile charge carriers in the conductor are free electrons. In the ionized gas they are electrons and positive ions. Positive and negative both types of ions are the mobile charge carriers in the electrolytes. In semi-conductors the flow of current is partly due to electrons and partly due to holes. (We shall study about semi-conductors and holes in the next chapters. At present, we will note that hole will behave like a positively charged particles.)

Comparing two equations (3.6.4) and (3.5.5) of current density,

$$nev_d = \sigma E$$

$$\therefore \frac{V_d}{E} = \frac{\sigma}{ne}$$

$\frac{V_d}{E}$ is the drift velocity of a charge carrier per unit electric field intensity. This quantity is known as mobility (μ) of a charge carrier.

$$\therefore \text{Mobility } \mu = \frac{V_d}{E} = \frac{\sigma}{ne} \qquad (3.6.7)$$

SI unit of mobility is $m^2V^{-1}s^{-1}$.

From equations (3.6.7)

$$\text{Conductivity } \sigma = ne\mu \qquad (3.6.8)$$

If charge carriers are electrons, then

$$\sigma_e = n_e e\mu_e \qquad (3.6.9)$$

$$\text{For holes, } \sigma_h = n_h e\mu_h \qquad (3.6.10)$$

In a semi-conductor, the holes and the electrons both constitute current in the same direction. The total conductivity,

$$\sigma = \sigma_e + \sigma_h$$

$$\sigma = n_e e\mu_e + n_h e\mu_h \qquad (3.6.11)$$

3.7 Temperature Dependence of Resistivity

The resistivity of a material is found to be dependent on the temperature. Different materials do not exhibit the same dependence on temperatures. Over a limited range of temperatures, that is not too large, the relationship between the resistivity of a metallic conductor and temperature is approximately given by,

$$\rho_\theta = \rho_{\theta_0} [1 + \alpha (\theta - \theta_0)] \qquad (3.7.1)$$

where, ρ_θ = resistivity at a temperature θ

ρ_{θ_0} = resistivity at a proper reference temperature θ_0.

and α is called the temperature co-efficient of resistivity and its unit is $(°C)^{-1}$ or K^{-1}.

The above equation can be written in the form of resistance as follows.

$$R_\theta = R_{\theta_0} [1 + \alpha (\theta - \theta_0)] \quad (3.7.2)$$

The resistivity (ρ) and temperature-coefficient (α) for some materials are given in Table 3.1.

Table 3.1 : The Value of ρ and α for various materials

(For information purpose only)

Material	At 0°C Temperature Resistivity (Ω m)	Temperature Co-efficient $(\alpha)(°C)^{-1}$
(A) Conductors		
Silver	1.6×10^{-8}	0.0041
Copper	1.7×10^{-8}	0.0068
Aluminium	2.7×10^{-8}	0.0043
Tungsten	5.6×10^{-8}	0.0045
Iron	10×10^{-8}	0.0065
Platinum	11×10^{-8}	0.0039
Mercury	98×10^{-8}	0.0009
Nichrome	$\sim 100 \times 10^{-8}$	0.0004
(B) Semi-conductors		
Carbon (graphite)	3.5×10^{-5}	– 0.0005
Germanium	0.46	– 0.05
Silicon	2300	–0.07
(C) Insulators		
Pure water	2.5×10^{5}	
Glass	$10^{10} - 10^{14}$	
Solid rubber	$10^{13} - 10^{16}$	
NaCl	$\sim 10^{14}$	
Fused Quartz	$\sim 10^{16}$	

From the above table, note that for metals, α is positive. Therefore the resistivity of the metal increases with temperature. For metallic conductors, the relation between resistivity (ρ) and temperature is non-linear at lower temperature (< 50 K) and the graph becomes linear near the room temperature. Finally at very higher temperature the graph again becomes non-linear, (Figure 3.7).

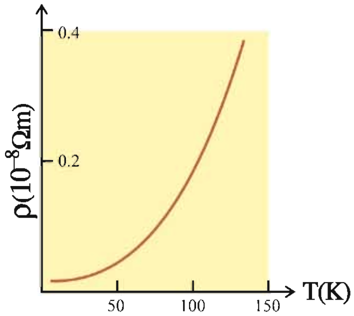


Figure 3.7 Graph of ρ → T for a Metal

Some materials like Nichrome (which is an alloy of nickel, iron and chromium) have very high value of resistivity, exhibit a very weak dependence of resistivity with temperature. (See figure 3.8). The resistivity of manganin (an alloy of copper, magnesene and nickel) is almost independent of temperature.

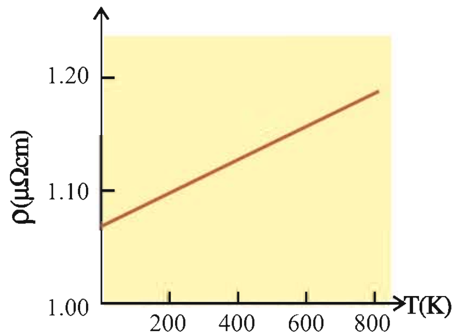


Figure 3.8 Graph of ρ → T for an Alloy

The resistivity of Nichrome does not become zero even at absolute zero temperature (0 K), while the resistivity of a pure metal becomes almost zero at absolute zero temperature. Using this fact the purity of the metal can be tested.

As shown in Table 3.1, semi-conductors like carbon, germanium and silicon have negative values of α. This means that the resistivity of such materials decreases with temperature (as shown in figure 3.9).

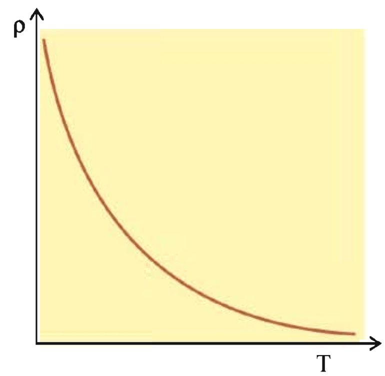


Figure 3.9 Graph of ρ → T for a Semiconductor

3.7.1 Classification of materials on the basis of resistivity

The materials are classified as conductors, semiconductors and insulators depending on their resistivities.

An ideal conductor has zero resistivity or infinite conductivity, while the ideal insulator has infinite resistivity (means zero conductivity).

Metals have low resistivities in the range of $10^{-8}\Omega$m to 10^{-6} Ωm. At the other end are insulators like ceramic, rubber and plastics having resisitivities 10^{18} times greater than metals or more.

Semiconductors lie between these two. They have resistivities characteristically decreasing with a rise in temperature. The resistivities of semiconductors are also affected by presence of small amount of impurities.

It is generally found that good conductors of electricity like the metals are also good conductors of heat (superconductors are an exception in this regard), while the bad conductors of electricity like ceramic, plastic etc. are also found to be bad conductors of heat.

Resistors used in laboratories are of two types.

(1) Wire Wound Resistors : Wire wound resistors are made by winding the wires of an alloy, viz., manganin, constantan, nichrome or similarones on a proper base. The resistivity of such materials does not change appreciably with temperature.

(2) Carbon Resistors : Carbon resistors are widely used in electronic circuits (like radio, television, amplifier etc.). The carbon resistors have very small dimension and it is very inexpensive. (Now a days thin film resistors are used very extensively in the electronic circuits.)

To make a carbon resistor, pure graphite mixed with resin is moulded into a cylinder at high temperature and pressure. Wire leads are attached to two ends of a cylinder and the entire resistor is enclosed in an insulating jacket (ceramic or plastic). Carbon resistors are available in the range of 1 Ω to 100 MΩ.

Colour Code for Carbon Resistors :

The value of the carbon resistor can be found from the colour bands, marked on the surface of the cylinder of carbon resistor. Let us refer the resistor and colour code shown in figure 3.10 in order to understand this.

Colour Code for Resistors (ohm)

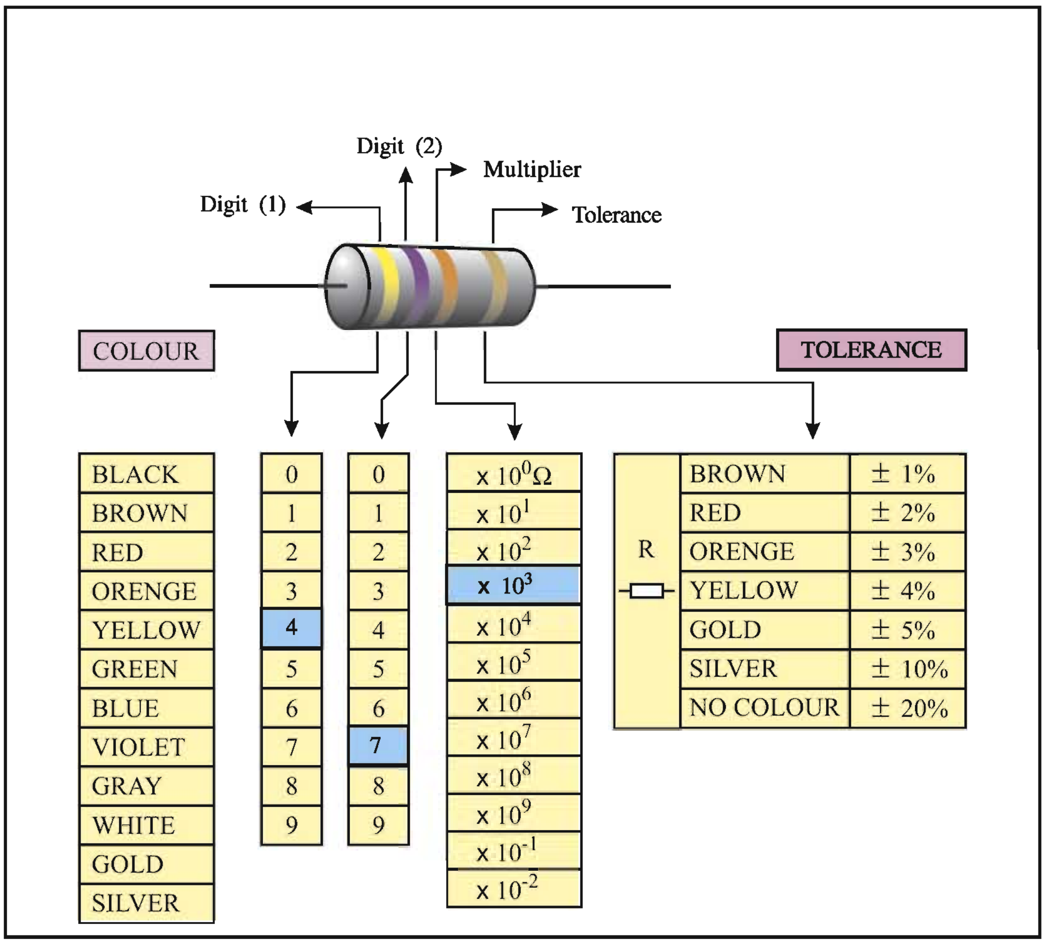


Figure 3.10

Colour of the first band on the resistor shows its value in **"tens"**. The colour of the second band shows its value in **"units"**. The digits for different colours are shown in the colourcode. (Figure 3.10)

The third band implies that the number formed by the first and second digit is to be multiplied by 10^n. The multiplier 10^n for different colours is given in the colourcode. The fourth colour band shows the possible deviation (tolerance) in the value of resistor.

Let us take the example of the resistor shown in figure 3.10. The first band on the resistor is yellow and the digit for this colour is 4. The second band is violet and its corresponding digit is 7. The number formed due to combination of this two digits is 47.

The third colourband is orange, which shows the multiplier 10^3. 47 multiplied by 10^3 gives the value of the resistance = $47 \times 10^3 = 47\ K\Omega$. The last colourband on the resistor is golden, which indicates that the value of the resistor calculated above can have a variation of 5%. Thus, the value of this resistor is ($47\ K\Omega \pm 5\%$)

Dear students, give the colourcode for the resistor $1K\Omega \pm 10\%$ using the colourcode given in Figure 3.10.

3.7.2 Super Conductivity

In 1911, Dutch physicist Kamerlingh Onnes experimentally discovered that the resistivity of mercury absolutely disappears at temperatures below about 4.2 K. As per his observation at 4.3 K temperature, the resistance of mercury is about 0.084Ω and at 3 K temperature it becomes $3 \times 10^{-6}\Omega$ (Which is about 10^6th part of the resistance at 0°C). This showed that,

"The resistance of certain materials reduces to almost zero, when its temperature is lowered below a certain definite temperature (which is known as critical temperature T_C). The material in this state is known as superconductor and this phenomenon is known as superconductivity."

Superconductivity is a specific state of the material. Most of the metals and alloys can achieve the state of superconductivity. Some of the semi-conductors like Si, Ge, Se and Te exhibit the state of suprconductivity under high pressure and low temperatures.

The current flowing through a super-conductor can be sustained over a long interval of time. The reason is that in an ordinary conductor the electrical energy is dissipated as heat energy due to the resistance offered by the conductor, while in super-conductor there will be no loss of electrical energy since the resistance of super-conductor is almost zero.

From the above discussion it seems that using superconductors, the problem of energy loss during the transmission of electrical energy can be solved. One of the important fact we have ignored is that the temperature of the material should be lowered to its critical temperature. Liquid helium and liquid nitrogen are used to achieve the temperature of the material below its critical temperature T_C. This situation can be best described by the proverb 'Penny wise pound foolish.'

In fact, the best of the normal conductors have higher critical temperature (but very much lower than room temperature) than oxide alloys. It shows that, compared to normal conductors, insulators like ceramic can easily achieve the state of superconductivity. Thus, superconductivity is a specific state of a material.

According to latest research, the critical temperature (T_C) of the compound Hg–Ba–Ca–Cu–O can be raised upto 164K.

Such superconductors are known as high temperature superconductors (HTS). HTS has applications in the areas of thin film devices, electric transmission over long distances, levitating trains (maglev trains) which can achieve speed of 550 km/h.

Illustration 6 : The resistance of the platinum wire of a platinum resistance thermometer at the icepoint is 5Ω and at steampoint is 5.23Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795Ω. Calculate the temperature of the bath.

Solution : $R_0 = 5\Omega$, $R_{100} = 5.23\Omega$ and $R_\theta = 5.795\Omega$

From equation, $R_\theta = R_{\theta_0}[1 + \alpha(\theta - \theta_0)]$

$R_\theta = R_0[1 + \alpha\theta]$ ($\because \theta_0 = 0$)

$\therefore R_\theta - R_0 = R_0\alpha\theta$

For steam, $R_{100} - R_0 = R_0\alpha(100)$ (1)

For heat bath, $R_\theta - R_0 = R_0\alpha\theta$ (2)

Dividing equation (2) by (1),

$$\frac{R_\theta - R_0}{R_{100} - R_0} = \frac{\theta}{100}$$

$$\therefore \theta = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100$$

$$= \frac{5.795 - 5}{5.23 - 5} \times 100$$

$\therefore \theta = 345.65$ °C

Illustration 7 : Two materials have the value of α_1 and α_2 as $6 \times 10^{-4} (°C)^{-1}$ and $-5 \times 10^{-4} (°C)^{-1}$ respectively. The resistivity of the first material $\rho_{20} = 2 \times 10^{-8}$. A new material is made by combining the above two materials. The resisitivity does not change with temperature. What should be the resistivity ρ_{20} of the second material ? Considering the reference temperature as 20°C assume that the resistivity of the new material is equal to the sum of the resistivity of its component materials.

Solution :

Here the reference temperature is 20 °C.

Resistivity of a material at temperature θ is,

$\rho_\theta = \rho_{20}[1 + \alpha(\theta - 20)]$

$$\therefore \frac{d\rho_\theta}{d\theta} = \rho_{20}\alpha$$

For material 1, $\left(\frac{d\rho_\theta}{d\theta}\right)_1 = (\rho_{20})_1 \alpha_1$

For material 2, $\left(\frac{d\rho_\theta}{d\theta}\right)_2 = (\rho_{20})_2 \alpha_2$

The resistivity of the mixture $\rho_\theta = (\rho_\theta)_1 + (\rho_\theta)_2$ does not change with temperature. Therefore,

$$\left(\frac{d\rho_\theta}{d\theta}\right) = \left(\frac{d\rho_\theta}{d\theta}\right)_1 + \left(\frac{d\rho_\theta}{d\theta}\right)_2 = 0$$

$$\therefore \left(\frac{d\rho_\theta}{d\theta}\right)_1 = -\left(\frac{d\rho_\theta}{d\theta}\right)_2$$

$\therefore (\rho_{20})_1 \alpha_1 = -(\rho_{20})_2 \alpha_2$

$$\therefore (\rho_{20})_2 = -\frac{(\rho_{20})_1\alpha_1}{\alpha_2}$$

$$= \frac{-(2\times10^{-8})(6\times10^{-4})}{-(5\times10^{-4})}$$

$\therefore (\rho_{20})_2 = 2.4 \times 10^{-8}\ \Omega$ m

Illustration 8 : The tungsten filament of bulb has resistance equal to 18 Ω at 20° C temperature. 0.185 A of current flows, when 30 V is connected to it. If $\alpha = 4.5 \times 10^{-3}\ K^{-1}$ for a tungsten, then find the temperature of the filament.

Solution : As per Ohm's Law,

$$I = \frac{V}{R} \quad \therefore R_\theta = \frac{V}{I} = \frac{30.0}{0.185} = 162\ \Omega$$

When the bulb is ON its resistance is 162 Ω,

Now, $R_\theta = R_{\theta_0}\ [1 + \alpha\ (\theta - \theta_0)]$

$\therefore\ 162 = 18[1 + 4.5 \times 10^{-3}\ (\theta - 293)]$

$$\therefore \frac{9-1}{4.5 \times 10^{-3}} = \theta - 293$$

$\therefore\ \theta = 2070.7\ K$

3.8 Electromotive Force and Terminal Voltage of a Cell

We have seen that current is constituted due to the motion of a charged particles. In order to bring them in motion, force must be exerted on them, in other words energy has to be supplied to the charged particles. The device, which serves the above purpose is called the source of electromotive force i.e. "emf". There are many ways in which force can be exerted on the charge. For example, the force exerted on the charge in an electric cell is due to the chemical processes, The force can be exerted on the charge due to the varying magnetic field and by temperature difference. All the above mentioned devices are the source of emf. A battery (cell) is also a source of emf. What does this emf mean ? We shall consider the example of an electric cell in order to understand the cell.

Figure 3.11 shows a schematic diagram of a battery. There are positive and negative charges present in the chemical of a battery. Due to certain chemical reactions occurring in the battery, force is exerted on these charges. Such a force is called chemical force or non-electrical force F_n. This force (F_n) drives positive charges towards one terminal (i.e. positive terminal) A and drives the negative charges towards the other terminal (i.e. negative terminal) B.

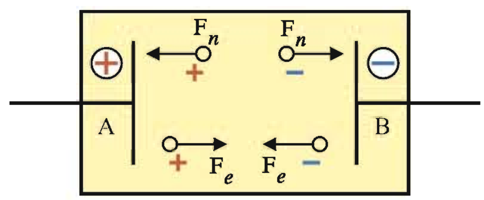


Figure 3.11 Schematic diagram of a battery

As the positive and negative charges build up on the positive and negative terminals A and B respectively, a potential difference (or electric field $\vec{E}$) is set up between them, which keep on increasing gradually. As a result of this, electric force $\vec{F_e}$ ($= q\vec{E}$) is exerted on the charge q in the direction opposite to $\vec{Fn}$. In the steady state, the charges stop accumulating further at the terminals A and B and $F_n = F_e$.

The work done by the non-electrical force in taking a unit positive charge from a negative terminal to the positive terminal is equal to $W = \int_{line} \vec{F_n} \cdot \vec{dl}$, where the line intergral is from negative to positive terminal. As per the definition of an emf, this work done is equivalent to the emf. Therefore the definition of emf can be given as follows.

When unit positive charge is driven from negative to positive terminal due to non-electrical forces, the energy gained by the charge (or work done by the non-electrical forces) is called an emf (ε) of a battery. The unit of emf is $\frac{\text{joule}}{\text{coulomb}}$ = volt. (in the memory of great scientist Volta) Remember that emf is not a force but energy per unit charge.

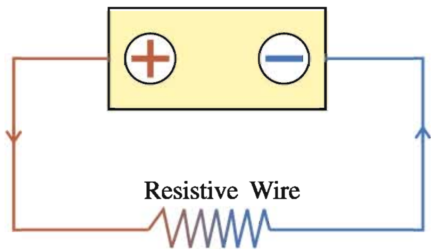


Figure 3.12 Terminal Voltage of a Battery

In the steady state of a battery (when $F_n = F_e$), the electric charge in the battery does not execute any motion i.e. no current is flowing through the battery. (I = 0). In this condition, battery is said to be in **open circuit condition.**

Let us consider a wire of resistance R connected across the two terminals of a battery as shown in figure 3.12. The electric field is thus established in the wire. As a result, positive charges which are at higher potential will move towards the negative terminal of a battery through the wire and constitute an electric current. The question then arises as to why didn't the positive charges move towards the negative terminal (inside the battery) rather than the longer route of the wire ? The reason for this is the non-electrical forces which oppose the motion of positive charges towards the negative terminal of the battery inside the battery.

The energy of the positive charge is consumed against the resistance of the wire. As it reaches the negative terminal, its energy becomes zero. This happens at every rotation of its motion.

During the flow of current, the positive charge is moving from negative terminal to the positive terminal because of non-electrical forces. During the motion, the charge has to pass through the chemical materials of a battery. In other words battery offers a resistance to the charge which is called the internal resistance (r) of a battery.

Due to this internal resistance (r), when a unit positive charge reaches to positive terminal, some part of its energy (which is gained due to the work done by the non-electrical forces.) is consumed against the internal resistance. If the current through the battery is I, then the energy consumed per unit charge against internal resistance = Ir.

Therefore, the energy of a unit positive charge at the positive terminal of battery is less by an amount Ir compared to the energy (ε) in the open circuit condition. Thus, the net energy per unit charge will be ($\varepsilon - Ir$). Thus, during the flow of current this energy is called the potential difference between two terminals of a battery or the terminal voltage (V) of a battery.

$\therefore V = \varepsilon - Ir$

3.8.1 Secondary Cell : Lead Accumulator

Electrochemical cells are of two types.

(1) Primary Cell : The cells which get discharged only are called primary cells. e.g. Voltaic cell. Primary cells cannot be recharged.

(2) Secondary Cell : The cells which can be restored to original condition by reversing the chemical processes (i.e. by recharging) are called secondary cells.

In a secondary cell, one can pass current in both directions.

(i) When (conventional) current leaves the cell at the positive terminal and enters the cell at the negative terminal, the cell is said to be discharging. This is the normal working of the cell during which chemical energy is converted into electrical energy.

(ii) If the cell is connected to some other source of larger emf, current may enter the cell at the positive terminal and leave it at the negative terminal. The electrical energy is then converted into chemical energy and the cell gets charged.

The most commonly used secondary cell is a lead accumulator.

Lead Accumulator : A lead accumulator consists of electrodes made of PbO_2 and of P*b* immersed in an electrolyte of dilute sulphuric acid (H_2SO_4). PbO_2 acts as the positive electrode and Pb as the negative electrode.

When the cell is in use, (i.e. when the cell is discharging) SO_4^{-2} ions move towards the Pb electrode, give up the negative charge and form $PbSO_4$ there. The H^+ ions move to the PbO_2 electrode, give up the positive charge and reduce PbO_2 to PbO.

The PbO so formed reacts with the H_2SO_4 to form $PbSO_4$ and water.

Thus, $PbSO_4$ is formed at both the electrodes and the concentration of the electrolyte decreases.

The concentration of the electrolyte can be measured by a device called hydrometer. When the cell is fully charged, the specific gravity of an electrolyte is 1.285 and emf of a cell is about 2.1 volt. In the discharged condition, the specific gravity falls to 1.15 and emf may fall to 1.8 V.

Charging : To charge a secondary cell of emf ε, direct current (d.c.) is passed through the cell as shown in figure 3.13. The positive terminal of the cell is connected to positive end of a d.c. source and the negative terminal is connected to negative of d.c. supply (opposing condition) for the charging of a cell. (Here V > E)

Due to the chemical reactions occurring in the cell during charging process, $PbSO_4$ deposited at the two electrodes is dissolved. Pb is deposited at the negative electrode and PbO_2 at the positive electrode, simultaneously H_2SO_4 is also formed. This restores the capacity of the cell to provide current.

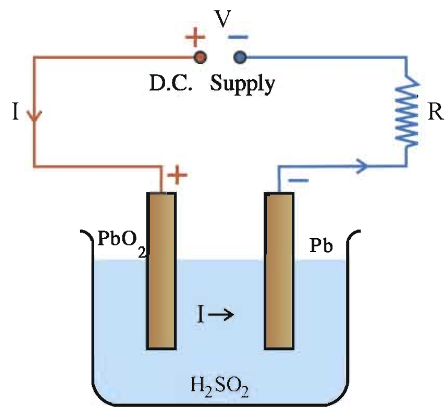


Figure 3.13 Charging of a Secondary Cell

Here, the elecrical energy VI*t* consumed by a *d.c.* source provides εIt energy for the charging of a cell and $I^2Rt + I^2rt$ energy dissipated in the external (series) resistance (R) and internal resistance (r) of a cell.

$$\therefore \ VIt = \varepsilon It + I^2Rt + I^2rt \qquad (3.8.2)$$

$$\therefore \ V = \varepsilon + I(R + r)$$

$$\therefore \ I = \frac{V - \varepsilon}{R + r} \qquad (3.8.3)$$

Above equation gives the charging current. Here, the resistance R is connected to control the current.

Illustration 9 : 6 batteries, each of 2.0 volts are connected in series so that they are helping each other. Internal resistance of each is 0.5 Ω. They are being charged using a direct voltage supply of 110 volts. To control the current, a resistance of 46 Ω. is used in the series. Obtain (1) power drawn from the supply, and (2) power dissipated as heat. Why are the two different ?

Solution : $V = \varepsilon. + Ir + IR$ gives

$$V = 6\varepsilon + 6\ Ir + IR$$

$V = 110$ V

$\varepsilon = 2.0$ V

$r = 0.50\ \Omega$

$R = 46\ \Omega$

Now, $I = \frac{V - 6\varepsilon}{6r + R} = \frac{110 - 12}{6 \times 0.50 + 46} = \frac{98}{49} \quad \therefore I = 2$ A

power drawn from the supply,

$W = V \times I = 110 \times 2 = 220$ W

power dissipated as heat $= 6I^2r + I^2R$

$= I^2(6r + R)$

$= 4 \times (6 \times 0.50 + 46)$

$= 4 \times (3 + 46)$

$= 196$ W

$\therefore$ Difference $= (220 - 196)$ W $= 24$ W. This power is used to charge the batteries.

3.9 Kirchoff's Rules

In different electronic circuits, components like resistors, inductors, capacitors and batteries are connected with each other in a complicated way. Such circuits cannot be considered as a simple series or parallel connections. Generally, such complicated circuits are known as network.

Ohm's law alone is not sufficient to analyze a network. There are several rules for the analysis of a network. Kirchoff's two rules are amongst them.

Let us try to understand the two terms concerning circuits before the discussion of Kirchoff's rules.

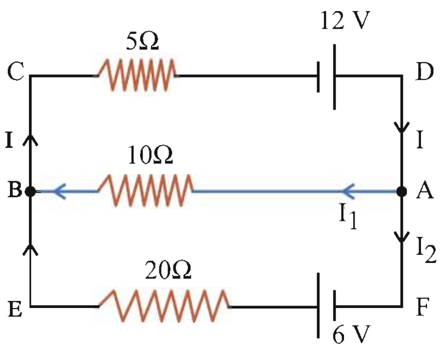


Figure 3.14 Network

Junction or Branch Point : The point in a network at which more than two conductors (minimum three) meet is called a junction or a branch point. (You will be aware that how many railway lines make a junction.) In figure 3.14 three conductors are meeting at points A and B. Therefore, points A and B are called junction or branch points.

Loop : A closed circuit formed by conductors is known as loop. As shown in figure 3.14, CDABC, AFEBA and CDAFEBC are some of the closed path of conductors known as loop.

In the analysis of a network, unknown quantities like V, I, R ...... in a given circuit can be determined from the known quantities.

Kirchhoff's Rules :

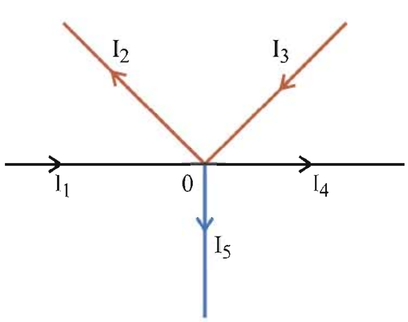


Figure 3.15

Kirchoff's First Rule : Kirchoff's first rule is the consequence of the law of conservation of charge.

Consider junction O of a network as shown in figure 3.15. The currents meeting at the junction point O are represented as I_1, I_2, ..., I_5. Their directions are represented by arrows in the Figure.

Let Q_1, Q_2, ..., Q_5 be electrical charges flowing through the cross-sectional area of the respective conductor in time interval t which constitutes current I_1, I_2, ..., I_5.

Hence, $I_1 = \frac{Q_1}{t} \rightarrow Q_1 = I_1 t$

$I_2 = \frac{Q_2}{t} \rightarrow Q_2 = I_2 t$

$I_5 = \frac{Q_5}{t} \rightarrow Q_5 = I_5 t$

It is evident from the Figure that the total electric charge entering the junction is $Q_1 + Q_3$, while $Q_2 + Q_4 + Q_5$ amount of electric charge is leaving the junction in the same interval of time.

As per the law of conservation of charge,

$Q_1 + Q_3 = Q_2 + Q_4 + Q_5$ (3.9.1)

$\therefore I_1 t + I_3 t = I_2 t + I_4 t + I_5 t$

$\therefore I_1 + I_3 + (-I_2) + (-I_4) + (-I_5) = 0$ (3.9.2)

$\therefore$ At the junction, $\Sigma I = 0$ (3.9.3)

Thus, **"The algebraic sum of all the electric currents meeting at the junctions is zero."** This statement is known as Kirchoff's first rule.

In the above sum I_1 and I_3 currents are positive while I_2, I_4 and I_5 are negative. Thus the electric currents entering the junction are considered as positive and the currents leaving the junction are considered as negative. One can also consider an opposite convention to arrive at the same result.

Kirchoff's Second Rule : Using law of conservation of energy and the concept of electric potential any closed circuit can be analyzed. Kirchoff's second rule is the essence of the above mentioned concepts. Let us consider a closed path ABCDEA as shown in figure 3.16.

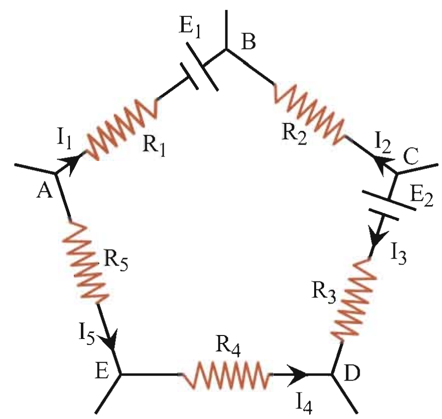


Figure 3.16

Here, resistors R_1, R_2, R_3, R_4, R_5 and batteris of emf's ε_1 and ε_2 form a closed loop ABCDEA. If the internal resistance of a battery is ignored, the rise in the electric potential while going from negative to positive terminal of a cell is equal to the emf (ε) of a battery. The potential difference across the ends of a resistor is equal to the product of the resistor and the current flowing through it ($V = IR$).

The electric potential at any point in a steady circuit does not change with time. If V_A is the electric potential at point A, and if we measure the changes in the electric potential while moving in clockwise or anticlockwise direction in a closed circuit and come back to point A, the potential V_A should remain unchaged. This is called the singlevaluedness of the electric potential. In fact, the singlevaluedness of the electric potential is a consequence of the law of conservation of energy.

The electric potential drops by an amount I_1R_1 when we move in a clockwise direction from A through the resistor R_1. Here, the direction of current is arbitrarily taken from A to B i.e. current flows through resistor R_1 from a point of higher potential (A) to lower potential. Hence there will be a drop in potential equal to I_1R_1 as we move from A to B. There is a rise in the potential ε_1 while going from the negative terminal to the positive terminal of a battery of emf ε_1. The potential rises by I_2R_2 when we go from B to C through resistor R_2. As the direction of current is assumed from C to B, the electric potential of point C is higher than B. Therefore, potential rises by an amount I_2R_2 while going from B to C.

In a similar way, there is a decrease in potential equal to ε_2 when we go from positive to negative terminal of a battery of emf ε_2. There is a potential drop I_3R_3 while passing through R_3, rise in potential I_4R_4 through R_4 and rise in potential I_5R_5 through R_5.

Taking the algebraic sum of all these changes, the potential at point A should remain V_A.

$\therefore\ V_A - I_1R_1 + \varepsilon_1 + I_2R_2 - \varepsilon_2 - I_3R_3 + I_4R_4 + I_5R_5 = V_A$

$$- I_1R_1 + \varepsilon_1 + I_2R_2 - \varepsilon_2 - I_3R_3 + I_4R_4 + I_5R_5 = 0 \quad (3.9.4)$$

Thus, the algebraic sum of all the changes in potential around any closed loop is zero.

$$\therefore\ (-I_1R_1) + I_2R_2 + (-I_3R_3) + I_4R_4 + I_5R_5 = (-\varepsilon_1) + \varepsilon_2 \quad (3.9.5)$$

$$\therefore\ \Sigma IR = \Sigma\varepsilon \quad (3.9.6)$$

This equation suggests that **"for any closed loop the algebraic sum of the products of resistances and the respective currents flowing through them is equal to the algebraic sum of the emf's applied along the loop."** This statement is known as Kirchhoff's second rule.

Sign convention for applying Kirchoff's rules :

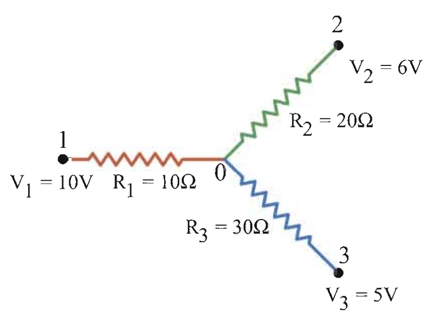
The following sign convention has to be followed in using equation (3.9.5).

(1) If our journey through the resistor is in the direction of flow of current which is arbitrarily chosen, IR should be considered negative and if the direction of journey and the direction of current is opposite to each other IR should be considered as positive.

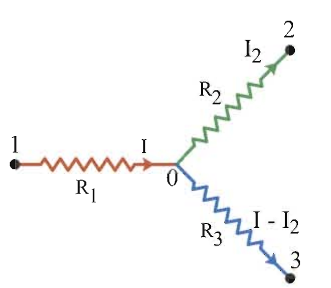
(2) The emf of a battery should be considered negative while moving from negative terminal of a battery to the positive terminal (while writing on the right hand side of the equation.) The emf of a battery is taken as positive while moving from positive to negative terminal of the battery.

The direction of the electric current can be arbitrarily chosen while using Kirchoff's rules to analyze any network. We shall get negative value of the current if the direction of current which is arbitrarily chosen is opposite to the actual direction of current.

Illustration 10 : Calculate the current flowing through the resistor R_1 in the given circuit. $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$ and $R_3 = 30\ \Omega$. The potentials of the points 1, 2 and 3 are respectively, $V_1 = 10$ V, $V_2 = 6$ V and $V_3 = 5$ V. Calculate the potential at the junction.



Solution : O is the junction point in the above circuit. The potential at point 1 is higher than the potential existing at point 2 and 3. Hence, the direction of the flow of the current is from point 1 to O, from O to 2 and from O to point 3. The Figure indicates the electric current and their direction.



Now for the 1O2 path, we have,

$V_1 - IR_1 - I_2R_2 = V_2$

$\therefore\ 10 - 10I - 20I_2 = 6$

$$\therefore\ 10I + 20I_2 = 4 \quad (1)$$

For the 1O3 path, we have,

$10I + 30\ (I - I_2) = 5$

$$\therefore\ 40I - 30I_2 = 5 \quad (2)$$

Solving equation (1) and (2), we have,

$I = 0.2$A

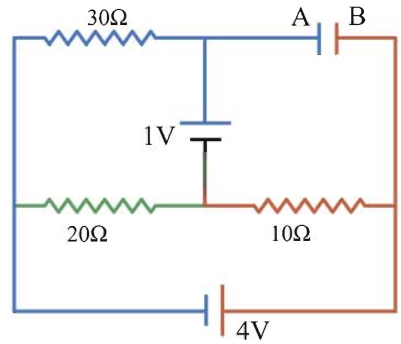
Let V_O, be the potential at point O, then

$10 - V_O = IR_1$

$\therefore\ 10 - V_O = 2$

$\therefore\ V_O = 8$ V

Illustration 11 : Calculate the potential difference between the plates A and B of the capacitor in the adjacent circuit.



Solution : The distributions of the current are shown in figure.

Applying Kirchhoff's Second Law to the closed loop abcdea, we have

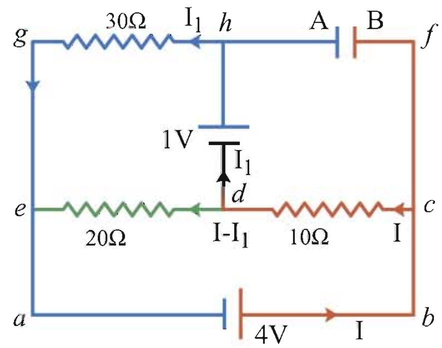
$- 10 I - 20 (I - I_1) + 4 = 0$

$\therefore 30 I - 20 I_1 = 4$ (1)

For the cdhge loop,

$20(I - I_1) + 1 - 30I_1 = 0$

$\therefore 20I - 50I_1 = -1$ (2)



Solving equation (1) and (2), we have,

$I_1 = 0.1$ A and $I = 0.2$ A.

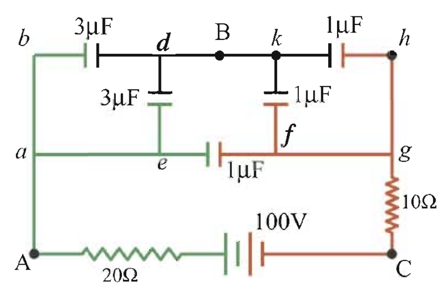
The p.d. between the two plates of the capacitor is equal to the p.d. between c and h point. Let V_c be the potential at point c and let V_h be the potential at point h. For the path cdh, we have

$\therefore V_c - 10 \times 0.2 + 1 = V_h$

$\therefore V_c - V_h = 2 - 1 = 1$

$\therefore$ The potential difference between the two capacitors = 1 V

Illustration 12 : Calculate the potential difference between points A and B as well as between, points C and B under a steady condition of the circuit shown in the figure.



Solution : e (or a or b) and d are the two ends of the capacitor 3 μF. 'k and g (or h or f) are the two ends of the capacitor 1 μF.

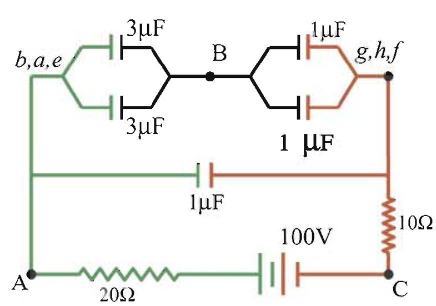
The equivalent circuit of the above circuit can be given as under :

There are two 3 μF capacitors connected in parallel.

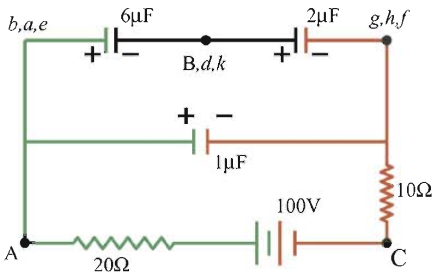
$\therefore$ Their equivalent capacitors = 6 μF

In a similar way equivalent capacitors of two 1 μF capacitors = 2 μF.

The above situation is represented in the figure below.



Since the circuit is in the steady state, no current flows through 20 Ω and 10 Ω resistances. It seems as if these resistors are not connected in the circuit. In this situation, the voltage of the battery (100 V) is applied between point a and h. The 6 μF and 2 μF capacitors are connected in a series combination between the two ends of the battery.



If the electrical charge on 6 μF and 2 μF capacitors is equal to q, then

$$V_1 + V_1 = V$$

$$\frac{q}{C_1} + \frac{q}{C_2} = V, \ \frac{q}{6} + \frac{q}{2} = 100$$

$$\therefore q = \frac{100\times12}{8} = 150 \ \mu C$$

Now, the potential difference between points A and B are equal to voltage developed across 6 μF capacitors,

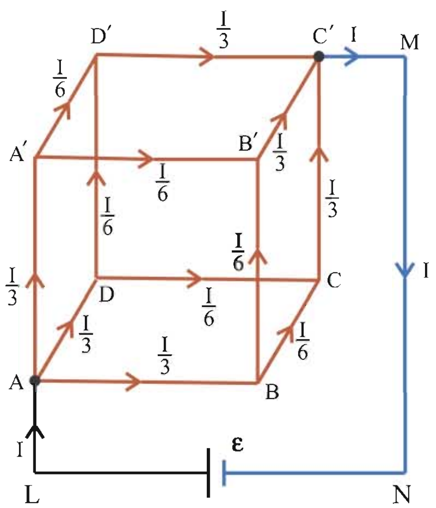
$$\therefore V_{AB} = \frac{150}{6} = 25 \ V$$

Now, the voltage between B and C

$V_{BC} = 100 - 25 = 75$ V

Illustration 13 : A cube is made by connecting 12 wires of equal resistance R. Find the equivalent resistance between any two of its diagonally opposite points.

Solution : Let I be the current through the cell.



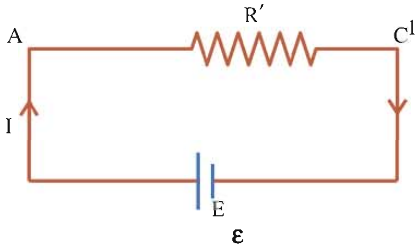
Since the paths AB, AD and AA' are symmetrical with respect to resistors, current through each of them is same (i.e. $\frac{I}{3}$). At the junctions B, D and A' the incoming current $\frac{I}{3}$ splits equally into the two outgoing branches, the current through each branch is $\frac{I}{6}$, as shown in Figure. At the junctions C, B' and D' these currents reunite and the currents along CC', B'C' and D'C' are $\frac{I}{3}$ each. These three currents reunite at the junction C' and the total current at junction C' is I again.

Applying Kirchoff's second rule to the closed loop AA'D'C'MNLA,

$$-\frac{I}{3}\cdot R - \frac{I}{6}R - \frac{I}{3}R = -\varepsilon$$

$$\therefore \varepsilon = \frac{5}{6}IR \qquad (1)$$

Let the equivalent resistance between two diagonally opposite points A and C' be R' this means that if R' is connected across the same battery (of emf ε) in place of the given network, the current I should remain same.



From the equivalent circuit shown in Figure,

$$\varepsilon = IR' \qquad (2)$$

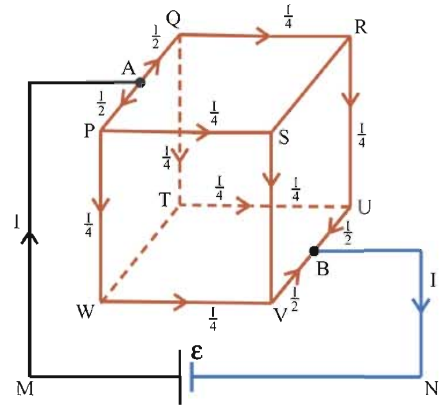
Comparing equation (1) and (2),

$$\frac{5}{6}IR = IR'$$

$$\therefore R' = \frac{5}{6}R$$

Illustration 14 : A cube is constructed by connecting 12 wires of equal resistance as shown in Figure. Find the equivalent resistance between the points A and B shown in the Figure. The resistance of each wire is of r Ω. A and B are the midpoints of the sides PQ and VU respectively.

Solution : Note that with reference to the line joining A and B, the pairs AP and UB, AQ and VB, PW and RU, QT and SV, WV and QR are symmetric branches. Hence current flowing through each of this symmetric pair must be same. e.g. if the current flowing through PW is $\frac{I}{4}$, the same current (i.e. $\frac{I}{4}$) will flow through RU. With this consideration the proportional currents through the various circuit branches are as assigned their values in figure.



Points W and T being symmetric about A are at the same potential, so no current will flow through WT and similarly also through SR.

Appyling Kirchoff's second rule to the closed loop APWVBNMA, taking r as the resistance of each wire.

$$-\frac{I}{2}\left(\frac{r}{2}\right) - \frac{I}{4}r - \frac{I}{4}r - \frac{I}{2}\left(\frac{r}{2}\right) = -\varepsilon$$

$\therefore$ IR = ε (1)

If the equivalent resistance is r',

then $Ir' = \varepsilon$ (2)

Comparing equations (1) and (2),

$r' = r$

3.10 Series and Parallel Connections of Resistors

Resistors can be connected in series or parallel or a mixed combination of both the types between any two points. You have studied the series and parallel connections of resistors in Standard–X. Here we will make note of their results.

Series Combination of Resistors

Resistors are said to be connected in series between any two points, if the same current is flowing through each resistor or in other words, there is only one path available for the flow of current.

Figure 3.17 shows the series connection of n resistors R_1, R_2, R_3, ..., R_n between two points A and B.

If the equivalent resistance of the series connection is R_s, then,

$$R_s = R_1 + R_2 + R_3 + ... + R_n = \sum_{i=1}^{n} R_i \quad (3.10.1)$$

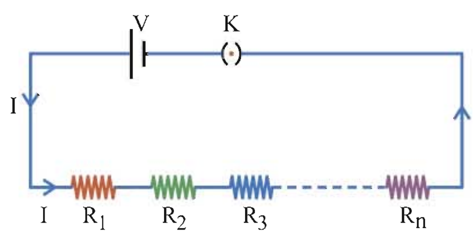


Figure 3.17 Series Connection of Resistors

Thus, the equivalent resistance of the series combination of the resistors is always greater than the greatest value of the resistors connected in series.

If n identical resistors each of value R are connected in series, the equivalent resistance is,

$$R_s = R + R + R + ... \ n \text{ times} = nR \quad (3.10.2)$$

Parallel Connection of Resistors : The resistors are said to be connected in parallel between two points if there are more than one path available for the flow of current and potential difference (V) across each of them is same.

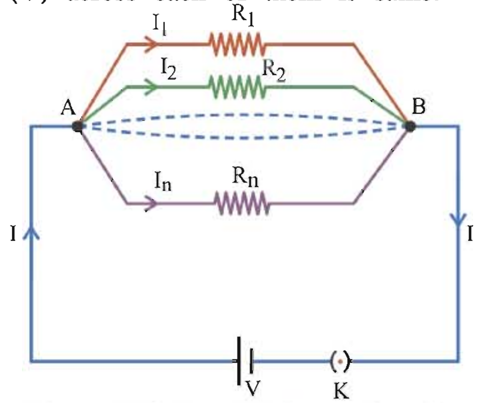


Figure 3.18 Parallel Connection of Resistors

In figure 3.18, n resistors R_1, R_2, R_3, ..., R_n are connected in parallel between two points A and B.

If the equivalent resistance of this parallel connection is R_P, then

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$

(3.10.3)

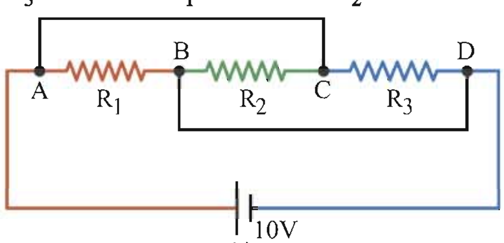
Thus, the equivalent resistance of a parallel combination of resistors is always smaller than the smallest value of resistors connected in parallel.

If n identical resistors having resistance R are connected in parallel, the equivalent resistance is,

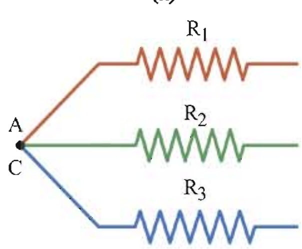
$$\frac{1}{R_P} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + ... n \text{ times} = \frac{n}{R}$$

$$\therefore R_P = \frac{R}{n} \tag{3.10.4}$$

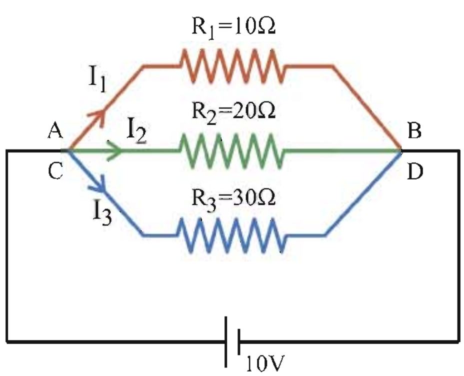
Illustration 15 : As shown in the figure (a), some current flows through resistors R_1, R_2 and R_3 resistors. $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$ and $R_3 = 30\ \Omega$ and the battery voltage is equal to 10 V.



(a)



(b)



(c)

Solution : Let us start from point A in order to obtain the equivalent circuit of the above given circuit. Here one end of the resistor R_1 is connected to point A. The common end of resistor R_2 and R_3 (point C) is connected to point A.

$\therefore$ The circuit (b) resembles partially the above circuit.

Similarly, the other end of the resistor R_1 and the common end of resistor R_2 and R_3 are connected to point B.

$\therefore$ The entire circuit can be represented by figure (c).

Hence, we have a situation in which 3 resistors are connected in parallel as shown in figure (c).

$\therefore$ The voltage developed across the two ends of each resistor will be equal to 10 V.

$\therefore$ Therefore, the current flowing through R_1, $I_1 = \frac{V}{R_1}$

$= \frac{10}{10} = 1A$, similarly the current flowing through R_2,

$I_2 = \frac{V}{R_2} = \frac{10}{20} = 0.5A$ and current flowing through R_3

$I_3 = \frac{V}{R_3} = \frac{10}{30} = 0.33$ A.

Illustration 16 : An electric current of 5A is divided in three branches forming a parallel combination. The lengths of the wires in the three branches are in the proportion 2 : 3 : 4 and their radii are in the proportion 3 : 4 : 5. Find the currents in each branch if the wires are of the same material.

Solution : Let the lengths of the wires be $2l$, $3l$ and $4l$ and their radii be $3r$, $4r$ and $5r$ respectively. Their respective resistances are,

$$R_1 = \rho.\frac{2l}{\pi(3r)^2}$$

$$R_2 = \rho.\frac{3l}{\pi(4r)^2}$$

and $$R_3 = \rho.\frac{4l}{\pi(5r)^2}$$

or, $R_1 : R_2 : R_3 = \frac{2}{9} : \frac{3}{16} : \frac{4}{25}$

The currents must be in the inverse proportion of resistances.

$$\therefore I_1 : I_2 : I_3 = \frac{9}{2} : \frac{16}{3} : \frac{25}{4}$$

$$= 54 : 64 : 75$$

$\therefore$ Current in the first branch $I_1 = \frac{54\times5}{193} = 1.40$ A

Current in the second branch $I_2 = \frac{64\times5}{193} = 1.66$ A

Current in the third branch $I_3 = \frac{75\times5}{193} = 1.94$ A

3.11 Series and Parallel Connections of Cells

Like resistors, cells can also be connected in series, parallel and combination of both between two points.

Cells in Series :

Suppose two cells having emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in series between two points A and B as shown in figure 3.19. An external resistance R is also connected across the connection.

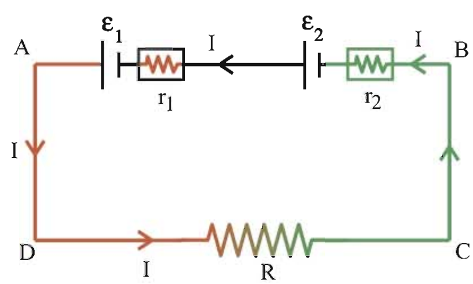


Figure 3.19 Series Connections of Cells

Applying Kirchhoff's second rule for the closed loop ABCDA,

$$-\varepsilon_1 + Ir_1 - \varepsilon_2 + Ir_2 + IR = 0$$

$$\therefore Ir_1 + Ir_2 + IR = \varepsilon_1 + \varepsilon_2$$

$$\therefore I[R + (r_1 + r_2)] = \varepsilon_1 + \varepsilon_2$$

$$\therefore I = \frac{\varepsilon_1+\varepsilon_2}{R+(r_1+r_2)} = \frac{\varepsilon_{eq}}{R+r_{eq}} \qquad (3.11.1)$$

where, I is the current through the resistor R.

Thus, the series combination of two cells acts as a single cell of emf $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ and internal resistance $r_{eq} = r_1 + r_2$. In this sense ε_{eq} is an equivalent emf and r_{eq} is the equivalent internal resistance of the series connection of cells.

If the polarity of one of the cells is reversed, the equivalent emf will be $|\varepsilon_1 - \varepsilon_2|$ but the equivalent internal resistance will remain $r_1 + r_2$.

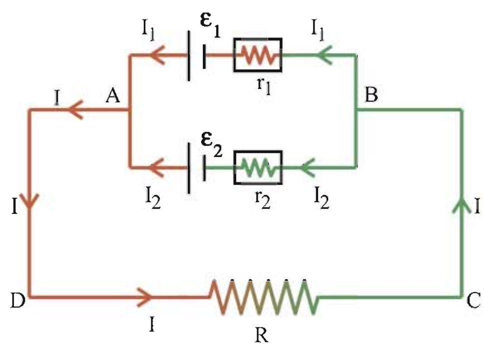


Figure 3.20 Parallel Connection of Cells

Cells in Parallel :

As shown in figure 3.20, suppose two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in parallel between two points A and B. The currents are also shown in the Figure.

We are interested in finding the current flowing through external resistor R.

At junction A, according to Kirchhoff's first rule,

$$I = I_1 + I_2 \tag{3.11.2}$$

Applying Kirchhoff's second rule to the closed loop ADRCBε_1A,

$$-IR - I_1 r_1 + \varepsilon_1 = 0$$

$$\therefore\ IR + I_1 r_1 = \varepsilon_1$$

$$\therefore\ I_1 = \frac{\varepsilon_1 - IR}{r_1} \tag{3.11.3}$$

Similarly for the closed loop ADRCBε_2A, we have

$$I_2 = \frac{\varepsilon_2 - IR}{r_2} \tag{3.11.4}$$

Substituting the values of I_1 and I_2 from equations (3.11.3) and (3.11.4) in equation (3.11.2), we have,

$$I = \left(\frac{\varepsilon_1 - IR}{r_1}\right) + \left(\frac{\varepsilon_2 - IR}{r_2}\right)$$

$$\therefore\ I = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - IR\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\therefore\ I + IR\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

$$\therefore\ I\left(1 + \frac{R}{r_1} + \frac{R}{r_2}\right) = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

$$\therefore\ I = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{1 + \frac{R}{r_1} + \frac{R}{r_2}} \tag{3.11.5}$$

$$\text{or, } I = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{R(r_1 + r_2) + r_1 r_2} \tag{3.11.6}$$

Dividing numerator and denominator of equation (3.11.6) by $(r_1 + r_2)$,

$$I = \frac{\frac{(\varepsilon_1 r_2 + \varepsilon_2 r_1)}{(r_1 + r_2)}}{R + \frac{r_1 r_2}{(r_1 + r_2)}} = \frac{\varepsilon_{eq}}{R + r_{eq}} \qquad (3.11.7)$$

Thus, the parallel combination of cells acts as a single cell whose emf is,

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \qquad (3.11.8)$$

and internal resistance is,

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \qquad (3.11.9)$$

$$\therefore \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \qquad \text{(from equation 3.11.9)} \qquad (3.11.10)$$

Taking the ratio of equations (3.11.8) and (3.11.9), we get

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \qquad (3.11.11)$$

If emf's of two cells are $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and internal resistances are $r_1 = r_2 = r$, then, $\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{2}$,

In figure 3.20, we had joined the positive terminals together (at point A) and similarly the two negative ones (at point B), so that the currents I_1 and I_2 flow out of positive terminals. If the negative terminal of the second is connected to positive terminal of the first, equations (3.11.10) and (3.11.11) would still be valid with $\varepsilon_2 \to -\varepsilon_2$

If there are n cells of emf $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$ and of internal resistances $r_1, r_2, ..., r_n$ respectively connected in parallel, the combination is equivalent to a single cell of emf ε_{eq} and internal resistance r_{eq}, such that

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n} \qquad (3.11.12)$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + ... + \frac{\varepsilon_n}{r_n} \qquad (3.11.13)$$

$$\text{and } I = \frac{\sum_{i=1}^{n} \frac{\varepsilon_i}{r_i}}{1 + R\sum_{i=1}^{n} \frac{1}{r_i}} \qquad (3.11.14)$$

If n cells of emf $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$ and internal resistances $r_1, r_2, ..., r_n$ are connected in series to form a row and m such rows are connected in parallel, the current in such connection (which is called mixed connection) is given by following formula.

$$I = \frac{\sum_{i=1}^{n} \varepsilon_i}{R + \frac{1}{m}\sum_{i=1}^{n} r_i} \qquad (3.11.15)$$

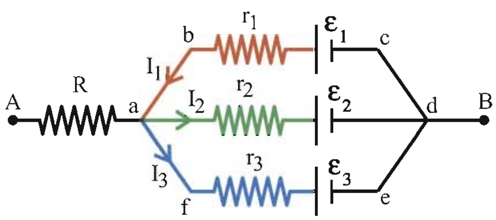
R = external resistance connected across the mixed connection

m = number of rows

n = number of cells in a row.

Illustration 17 : In the circuit shown in Figure, $\varepsilon_1 = 3V$, $\varepsilon_2 = 2V$, $\varepsilon_3 = 1V$ and $R = r_1 = r_2 = r_3 = 1\Omega$.. Find the current through each branch and potential difference between the points A and B.

Solution : Let I_1, I_2 and I_3 be the currents through the resistors r_1, r_2 and r_3 respectively as indicated in figure. Using Kirchhoff's second rule to loops abcda and abcdefa, we have,



$+I_1r_1 - \varepsilon_1 + \varepsilon_2 + I_2r_2 = 0$ .....(1)

and $I_1r_1 - \varepsilon_1 + \varepsilon_3 + I_3r_3 = 0$ .....(2)

From equation (1) and (2),

$\varepsilon_1 - I_1r_1 = \varepsilon_2 + I_2r_2 = \varepsilon_3 + I_3r_3$ .....(3)

Appyling Kirchhoff's first rule to junction a, we have

$I_1 = I_2 + I_3$ .....(4)

Using equation (4) in (3), we get

$\varepsilon_1 - (I_2 + I_3)r_1 = \varepsilon_3 + I_3r_3$

or, $2I_3 + I_2 = 2$ .....(5)

Also $\varepsilon_2 + I_2r_2 = \varepsilon_3 + I_3r_3$

or, $I_3 - I_2 = 1$ .....(6)

From equations (4), (5) and (6),

$I_1 = 1A$, $I_2 = 0$ A and $I_3 = 1A$

Potential difference between A and B

= Potential difference between *a* and *d*

$= \varepsilon_1 - I_1r_1$

$= 3 - 1 \times 1$

$= 2V$

3.12 Wheatstone Bridge

In 1843, Charles Wheatstone developed a circuit to measure unknown resistor with reference to standard known resistance. This circuit is known as Wheatstone bridge. Wheatstone bridge network is shown in figure 3.21. The bridge has four resistor arms R_1, R_2, R_3 and R_4 connected to form a closed loop. The source of emf (battery) is connected between A (common point of R_1 and R_3) and C (common point of R_2 and R_4) and sensitive galvanometer is connected between B (common point of R_1 and R_2) and D (common point of R_3 and R_4)

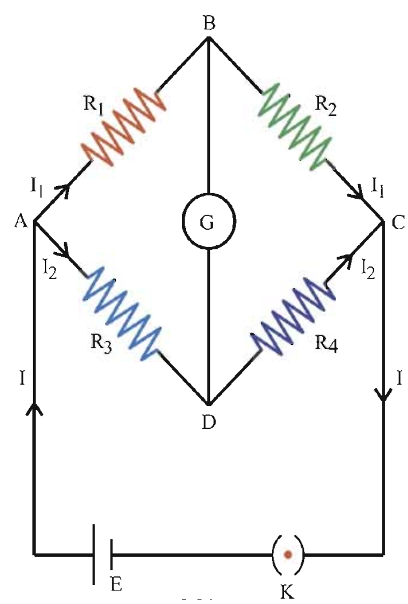


Figure 3.21 Wheatstone Bridge

Three resistors out of the four are known and the fourth one is unknown. The three resistors are chosen in such a way that galvanometer shows zero deflection. In this condition the potential at point B and D are same hence there will be no flow of current through the galvanometer. This condition of Wheatstone bridge is said to be balanced condition.

Applying Kirchhoff's second rule to loop ABDA in a balanced condition,

$-I_1R_1 + I_2R_3 = 0$

$\therefore\ I_1R_1 = I_2R_3$ (3.12.1)

Similarly, applying Kirchhoff's second rule to the loop BCDB,

$-I_1R_2 + I_2R_4 = 0$

$\therefore\ I_1R_2 = I_2R_4$ (3.12.2)

Dividing equation (3.12.1) by equation (3.12.2) we have,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad (3.12.3)$$

By knowing three resistors, the fourth unknown resistance can be found.

Meterbridge :

Meterbridge is the simplest practical device based on the principle of Wheatstone bridge. It is used to measure an unknown resistance experimentally. The Meterbridge used in the laboratory is shown in figure 3.22.

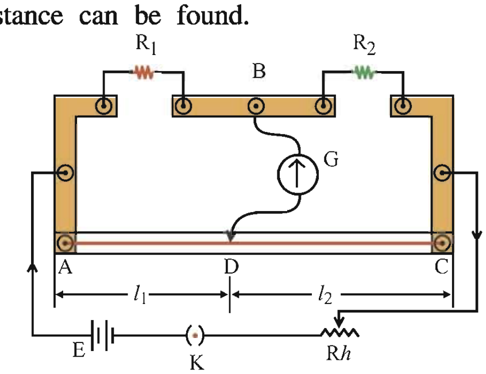


Figure 3.22 Meterbridge

Meterbridge consists of a constantan wire of length 1 m and of uniform cross sectional area which is used in place of resistors R_3 and R_4. This wire is stretched taut and clamped on a meterscale which is mounted on a wooden plateform. Two thick copper strips bent at right angles are connected at two ends A and C of the wire as shown in Figure. The connecting terminals are provided on this metallic strip (at the end points of a wire) where a battery can be connected. Another copper strip is fixed between two thick copperstrips in such a way that the metallic strip has two gaps across which resistors can be connected. One end of a sensitive galvanometer is connected to the copper strip midway (at point B) between the two gaps. The other end of the galvanometer is connected to a 'jockey' D which can slide over the wire to make electrical connection.

As shown in figure 3.22 unknown resistance R_1 is connected across one of the gaps and a standard known resistance R_2 is connected across the other gap.

For one value of known resistor R_2, the jockey is slided along the wire to get the position (say D) where the galvanometer will show no current. Point D is called balance (null) point.

Let the distance of the jockey from the end A at the balance point is AD = l_1 and the length of the wire DC is l_2, then from equation (3.12.3), we have,

$$\frac{R_1}{R_2} = \frac{\text{Resistance of Wire AD}}{\text{Resistance of Wire DC}}$$

$$\frac{R_1}{R_2} = \frac{l_1\rho}{l_2\rho} = \frac{l_1}{l_2} \quad (3.12.4)$$

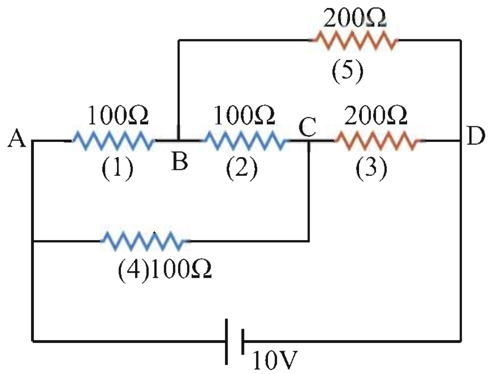
where, ρ = resistance per unit length of the wire

$$\frac{R_1}{R_2} = \frac{l_1}{(100-l_1)}$$

$$\therefore\ R_1 = R_2 \cdot \frac{l_1}{(100-l_1)} \quad (3.12.5)$$

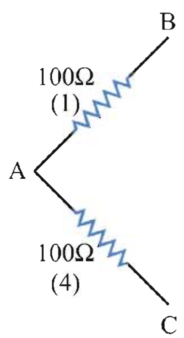
By choosing various values of known resistance R_2, the value of $\frac{l_1}{l_2}$ is calculated each time and the average value of unknown resistance R_1 can be found. The above method gives accurate valve of unknown resistance R_1 but this method is not useful for the measurement of small resistance.

Illustration 18 : Calculate the current flowing through the BC wire for the given circuit (a) shown here.

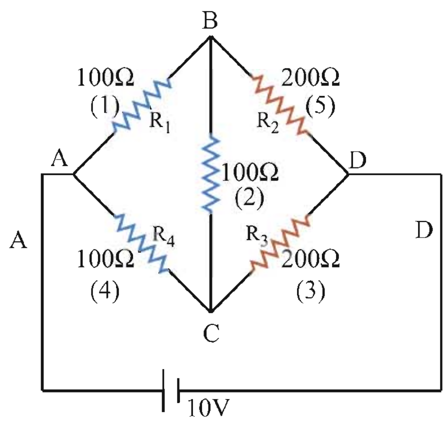


(a)

Solution : Kirchhoff's Law can be used to solve the above problem. We shall redraw the above circuit in a different way. The four points ABCD are common to two different resistances. Let us start from point A. At point A one end of resistances (1) and (4) is common. (Figure (b)). A 100 Ω resistance is connected between B and C. Resistance 200 Ω is connected between points C and D (resistance 3). A 200 Ω resistance present between B and D (resistance 5). The original circuit can be redrawn as shown in the figure (c).



(b)



(c)

In the above circuit a 10 V battery is connected between points A and D.

We form a close loop when we go from point A to B to D to C and back to A. A battery is connected between points A and D.

Under the balanced condition of Wheatstone bridge no current will flow from point B to C, since

$\frac{R_1}{R_2} = \frac{R_4}{R_3}$ condition is satisfied.

$\therefore$ No current flows through the resistor connected between the points B and C.

$\therefore$ $I_{BC} = 0$

Illustration 19 : 200 Ω resistor is connected in one of the gaps of the Meterbridge. Series combination of X Ω and 50 Ω resistors is connected in the second gap. Here unknown resistance X Ω is kept in a Heat bath at a certain temperature. Calculate the unknown resistance and its temperature if the balance point is obtained at 50 cm. The total length of the wire of the Meterbridge is equal to 1 meter. The resistance of the unknown resistance at 0 °C temperature is equal to 100 Ω $\alpha = 0.5 \times 10^{-3}$ $°C^{-1}$ for the material of the X Ω resistors.

Solution : Here, we have $\frac{R_1}{R_2} = \frac{l_1}{l_2}$

$\therefore \frac{200}{X+50} = \frac{50}{50}$ $\qquad R_1 = 200\ \Omega$

$\therefore X = 150\ \Omega$ $\qquad R_2 = (X + 50)\Omega$

Now, $X = X_0[1 + \alpha(\theta - 0)]$ $\qquad l_1 = 50$ cm

$\therefore 150 = 100[1 + 5 \times 10^{-3}\theta]$ $\qquad l_2 = 100 - 50 = 50$ cm

$\therefore 1.5 = 1 + 5 \times 10^{-3}\theta$

$\therefore \theta = 100\ ^{\circ}C$

Note : We can understand from the above example that temperature of the resistor can be measured using wheatstone bridge. (The varying temperature of the resistor can also be measured.)

The thermometer can be constructed by knowing the relationship between resistance and temperature. Such a thermometer is known as **resistance thermometer.** The manufacturer of such a thermometer provides us with R $\rightarrow$ T graph. Presently thermometer with digital display is available. The resistance thermometer is an example of a transducer. In a transducer the physical quantity is converted into an electrical quantity or vice-versa.

3.13 Potentiometer

(A) The Requirement of Potentiometer : We have seen that the terminal voltage of a battery is given by,

$$V = \varepsilon - Ir \qquad (3.13.1)$$

where ε = emf of battery

and r = internal resistance of a battery.

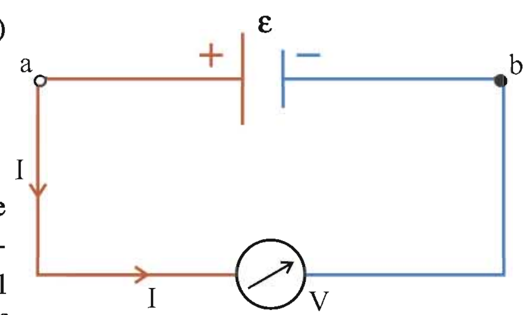


Figure 3.23

As shown in figure 3.23 if voltmeter used in the laboratory (table voltmeter) is connected across two terminals (between points a and b) of a battery, then it will measure the potential difference between two terminals of a battery or a terminal voltage (V).

The equation (3.13.1) reduces to $V = \varepsilon$ when the internal resistance of a battery is zero (i.e. $r = 0$) or no current flows through the battery (i.e. $I = 0$). But the internal resistance (r) of a battery can never be zero. Therefore voltmeter can measure the emf (ε) of a battery only if no current is drawn from it. (i.e. $I = 0$ open circuit condition.)

The resistance of an ordinary voltmeter is approximately in the range of 5000 Ω to 6000 Ω. Hence a small amount of current flows through the battery when connected to voltmeter. **This means that the voltmeter measures only the terminal voltage (V) and not the emf (ε) of a battery.**

Thus, in order to measure the emf of a battery we have to design a new device in which open circuit condition ($I = 0$) is achieved. Such a device is called potentiometer.

Potentiometer is such a device in which one can obtain a continuously varying potential difference between any two points which can be measured simultaneously. This can be understood from the principle of potentiometer.

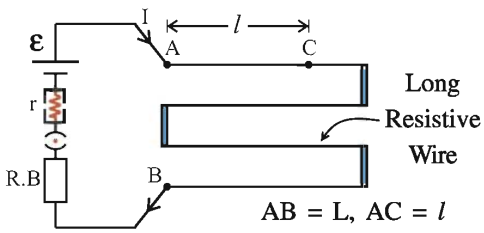


Figure 3.24 Principle of Potentiometer

(B) Principle of Potentiometer : As shown in figure 3.24 a battery of emf ε and having internal resistance r is connected in series with a resistance box R and a long resistive wire of uniform cross-sectional area (i.e. the resistance per unit length of the wire is same throughout the length of the wire.) It is to be noted that the resistance box R is not always necessary to connect.

(**Note :** In potentiometer a long piece of uniform wire few meters in length across which a battery is connected is clamped on a meterscale which is mounted on a wooden plateform.)

Let L be the length of the potentiometer wire AB and ρ be the resistance per unit length of the wire. Therefore the resistance of the wire AB = Lρ. If R is the resistance of the resistance box then the current flowing through wire AB can be given by Ohm's law as follows.

$$I = \frac{\varepsilon}{R+L\rho+r} \tag{3.13.2}$$

If l = length of the wire from A to C then $l\rho$ = resistance of the AC part of wire,

Therefore, the potential difference between A and C is = $Il\rho$

This potential difference is denoted by V_l

$$\therefore V_l = Il\rho \tag{3.13.3}$$

Substituting the value of I from equation (3.13.2) into equation (3.13.3),

$$V_l = \left(\frac{\varepsilon}{R+L\rho+r}\right)l\rho$$

$$\therefore V_l = \left(\frac{\varepsilon.\rho}{R+L\rho+r}\right)l \tag{3.13.4}$$

$$\therefore V_l \propto l \tag{3.13.5}$$

Principle : The potential difference between any two points of a potentiometer wire is directly proportional to the distance between that two points. By taking different values of l, different potential difference can be obtained. Points A and C of the wire behave as if they are positive and negative terminals of a battery. By changing the position of C (with the help of jockey) the emf of such a battery can be continuously varied.

From equation (3.13.4),

$$\sigma = \frac{V_l}{l} = \frac{\varepsilon.\rho}{R+L\rho+r} \tag{3.13.6}$$

The potential difference per unit length of the wire $\frac{V_l}{l} = \sigma$ is called potential gradient.

Its unit is Vm^{-1}.

The sensitivity of the potentiometer depends on the potential gradient along the wire. Smaller the potential gradient, greater will be the sensitivity of potentiometer.

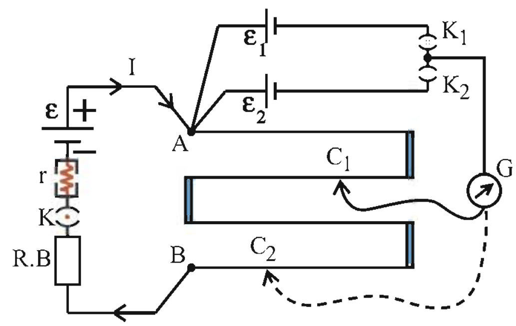
For a given V_{AB}, the sensitivity of a potentiometer can be increased by increasing the length of the potentiometer wire.

(C) Uses of Potentiometer :

(i) Comparison of emf's of two cells : Let ε_1 and ε_2 be the emf's of the two cells which are to be compared using potentiometer. For this purpose, the emf (ε) of the driver cell (main battery) in potentiometer should be greater than emf's of cells (ε_1 and ε_2) to be determined.

As shown in figure 3.25, firstly the positive terminal of cell ε_1 is connected to the end A of the potentiometer wire and the negative terminal of ε_1 is connected to jockey through a sensitive galvanometer. For this connection, plug key k_1 is inserted.

The jockey is moved along the wire AB till the galvanometer shows no deflection. Let the position of the jockey be C_1. In this condition no current is flowing through cell ε_1 and hence its terminal voltage is equal to its emf (ε_1). Such a point on the wire is called **null point**. Suppose, null point C_1, is at a distance l_1 from point A of wire. In the balanced condition, potential difference between point A and C_1 of the wire should be equal to emf of cell ε_1.



$AC_1 = l_1$,
$AC_2 = l_2$

Figure 3.25 Comparision of emfs of Two Cells

From equation (3.13.4), we have

$$V_{AC_1} = \varepsilon_1 = \sigma l_1 \qquad (3.13.7)$$

where, $\sigma = \left(\dfrac{\varepsilon.\rho}{R + L\rho + r}\right)$ respresents the potential gradient.

Now by inserting plug key K_2 battery ε_2 is connected in place of ε_1. The null point (C_2) is again obtained for cell ε_2 by sliding jockey on the wire. Let the balancing length be $AC_2 = l_2$, then,

$$V_{AC_2} = \varepsilon_2 = \sigma l_2 \qquad (3.13.8)$$

Taking the ratio of equation (3.13.7) and (3.13.8), we get

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} \qquad (3.13.9)$$

Using the above equation the emfs of two cells can be compared.

In practice, the emf of a cell is determined by comparing it with the emf of a standard cell and equation (3.13.9) is employed.

One can obtain desired value of potential difference between any two points of the wire by choosing appropriate value of R from the resistance box. The potential difference of the order of 10^{-6}V (=1 μV) or of the order of 10^{-3}V(= 1mV) can be obtained. Thus, potentiometer can also be used to measure a very small emf.

Note : If two cells of emfs ε_1 and ε_2 are first connected in helping condition and then in opposing condition, the lengths of the null point is respectively l_3 and l_4, then,

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_3 + l_4}{l_3 - l_4} \qquad (3.13.10)$$

(ii) To Determine the Internal Resistance of a Cell :

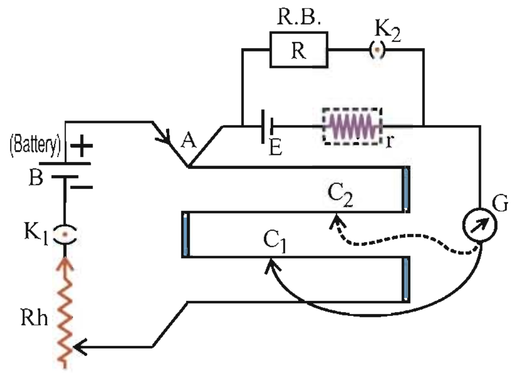


Figure 3.26 Internal Resistance of a Cell

We can also use a potentiometer to measure the internal resistance of a cell. For this as shown in figure 3.26 the cell (emf ε) whose internal resistance (r) is to be determined is connected across a small resistance box R through a key K_2.

The null point C_1 is obtained on the potentiometer wire when key K_2 is open. (i.e. when resistance box is not connected.) At this time there will be no flow of current through a cell (ε) which is called open circuit condition. If the null point C_1 is obtained at a distance l_1 from point A of the wire, then

$$V_{AC_1} = \varepsilon = \sigma l_1 \quad (3.13.11)$$

When key K_2 is closed, resistance box comes in the circuit. Null point C_2 is again obtained on the wire for an appropriate value of R. If the terminal voltage of a cell is V and null point is obtained at $AC_2 = l_2$,

$$V_{AC_2} = V = \sigma l_2 \quad (3.13.12)$$

$$\therefore \frac{\varepsilon}{V} = \frac{l_1}{l_2} \quad (3.13.13)$$

From Ohm's law $\varepsilon = I(R + r)$

and $V = IR$

This gives, $\frac{\varepsilon}{V} = \frac{R+r}{R}$ (3.13.14)

Using equation (3.13.14) into equation (3.13.13),

$$\frac{R+r}{R} = \frac{l_1}{l_2}$$

$$\therefore r = R\left(\frac{l_1}{l_2} - 1\right) \quad (3.13.15)$$

Using equation (3.13.15) we can find the internal resistance of a given cell.

3.14 Electrical Energy, Power : Joule's Law

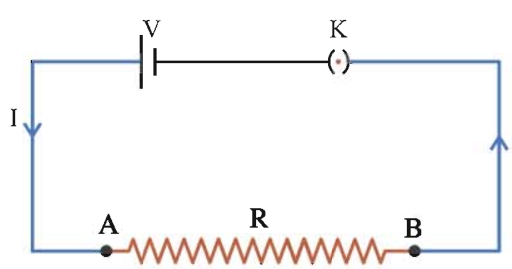


Figure 3.27

Consider figure 3.27 A battery having terminal voltage of V volt is connected to a resistance R and the circuit is completed. As explained above at end A of the resistor, energy of 1 C positive charge is V Joule. This energy per unit positive charge represents the electric potential at point A.

If the electric current is considered due to the motion of electrons (which is true in reality), it can be said that a unit negative charge possesses an electrical energy of V Joule at end B of the resistor.

We have also studied that when the electrons acquire drift velocity they experience collisions with the positive ions oscillating about their mean position, and the energy acquired by the electrons is partly transferred to the ions making their oscillations faster and more random. In Standard 11 we have already seen that heat energy is the kinetic energy associated with the random motion of constituent particles of a substance. Accordingly, this increase in the energy of oscillations of the ions due to collisions with electrons manifests as heat energy.

The heat energy released in a conductor on passing of an electric current is called the "Joule heat" and the effect is called the "Joule effect".

The p.d. of V volt applied between two ends of a conductor means that V joule energy is utilized when a unit charge passes through the conductor.

If Q coulomb charge passes through the conductor in t seconds, the electrical energy consumed in t second = heat energy produced during this time,

$$W = V Q \quad (3.14.1)$$

This is the heat energy produced in time t.

Let a steady current of I ampere be produced due to this charge then,

$$I = \frac{Q}{t}$$

$$\therefore Q = It$$

$$\therefore W = V I t \quad (3.14.2)$$

Now according to Ohm's law, $V = IR$

$$\therefore W = I^2Rt \quad (3.14.3)$$

$\therefore$ Electrical energy converted into the heat energy per unit time (power) is given by

$$P = I^2R \quad (3.14.4)$$

Here, R is the Ohmic resistance of the conductor, value of which does not depend upon V or I. Considering R as a constant, the heat energy produced per second.

$$P \propto I^2 \quad (3.14.5)$$

This equation is known as Joule's law.

Joule's Law : "The heat produced per unit time, on passing electric current through a conductor at a given temperature, is directly proportional to the square of the electric current."

The heat energy produced here is in joules.

We must know the relation between joule and calorie if we want to express heat energy in calories. Such a relation was given by Joule (James Prescott Joule, 1818–1889) according to which W = JH. Where W is in joule and H is in calorie. Here J is called joule's constant or mechanical equivalent of heat and its value is $J = 4.2 \ J \ cal^{-1}$.

$$\therefore H = \frac{I^2Rt(\text{joule})}{J(\text{Joule/cal})} = \frac{I^2Rt}{J} \text{ cal} \quad (3.14.6)$$

3.15 Practical Applications of Joule Heating

Generation of heat on passing electric current through a conductor is an inevitable phenomenon. In most cases it is unwanted, as electrical energy gained by charges is wasted in the form of heat energy. This is known as **'Ohmic dissipation'** or **'Ohmic loss'**. For example, a considerable part of electrical power supplied to an electric motor used to pump water to overhead tank in our houses, is wasted in the form of heat. Moreover, when current is passed through a circuit, properties of some components in the circuit change due to heat produced. Long distance electric transmission is done at very high voltage to reduce this Ohmic loss.

Joule heat is useful in case of some domestic applications also. Usefulness of Joule heat will be immediately clear if you think of electrical appliances used such as electric iron, electric toaster, electric oven, electric kettle, room heater etc. Joule heat is also used in electric bulbs to produce light. When electric current is passed through the filament of a bulb, its temperature rises considerably due to the heat produced, and hence it emits light. The filament should consist of a metal of very high melting point (e.g. tungsten's melting point is 3380^{o} C). As far as possible this filament should be thermally isolated from the surrounding.

Note that only a very small fraction of electrical power supplied converts into light. Normally bulbs emit 1 Candela of light energy per 1 W electrical power consumed.

A very common application of Joule heat is fuse wires used in circuits (and in our houses.) A fuse consists of a piece of wire of metals having low melting point (such as aluminium, iron, lead etc.) and is connected in series with an appliance. If a current larger than a Pre-decided value flows, the fuse wire melts and breaks the circuit and thus protects the appliance.

Illustration 20 : Electric current divides among two resistors connected in parallel in such a way that the joule heat developed becomes minimum. Using this fact, obtain the equation of division of currents.

Solution : Suppose total current I divides into two parts among two resistances R_1 and R_2 connected in parallel. Let current passing through R_1 be I_1, then the current passing through R_2 will be $I_2 = I - I_1$. Joule heat produced in unit time in this case will be,

$$H = I_1^2R_1 + (I - I_1)^2R_2$$

For this heat to be minimum, we should have $\frac{dH}{dI_1} = 0$

$$\therefore \frac{dH}{dI_1} = 2I_1R_1 + 2(I - I_1)(-1)R_2 = 0$$

On simplifying, we get $I_1 = \frac{IR_2}{R_1+R_2}$

Also, $I_2 = I - I_1 = I - \frac{IR_2}{R_1+R_2}$ $\therefore I_2 = \frac{IR_1}{R_1+R_2}$

Note : How does electric current know that some resistance is low so that more of it should pass through it ! This is due to a fundamental principle of physics which you will study in future. That fundamental principle is reflected here.

Illustration 21 : When two resistors are connected with voltage V individually, the powers obtained are P_1 and P_2 respectively. Then,

(i) they are connected in series

(ii) they are connected in parallel

Shown that the product of powers in (i) and (ii) is P_1P_2.

Solution : Here suppose that the given resistances are R_1 and R_2.
When they are connected separately,

$$P_1 = \frac{V^2}{R_1} \text{ and } P_2 = \frac{V^2}{R_2} \quad (1)$$

$$\therefore R_1 = \frac{V^2}{P_1} \text{ and } R_2 = \frac{V^2}{P_2} \quad (2)$$

Their combined resistance in series connection is, $R_1 + R_2$.
This combination is connected with voltage V.

$\therefore$ The power in case of series connection, $P_s = \frac{V^2}{R_1+R_2}$.

Substituting values of R_1 and R_2 from equations (2),

$$P_s = \frac{V^2}{\frac{V^2}{P_1}+\frac{V^2}{P_2}} = \frac{P_1P_2}{P_1+P_2} \quad (3)$$

Equivalent resistance for their parallel connection $= \frac{R_1R_2}{R_1+R_2}$

$\therefore$ The power for this combination, $P_p = \frac{V^2}{\left(\frac{R_1R_2}{R_1+R_2}\right)} = \frac{V^2}{R_1R_2}(R_1 + R_2)$

Using equation (2) in above equation,

$$P_p = \frac{V^2\left(\frac{V^2}{P_1}+\frac{V^2}{P_2}\right)}{V^4\left(\frac{1}{P_1}\times\frac{1}{P_2}\right)}$$

$$\therefore P_p = \frac{P_1P_2\times(P_1+P_2)}{P_1P_2}$$

$$\therefore P_p = P_1 + P_2 \quad (4)$$

Note : As the voltage across both the resistors is the same in parallel connection, we could have obtained equation (4) directly also !

From equations (3) and (4),

$$P_s \times P_p = P_1 \times P_2$$

Illustration 22 : A battery having an emf ε and an internal resistance r is connected with a resistance R. Prove that the power in the external resistance R is maximum when $R = r$.

Solution : Power in the external resistance $P = I^2R$

$$\therefore P = \left(\frac{\varepsilon}{R+r}\right)^2 R$$

$$\therefore \frac{dP}{dR} = -\frac{2\varepsilon^2R}{(R+r)^3} + \frac{\varepsilon^2}{(R+r)^2} = 0$$

(being the condition for maximum or minimum P)

$\therefore R = r$

(It can readily be shown by the second differentiation of P with respect to R, that for $r = R$ it is negative. This shows that for $r = R$, Power P is maximum.)

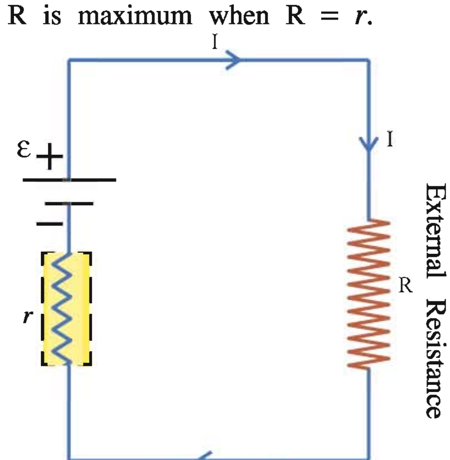


Figure 3.28

SUMMARY

1. **Electric Current :** Charges in motion constitute an electric current.
The amount of charge flowing per unit time across any cross-sectional area of a conductor held perpendicular to the direction of flow of charge is called current (I).

For a steady flow of charge, $I = \frac{Q}{t}$

If the rate of flow of charge varies with time,

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

2. **Electric Current Density :** It is the amount of electric current flowing (electric charge flowing per unit time) per unit cross–sectional area perpendicular to the current at that point. If a cross-sectional area is not perpendicular to the current, then the current density at any point,

$$J = \frac{dI}{da\cos\theta}$$

$$\therefore\ dI = Jda\ cos\theta = \vec{J} \cdot d\vec{a}$$

If the cross-sectional area is perpendicular to the current and if J is constant over the entire cross-section then,

$$I = \int_a \vec{J} \cdot d\vec{a} = J\int da$$

$$\therefore\ I = JA$$

$$\therefore\ J = \frac{I}{A}$$

3. **Ohm's Law :** "Under a definite physical condition (e.g. constant temperature) the current (I) flowing through the conductor is directly proportional to the potential difference (V) applied across its ends."

From this, $\frac{V}{I} = R$ or $V = IR$

The reciprocal of a resistance i.e. $\frac{1}{R}$ is called the conductance of the material.

4. **Resistivity :** The resistance of a conductor,

$$R = \rho . \frac{l}{A}$$

$$\therefore\ \text{resistivity } \rho = \frac{RA}{l}$$

The reciprocal of a resistivity is called conductivity of the material.

$$\therefore\ \text{Conductivity } \sigma = \frac{1}{\rho}$$

5. **Drift Velocity and Relaxation Time :** The velocity of electron corresponding to the effective (drift) displacement of the electron in the presence of electric field is known as the drift velocity.
Relaxation Time : The average time between two successive collisions of the electron with the ions is called relaxation time (τ).
The drift velocity achieved by the electron during the relaxation time (τ) is,

$$v_d = a\tau = \left(\frac{E.e}{m}\right)\tau$$

The relation between the drift velocity and current is, $I = nAv_d e$.

The relation between the drift velocity and current density is, $J = \frac{I}{A} = nev_d$.

6. The Relation between the Resistivity (ρ) and Conductivity (σ)

$\sigma = \frac{ne^2\tau}{m}$ and $\rho = \frac{m}{ne^2\tau}$

7. Mobility : It is the drift velocity of a charge carrier per unit electric field intensity.

$\mu = \frac{v_d}{E} = \frac{\sigma}{ne}$

$\therefore \sigma = ne\mu$

The conductivity of a semiconductor,

$\sigma = n_e e\mu_e + n_h e\mu_h$

8. Temperature dependence of resistivity :

The relation between the resistivity of a metallic conductor and temperature is given by the following empirical formula.

$\rho_\theta = \rho_{\theta_0}[1 + \propto(\theta - \theta_0)]$

where, θ_0 = reference temperature

For a resistance,

$R_\theta = R_{\theta_0}[1 + \propto(\theta - \theta_0)]$

$\propto$ = temperature coefficient of resistance

For metals $\propto$ is positive i.e., resistivity of metals increases with the increase in temperature.

For semiconductors $\propto$ is negative i.e., their resistivity decreases with the increase in temperature.

9. Super Conductivity : "The resistance of certain materials reduces to almost zero, when its temperature is lowered below a certain definite temperature (which is known as critical temperature T_C). The material in this state is known as superconductor and this phenomena is known as a superconductivity.

10. The emf of a Cell and Terminal Voltage : When unit positive charge is driven from negative terminal to the positive terminal due to non-electrical forces, the energy gained by the charge (or work done by the non-electrical forces) is called an emf (ε) of a battery.

The potential difference between the two terminals of a battery is called the terminal voltage (V).

The terminal voltage of a battery is, $V = \varepsilon - Ir$

11. Secondary Cell : The cells which can be restored to original condition by reversing chemical processes (i.e. by recharging) are called secondary cells. e.g. lead accumulator.

12. Charging : If the secondary cell is connected to some other source of larger emf, current may enter the cell at the positive terminal and leave it at the negative terminal. The electrical energy is then converted into chemical energy. This is called charging of the cell.

For the charging of a lead storage cell (lead accumulator),

$VIt = \varepsilon It + I^2Rt + I^2rt$ and $I = \frac{V-\varepsilon}{r+R}$

13. **Junction or Branch Point :** It is the point in a network at which more than two conductors (minimum three) meet.

14. **Loop :** A closed circuit formed by conductors is known as loop.

15. **Kirchoff's Rules :**

First rule : "The algebraic sum of all the electric currents meeting at the junction is zero."

$\therefore\ \Sigma I = 0$

Second rule : "For any closed loop the algebraic sum of the products of resistances and the respective currents flowing through them is equal to the algebraic sum of the emfs applied along the loop."

$\Sigma IR = \Sigma \varepsilon$

16. **Connections of Resistors**

Series Connection :

$R_s = R_1 + R_2 + R_3 + \ldots\ldots + R_n$

where, R_s = Equivalent resistance of n resistors connected in series.

Parallel connection :

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

where, R_P = Equivalent resistance of n resistors connected in parallel.

17. **Series Connection of Cells :** For the series connections of two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2,

$$I = \frac{\varepsilon_1 + \varepsilon_2}{R + (r_1 + r_2)} = \frac{\varepsilon_{eq}}{R + r_{eq}}$$

where, I = Current flowing through the external resistance R connected across the series connection.

Equivalent emf $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$

Equivalent internal resistance $r_{eq} = r_1 + r_2$

18. **Parallel Connection of Cells :** If two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in parallel, then we have,

$$I = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{1 + \frac{R}{r_1} + \frac{R}{r_2}} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{R(r_1 + r_2) + r_1 r_2}$$

$$\therefore\ I = \frac{\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{(r_1 + r_2)}}{R + \frac{r_1 r_2}{(r_1 + r_2)}} = \frac{\varepsilon_{eq}}{R + r_{eq}}$$

Equivalent emf $\varepsilon_{eq} = \dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$

Equivalent internal resistance $r_{eq} = \dfrac{r_1 r_2}{r_1 + r_2}$

19. **Wheatstone Bridge :** In the balanced condition of Wheatstone bridge,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

20. **Potentiometer :** It is a device in which one can obtain a continuously varying potential difference between any two points which can be measured simultaneously.

Principle : The potential difference between any two points of a potentiometer wire is directly proportional to the distance between that two points.

$$\therefore \ V_l \propto l$$

$$V_l = \left(\frac{\varepsilon.\rho}{R+L\rho+r}\right) \cdot l$$

where, $\sigma = \frac{V_l}{l} = \left(\frac{\varepsilon.\rho}{R+L\rho+r}\right)$ = Potential gradient

21. **Joule Effect :** "The heat energy released in a conductor on passing an electric current is called the 'Joule heat' and this effect is called the 'Joule effect' ".

Joule heat $W = I^2Rt$ (Joule)

$$H = \frac{I^2Rt}{J} \text{ (cal)}$$

Electrical energy consumed per unit time or heat energy produced per unit time i.e. electric power

$P = I^2R$

$P \propto I^2$

Joule's Law : "The heat energy produced per unit time, on passing electric current through a conductor at a given temperature, is directly proportional to the square of the electric current.

22. **Ohmic loss :** On passing electric current through a conductor, an electrical energy gained by charges is wasted in the form of heat energy. This is known as "Ohmic loss".

EXERCISE

For the following statements choose the correct option from the given options :

1. In a hydrogen atom, the electron is moving in a circular orbit of radius 5.3×10^{-11} m with a constant speed of 2.2×10^6 ms^{-1}. The electric current formed due to the motion of electron is ....

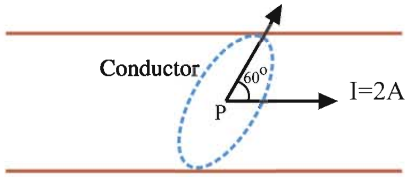
(A) 1.12 A (B) 1.06 mA (C) 1.06 A (D) 1.12 mA

2. A ring of radius R and linear charge density λ on its surface is performing rotational motion about an axis perpendicualr to its plane. If the angular velocity of the ring is ω, how much current is constituted by the ring ?

(A) $R\omega\lambda$ (B) $R^2\omega\lambda$ (C) $R\omega^2\lambda$ (D) $R\omega\lambda^2$

3. A cell supplies a current of 0.9 A through a 2 Ω resistor and current of 0.3 A through a 7 Ω resistor. What is the internal resistance of the cell ?

(A) 0.5 Ω (B) 1.0 Ω (C) 1.2 Ω (D) 2.0 Ω

4. 

The cross-sectional area of the plane shown in the figure is equal to 1 cm^2. 2A current flows through a conductor. The current density at point P in the conductor will be ....

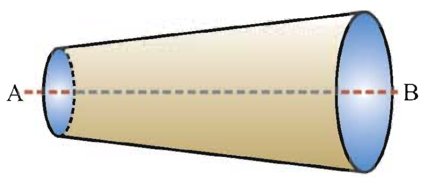
(A) $\frac{4}{\sqrt{3}} \times 10^4 Am^{-2}$ (B) $\frac{\sqrt{3}}{2} \times 10^4 Am^{-2}$

(C) $\frac{\sqrt{3}}{2} \times 10^{-4} Am^{-2}$ (D) $\frac{\sqrt{3}}{4} \times 10^{-4} Am^{-2}$

5. A current density of 2.5 Am^{-2} is found to exist in a conductor when an electric field of $5 \times 10^{-8} Vm^{-1}$ is applied across it. The resistivity of a conductor is ....

(A) $1 \times 10^{-8}\ \Omega m$ (B) $2 \times 10^{-8}\ \Omega m$

(C) $0.5 \times 10^{-8}\ \Omega m$ (D) $12.5 \times 10^{-8}\ \Omega m$

6. 

A wire has a non-uniform cross-section as shown in figure. A steady current is flowing through it. Then the drift speed of the electrons while going from A to B ....

(A) is constant throughout the wire (B) decreases

(C) increases (D) varies randomly

7. A resistive wire is stretched till its length is increased by 100 %. Due to the consequent decrease in diameter, the change in the resistance of a stretched wire will be ....

(A) 300 % (B) 200 % (C) 100 % (D) 50 %

8. At what temperature would the resistance of a copper conductor be double its resistance at 0°C ? Given ∝ for copper = $3.9 \times 10^{-3}\ °C^{-1}$

(A) 256.4°C (B) 512.8°C (C) 100°C (D) 256.4 K

9. You are given n identical resistors each of resistance $r\Omega$. First they are connected in such a way that the possible minimum value of resistance is obtained. Then they are connected in a way to get maximum possible resistance. The ratio of minimum and maximum resistance obtained in these ways is ....

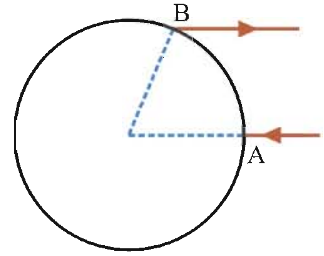
(A) $\frac{1}{n}$ (B) n (C) n^2 (D) $\frac{1}{n^2}$

10. P and Q are two points on a uniform ring of resistance R. O is the centre of the ring. If the part PQ of the ring subtends an angle θ at the centre O of the ring (i.e. ∠POQ = θ), the equivalent resistance of the ring between the points P and Q will be ....

[radius of the ring = r and resistance per unit length of the ring = ρ]

(A) $\frac{R\theta}{4\pi^2}(2\pi - \theta)$ (B) $R\left(1-\frac{\theta}{2\pi}\right)$ (C) $\frac{R\theta}{2\pi}$ (D) $R\left(\frac{2\pi-\theta}{4\pi}\right)$

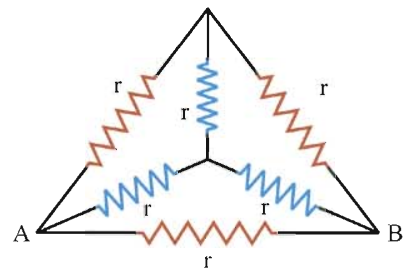
11. A wire in a circular shape has 10 Ω resistance. The resistance of wire per 1 m length is 1 Ω. If the equivalent resistance betwen A and B is 2.4 Ω, then the length of the chord AB will be equal to .... meter.



(A) 2.4 (B) 4

(C) 4.8 (D) 6

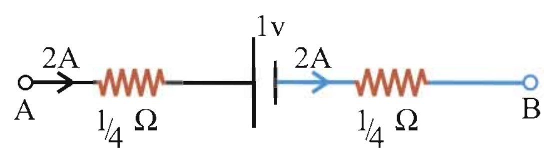
12. In the circuit shown in figure, what will be the effective resistance between points A and B ?



(A) r (B) $\frac{r}{2}$

(C) $\frac{r}{3}$ (D) $2r$

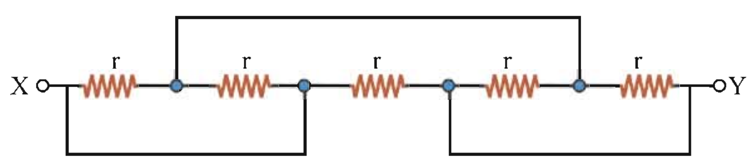
13. Figure shows a part of a closed circuit. If the current flowing through it is 2A, what will be the potential difference between points A and B ?



(A) +2 V (B) +1 V

(C) −2 V (D) −1 V

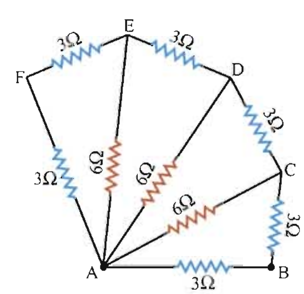
14. In the network shown in figure, the equivalent resistance between points X and Y will be ....



(A) r (B) $\frac{r}{2}$

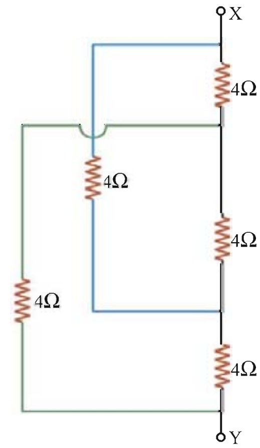
(C) $2r$ (D) $\frac{r}{3}$

15. The effective resistance between points A and B of the network shown in figure ....



(A) 2 Ω (B) 3 Ω

(C) 6 Ω (D) 12 Ω

16. 

The equivalent resistance between points X and Y in the following figure is ....

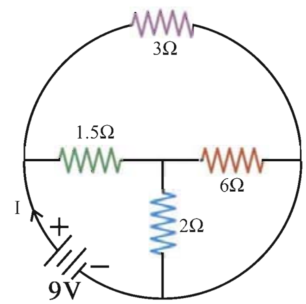
(A) 4 Ω

(B) 2 Ω

(C) 1 Ω

(D) 3 Ω.

17. What is the total current supplied by the battery to the circuit shown in the adjoing figure ?



(A) 2 A

(B) 4 A

(C) 6 A

(D) 9 A

18. A uniform conductor of resistance R is cut into 20 equal pieces. Half of them are joined in series and the remaining half of them are connected in parallel. If the two combinations are joined in series, the effective resistance of all the pieces is :

(A) R (B) $\frac{R}{2}$ (C) $\frac{101R}{200}$ (D) $\frac{201R}{200}$

19. What will be the time taken by electron to move with drift velocity from one end to the other end of copper conductor 3 metre long and carrying a current of 3 A ?

[The cross-sectional area of the conductor = 2×10^{-6} m^2 and electron density for copper $n = 8.5 \times 10^{28}$ m^{-3}]

(A) 2.72×10^3s (B) 2.72×10^4s (C) 2.72s (D) 2.72×10^{-4}s

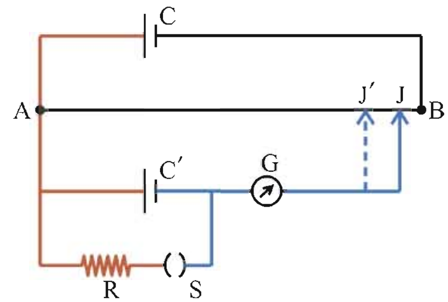
20. Masses of three wires of copper are in the ratio 5 : 3 : 1 and their lengths are in the ratio 1 : 3 : 5. The ratio of their electrical resistances are ....

(A) 5 : 3 : 1 (B) $\sqrt{125}$: 15 : 1 (C) 1 : 15 : 125 (D) 1 : 3 : 5

21. The resistance of a 10 m long potentiometer wire is 20 **Ω**. It is connected in series with a 3 V battery and 10 **Ω** resistor. The potential difference between two points separated by distance 30 cm is equal to ....

(A) 0.02 V (B) 0.06 V (C) 0.1 V (D) 1.2 V

22. In the potentiometer circuit shown in figure, the balance length AJ = 60 cm when switch S is open. When switch S is closed and the value of R = 5 Ω, the balance length AJ' = 50 cm. What is the internal resistance of cell C' ?



(A) 0.5 Ω (B) 1 Ω

(C) 1.5 Ω (D) 0.1 Ω

23. n identical cells each of emf ε and internal resistance r are connected in parallel with resistor R. The current flowing through resistor R is, ...

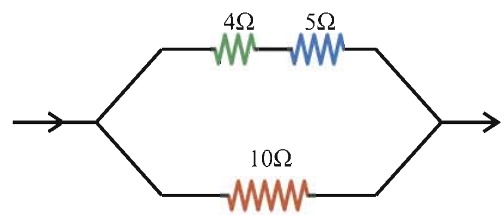
(A) $\frac{n\varepsilon}{R+nr}$ (B) $\frac{n\varepsilon}{nR+r}$ (C) $\frac{\varepsilon}{R+r}$ (D) $\frac{\varepsilon}{nR+r}$

24. A wire is uniformly stretched to make its area of cross-section $\frac{1}{n}^{th}$ times ($n > 0$). What will be its new resistance ?

(A) $\frac{1}{n^2}$ times (B) n^2 times (C) $\frac{1}{n}$ times (D) n times

25. If the current in an electric bulb increases by 1 %, what will be the change in the power of a bulb ?
[Assume that the resistance of the filament of a bulb remains constant.]
(A) increases by 1 % (B) decreases by 1 %
(C) increases by 2 % (D) decreases by 2 %

26. In the following circuit if the heat evolved in the 10 Ohm resistor is 10 cal/s. The heat evolved in the 4 Ω resistor is approximately .... cal/s.



(A) 4 (B) 5

(C) 10 (D) 20

27. Two bulbs of 220 V and 100 W are first connected in series and then in parallel with a supply of 220 V. Total power in both the cases will be ....
(A) 50 W, 100 W (B) 100 W, 50 W
(C) 200 W, 150 W (D) 50 W, 200 W

ANSWERS

1. (B)	2. (A)	3. (A)	4. (A)	5. (B)	6. (B)
7. (A)	8. (A)	9. (D)	10. (A)	11. (B)	12. (B)
13. (A)	14. (B)	15. (A)	16. (A)	17. (C)	18. (C)
19. (B)	20. (C)	21. (B)	22. (B)	23. (B)	24. (B)
25. (C)	26. (B)	27. (D)			

Answer the following questions in brief :

1. Why is an electric current density defined ?
2. How many electrons are present in 1 nenocoulomb (1 *nc*) charge ?
3. The internal resistance of a battery of 2 V terminal voltage is 0.2 Ω. If the current flowing through the battery is 0.5 A, what will be the emf of battery ?
4. Define the mobility of a charge carrier.
5. Give the relation between the drift velocity and current flowing through the conductor.
6. The drift velocity of the electron is v when current I is flowing through a conductor of radius r. What will be the drift velocity of electron in a similar conductor of radius 2 r if the same current (I) is flowing through it ?
7. Give the empirical formula showing the relation between the resistivity of a metallic conductor and temperature.
8. Resistance of a wire is 10 Ω. What will be the required change in the length of it to increase its resistance to 1000 Ω ?
9. State the law of conservation of charge.
10. Kirchoff's second law is the consequence of which law ?
11. Why the current in a superconductor can be sustained over a long interval of time ?
12. Why the emf (ε) of a battery cannot be measured using table voltmeter ?
13. State the principle of potentiometer.
14. Write Joule's law.
15. Give the examples of "Ohmic loss".
16. What are the changes made in the temperature of a semiconductor in order to reduce its conductivity.

Answer the following questions :

1. Define an electric current density. Clarify the differences between the electric current and electric current density.
2. Explain the emf of a battery. When the battery is said to be in "open circuit condition" ?
3. Write Ohm's law. Explain the I–V characteristics for a conductor obeying Ohm's law.
4. Using necessary diagram explain the drift velocity of electron in a conductor in the presence of external electric field.
5. Explain the mobility of a charge carrier and obtain the formula for the conductivity of a semiconductor.
6. Obtain the relation between the drift velocity and current density.
7. Accepting the single valuedness of electric potential in an appropriate closed circuit, derive Kirchoff's second rule by drawing necessary circuit diagram.
8. Deduce the principle of potentiometer with the help of necessary circuit diagram.
9. Explain the method of finding the internal resistance of a cell using potentiometer.
10. Giving appropriate circuit diagram, describe the charging process of a lead storage cell (accumulator). Obtain the formula for the charging current.
11. Derive the expression to find the unknown resistance in the balanced condition of Wheatstone bridge.
12. Obtain an expression for the equivalent emf and equivalent internal resistance in the parallel connection of two cells.
13. State the limitations of Ohm's law.
14. Write notes on superconductivity.
15. What is Joule heat and Joule effect ? Obtain Joule's law for Joule heating.

Solve the following examples :

1. A stream of electron moves from the electron gun to a screen of a television. The electric current of the 10 μ A is constituted. Calculate the number of electrons striking the screen at every second. Also calculate magnitude of the charges striking the screen in one minute.

[**Ans. :** $n = 6.25 \times 10^{13}$ electron/Sec., $Q = -600\ \mu C$]

2. An electron in the hydrogen atom is revolving around a proton with a speed of $\frac{e^2}{\hbar}$. The radius of the electron orbit is equal to $\frac{\hbar^2}{me^2}$. Obtain the formula for the electric current in the above case. Mass of the electron = m, charge on electron = e. (**Hint :** $\hbar = \frac{h}{2\pi}$)

[**Ans. :** $I = \frac{4\pi^2 me^5}{h^3}$]

3. A current of 1.0 A is flowing through a copper wire of length 0.1 m and corss-section 1.0×10^{-6} m^2.

(i) If the resistivity of copper be $1.7 \times 10^{-8}\ \Omega$ m, calculate the potential difference across the ends of a wire.

(ii) Determine the drift velocity of electrons.

[Density of copper = 8.9×10^3 kg m^{-3}, valancy of Cu = 1, atomic weight of copper = 63.5 g mol^{-1}, $N_A = 6.02 \times 10^{23}$ mol^{-1}]

[**Ans. :** $V = 1.7 \times 10^{-3}$ V and $v_d = 7.4 \times 10^{-5}$ ms^{-1}]

4. An n-type semi-conductor has 4×10^{-3} meter width, 25×10^{-5} metre thickness and 6×10^{-2} metre length. 4.8 mA current is flowing through it. Here voltage is applied parallel to the length of the semi-conductor. Calculate the current density. The density of the free electron is equal to 10^{22} m^{-3}. What will be the time taken by the electron across the length of the semi-conductor ?

[**Ans. :** 4.8×10^3 Am^{-2}, 2×10^{-2} s]

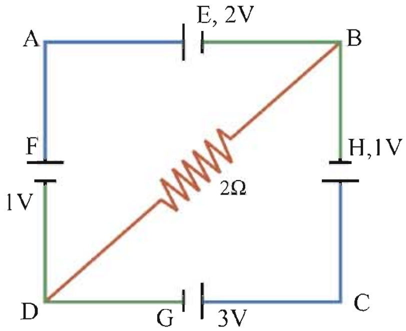
5. A cylindrical wire is stretched to increase its length by 10%. Calculate the percentage increase in resistance. [**Ans. :** 21%]

6. One conducting wire of length 1 m is cut into two unequal part P and Q respectively. Now, part P is stretched to double its length. Let the modified wire be R. If the resistance of the R and Q wires are same, then calculate the length of P and Q wires.

[**Ans. :** Length of the P wire is $\frac{1}{5}$ meter, Length of Q wire is $\frac{4}{5}$ meter]

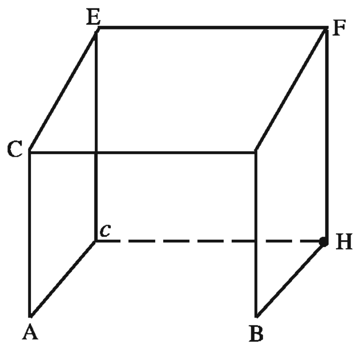
7. The resistance of one aluminium and one copper wires, having identical lengths is equal. Which of the two wires will be lighter ? $\rho_{Al} = 2.63 \times 10^{-8}\ \Omega$ m, $\rho_{Cu} = 1.72 \times 10^{-8}\ \Omega$ m the density of the aluminium is 2.7×10^3 kg m^{-3} and density of cooper is 8.9×10^3 kg m^{-3}.

[**Ans. :** aluminium]

8. 

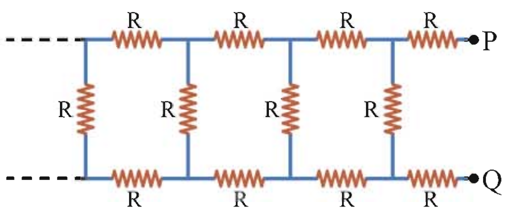
The emf of the batteries E, F, G and H are 2V, 1V, 3V and 1V respectively. Their internal resistance are respectively 2 Ω, 1 Ω, 3 Ω respectively. Calculate p.d. between B and D.

[**Ans.** : $\frac{2}{13}$ V]

9. 

Find the effective resistance between the points A and B in the network given below. All wires in the network have the same resistance 'r' Ohm.

[**Ans.** : $\frac{7r}{5}$ Ohm]

10. 

Consider an infinite network as shown in the figure. The resistance of each of the wires of the network is equal to R. Calculate the resultant resistance between points P and Q.

[**Ans.** : R $(1 + \sqrt{3})$]

11. The length of a potentiometer wire is 200 cm. For a given cell, the null point is obtained at 80 cm. What will be the length of wire required for balancing the cell if the length of the same wire is made 300 cm ? [**Ans.** : 120 cm]

12. A battery having an emf of 12 volt and an internal resistance of 2 Ω is connected to another battery having an emf of 18 volt and an internal resistance of 2 Ω in such a way that they are opposing each other and the circuit is closed. Calculate the following :
(1) current flowing in the circuit.
(2) electrical power in the two batteries.
(3) terminal voltage of the two batteries.
(4) electric power consumed in the batteries.

[**Ans.** : (1) 15 A (2) 18 W, 27 W (3) 15 V, 15 V (4) 4.5 W, 4.5 W]

13. An electric kettle has two heating coils. When one of the coils is switched on, a given quantity of water in the kettle starts boiling in 6 minutes. When the other coil only is switched on, then the same amount of water starts boiling in 8 minutes. If the two coils are switched on in parallel how much time will the same amount of water take to boil ? Each time the voltage applied is the same. [**Ans.** : 3.43 min]

14. Two wires which are made of the same material have the same cross-sectional area, but different lengths l_1 and l_2. Prove that if they are used as fuse wires, they will melt for the same value of the current flowing through them, in the same time.

15. A and B are two electric bulbs with their ratings respectively 40 W, 110 V and 100 W and 110 V. Find their respective filament resistances. If the bulbs are connected in series with a supply of 220 V, which bulb will fuse ? [**Ans.** : R_A = 302.5 Ω, R_B = 121 Ω, bulb A]