4

MAGNETIC EFFECTS OF ELECTRIC CURRENT

4.1 Introduction

Branches of electricity and magnetism have been known for more than 2000 years. Danish physicist Oersted's observation and contributions given by Rowland, Faraday, Maxwell and Lorentz, unified these two branches, initially developed independently.

New concept was developed when the Laws obtained from the experimental studies of electricity and magnetism were presented mathematically and led to fundamental unification of these two branches. This helped in understanding nature of light and production of electromagnetic waves and its propagation become possible. As a result of this revolution is created in communication.

The branch of physics which envelops a comprehensive study of electricity and magnetism is called **electrodynamics**. In the modern technology of communication electrodynamics is of prime importance.

In the present chapter we will study, magnetic field produced due to electric current, force on electric charge moving in the magnetic field, force on current carrying conductor placed in magnetic field, cyclotron, galvanometer etc.

4.2 Oersted'd Observation

Some experimental observations are involved in the development process of the study of electricity and magnetism. One of theses observations is the Danish physicist Oerted's observation. In the year 1819 A.D. he made (Hans Christian Oersted 1771–1851) the following observation. he was a school teacher in Denmark.

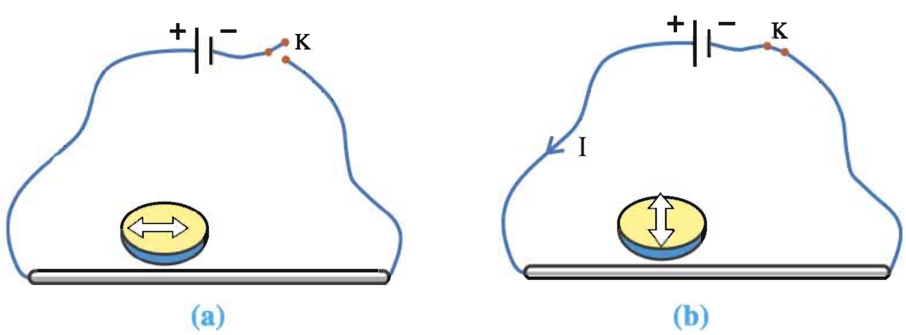


Figure 4.1 Oersted's Observation

Arrange the conductor (wire) parrallel to the magnetic needle such that it remains below the wire as shown in figure 4.1(a). On completing electric circuit shown in figure 4.1(a) current passes through the conducting wire and magnetic needle gets deflected and becomes perpendicular to the conducting wire see figure 4.1(b).

Thus in this observation of the experiment, he noted that when electric current passes through the conducting wire magnetic field is produced arround it.

This observation was presented to the French Academy by scientist Arago on 11th September 1820.

4.3 Biot–Savart's Law

When Biot and Savart, in Paris, came to know about Oested's above mentioned discovery, they, from the analysis of experimental studies, presented a Law for magnetic field produced due to electric current element in the following form.

The intensity of magnetic field due to an electric current element $I\vec{dl}$ at a point having position vector $\vec{r}$ with respect to the electric current element is given by the formula.

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2} \qquad (4.3.1)$$

Here, $I\vec{dl}$ = Current element i.e. the product of electric current and length of line element $\vec{dl}$ of a conductor of very small length

μ_0 = magnetic permeability of vacuum

$= 4\pi \times 10^{-7}$ tesla meter ampere^{-1} (T m A^{-1})

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ unit vector along the direction of $\vec{r}$

Equation (4.3.1) can also be written as

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}}{r^3} \qquad (4.3.2)$$

From equation (4.3.1) $|\vec{dB}| = \frac{\mu_0}{4\pi}\frac{Idl\ \sin\theta}{r^2}$ (4.3.3.)

Where θ is the angle between $\vec{dl}$ and $\vec{r}$.

Explanation : Consider a current currying conducting wire of any arbitrary shape as shown in figure 4.2. Suppose we wish to find magnetic field produced due to this current carrying conductor at any point P.

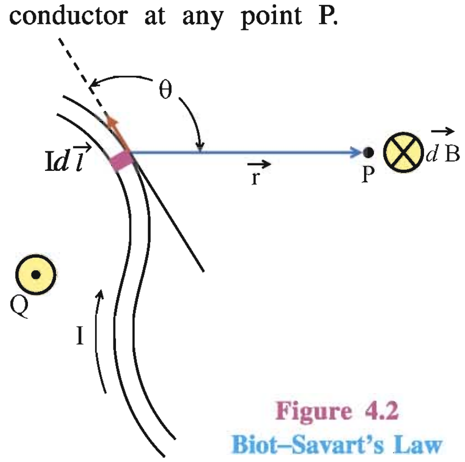


Figure 4.2
Biot–Savart's Law

We can think of the wire to be consisting of line elements dl_1, dl_2 ..... dl_n of infinitesimal lengths. Here, each element is so small that it can be locally considered straight and parallel to the direction of electric current. One such line element is shown in Figure 4.2 by $\vec{dl}$. $\vec{r}$ is the position vector of point P with respect to the current element $I\vec{dl}$. Intensity of magnetic field ($\vec{dB}$) at point P, due to this current element, can be found using equation (4.3.1).

Direction of $\vec{dB}$ is perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$ given by right hand screw rule. As $\vec{dl}$ and $\vec{r}$ taken in the plane of page of the book, the direction of $\vec{dB}$ at point

P is perpendicular to the plane of page of the book and going inside it shown by symbol ⊗ (As shown in the Figure, the direction of the magnetic field at Q is perpendicular to the plane of the page of the book towards the observer, and is shown by the symbol ⊙.)

To find the total magnetic field at point P, we will have to take the vector sum of magnetic field at P due to various current elements. As the current elements are continuous, the vector addition can be written as a line integral, as under.

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}\times\hat{r}}{r^2} \quad \text{or} \tag{4.3.4}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}\times\vec{r}}{r^3} \tag{4.3.5}$$

Here, the line integral is taken over the entire path formed by the conducting wire.

Note that Biot-Savart's Law is an inverse square Law like Coulomb's Law and Newton's universal Law of gravitation.

The use of Biot-Savart's Law becomes simple in case of current carrying conductor of a simple geometrical shape.

Here, it is clear for the straight current currying conductor kept perpendicular to a plane, magnetic field at the equidistance point in this plane from the conductor will be same. That is as shown in the figure 4.3 magnetic field is equal of every point on the circumference of circle at radius OP and is along the tangent. For finding the direction of the magnetic field right hand thumb rule is as follows.

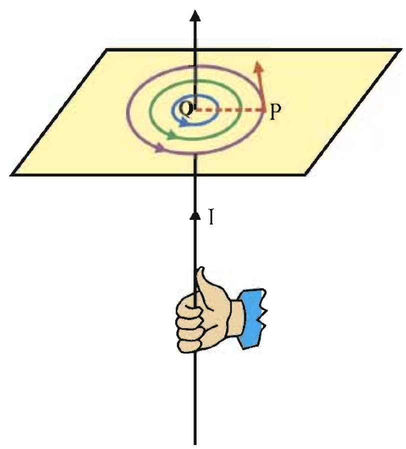


Figure 4.3 Right Hand Thumb Rule

Hold the wire in such a way that the thumb is in the direction of electric current, the fingers encircling the wire indicate the direction of magnetic field as shown in figure 4.3.

4.4 Magnetic Field at a Point on the Axis of a Circular Ring Carrying Current

Consider a ring of thin wire carrying current I as shown in figure 4.4. Its radius is a. X-axis is taken along the axis of the ring. Suppose a point P is at a distance x from the centre of the ring on the axis of the ring.

Let the position vector of point P with respect to an element $\vec{dl}$ of wire be $\vec{r}$. The magnetic field $d\vec{B}$ at point P due to the current element $I\vec{dl}$ is in a direction perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$.

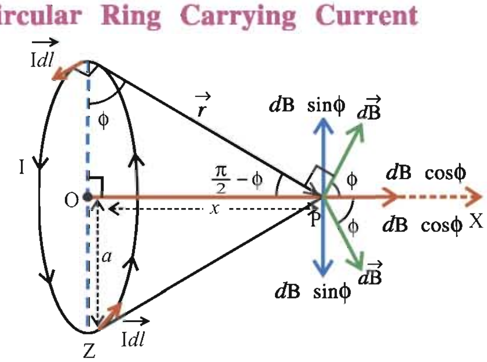


Figure 4.4 Magnetic Field Produced due to Circular Ring

Two mutually perpendicular components of this field $d\vec{B}$ are (1) a component $dB\cos\phi$ parallel to the X-axis and (2) a component $dB\sin\phi$ perpendicular to the X-axis. One thing is clear from the Figure 4.4 that when vector sum of magnetic field due to all such elements are considered, component $dB\sin\phi$ due to the diametrically opposite elements, which are in mutually opposite directions, will nullify each other.

Hence all axial components $dB\cos\phi$ will be in the X-direction and can be added together. Using Biot-Savart's Law

$$|\vec{dB}| = \left|\frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}}{r^3}\right| = \frac{\mu_0}{4\pi}\frac{Idl\, r\sin\theta}{r^3} = \frac{\mu_0 Idl\sin\theta}{r^2}$$

where θ is angle between $\vec{dl}$ and $\vec{r}$.

But $\vec{dl} \perp \vec{r}$ $\therefore$ $\sin\theta = \sin\frac{\pi}{2} = 1$

$$\therefore\ |d\vec{B}| = \frac{\mu_0}{4\pi}\cdot\frac{Idl}{r^2} \qquad (4.4.1)$$

Now, point P is at a distance x from centre of the circular ring.

$$\text{Hence } dB(x) = |d\vec{B}|\cos\phi \qquad (4.4.2)$$

Using equation (4.4.1) in (4.4.2)

$$dB(x) = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\cos\phi = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\frac{a}{r} \quad (\because \text{ from Figure } \cos\phi = \frac{a}{r})$$

Line integration should be taken over the circumference of the ring to find resultant magnetic field $B(x)$ at point P.

$$\therefore\ B(x) = \oint dB(x) = \frac{\mu_0 Ia}{4\pi r^3}\oint dl$$

Here $\oint dl$ is the line integral taken over the whole ring. $\therefore$ $\oint dl = 2\pi a$.

$$\therefore\ B(x) = \frac{\mu_0 Ia}{4\pi r^3}.2\pi a \text{ ring.}$$

But from the geometry of the Figure.

$$r^2 = a^2 + x^2 \Rightarrow r^3 = (a^2+x^2)^{\frac{3}{2}}$$

$$B(x) = \frac{\mu_0 Ia^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

The magnetic field is along the X-axis.

If the ring consists of N closely wound turns, we can write.

$$B(x) = \frac{\mu_0 NIa^2}{2(a^2+x^2)^{\frac{3}{2}}} \qquad (4.4.3)$$

Magnitude of the field at the centre of the ring is obtained by sustituting $x = 0$ in the equation (4.4.4). Thus the magnetic field B (centre) at centre of the ring.

$$B_{(centre)} = \frac{\mu_0 NI}{2a} \qquad (4.4.4)$$

For a point far away from the centre of the ring compared to its radius, we have $x >> a$. Neglecting a^2 in comparison to x^2 in equation (4.4.4)

$$B(x) = \frac{\mu_0 N I a^2}{2(x^2)^{\frac{3}{2}}} = \frac{\mu_0 N I a^2}{2x^3} \quad \text{(where } x >> a\text{)} \tag{4.4.5}$$

Illustration 1 : Electron is rotating in circular orbit with radius 5.2×10^{-11}m and with linear speed 2×10^6 m s^{-1} in an Hydrogen atom arround the proton. Find the magnetic field produced at the centre of the orbit.

Solution : Here $v = 2 \times 10^6$ m s^{-1}

$r = 5.2 \times 10^{-11}$ m

$e = 1.6 \times 10^{-19}$ C

Frequency of electron in the orbit f (No. of rotations completed in 1 second).

$$f = \frac{v}{2\pi r}$$

Electric current $I = f.e$

$$= \frac{v}{2\pi r} \times e$$

$$= \frac{2 \times 10^6}{2 \times 3.14 \times 5.2 \times 10^{-11}} \times 1.6 \times 10^{-19} = 9.8 \times 10^{-4} \text{A}$$

Magnetic field produced at the centre of the circular orbit.

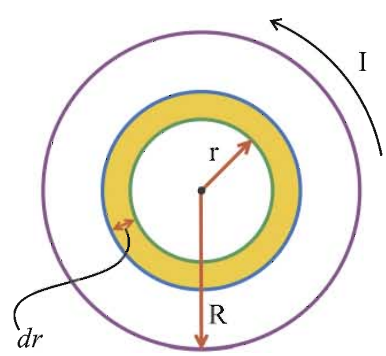
$$B = \frac{\mu_0 I}{2r}$$

$$= \frac{4 \times 3.14 \times 10^{-7} \times 9.8 \times 10^{-4}}{2 \times 5.2 \times 10^{-11}}$$

$$= 11.8 \text{ T}$$

Illustration 2 : A charge Q is uniformly spread over a disc of radius R made from non-conducting material. This disc is rotated about its geometrical axis with frequency f. Find the magnetic field produced at the centre of the disc.

Solution : Suppose the disc with radius R is devided into the concentric rings with various radii. Consider one of these rings with radius r and width dr. Total charge on the disc is Q.



Hence charge per unit area $= \frac{Q}{\pi R^2}$

$\therefore$ The charge on the ring with radius r.

= (area of the ring) (charge per unit area)

$$= (2\pi r dr)\left(\frac{Q}{\pi R^2}\right)$$

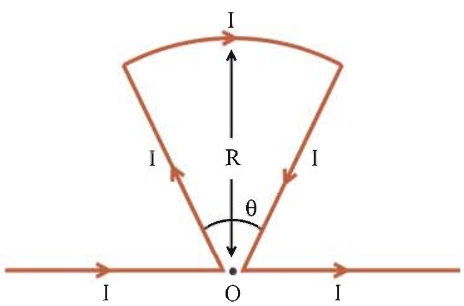
If the ring is rotating with frequency f, then current produced $I = \frac{Q}{\pi R^2} 2\pi r dr f$ and magnetic field produced at the centre due to this current $dB = \frac{\mu_0 I}{2r} = \frac{\mu_0 Q 2\pi}{\pi r^2} \frac{dr}{2r} f = \frac{\mu_0 Q f}{R^2} dr$

$\therefore$ Magnetic field B produced at the centre due to the whole disc.

$$B = \int dB = \int_0^R \frac{\mu_0 Qf}{R^2} dr = \frac{\mu_0 Qf}{R^2} \int_0^R dr$$

$$\therefore \ B = \frac{\mu_0 Qf}{R}$$

Illustration 3 : Find the intensity of magnetic field at point P shown in the figure. At point O, the wires do not touch each other. Corners of the two wires are very close to point O.



Solution : Here point O is on the line of horizontal currents, hence the magnetic field is not developed due to them. It also lies on the directions of radial currents hence magnetic fields due to them is also zero. So the magnetic field is produced only due to the arc. To find this, the formula of magnetic field at the center of a ring having n turns and radius R can be used. According to this equation,

$$B = \frac{\mu_0 nI}{2R} \text{ (in a direction going in to the plane of paper)} \quad (1)$$

In the present case, the length of the arc is = $R\theta$

For one complete turn, the length of the arc is $2\pi R$, then the number of turns for length $R\theta$ will be,

$2\pi R$: 1 turn

$R\ \theta$: ? $\Rightarrow$ number of turns, $n = \frac{R\theta}{2\pi R} = \frac{\theta}{2\pi}$

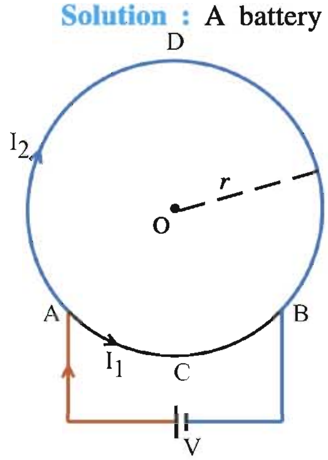
Using this in equation (1),

$$B = \frac{\mu_0 I\theta}{2R \times 2\pi}$$

$$\therefore \ B = \frac{\mu_0 I\theta}{4\pi R} \text{ (going in to the plane of figure)}$$

Illustration 4 : A circular loop is prepared from a wire of uniform cross section. A battery is connected between any two points on its circumference. Show that the magnetic induction at the centre of the loop is zero.

Solution : A battery is joined between points A and B of the loop as shown in the figure.



Since the cross-section of the wire is uniform, the resistance of the part of wire is proportional to the length of that part ($\because R = \rho \frac{l}{A}$).

Let the resistance **per unit length** be R'.

Length of wire ACB = l_1

Length of wire ADB = l_2

$\therefore$ Resistance of wire ACB = $R_1 = R' l_1$

Resistance of wire ADB = $R_2 = R' l_2$

Current in wire ACB $= I_1$

Current in wire ADB $= I_2$

These two parts ACB and ADB are in parallel between A and B points.

$V = I_1R_1 = I_2R_2$

$I_1(R'l_1) = I_2(R'l_2)$

$\therefore\ I_1l_1 = I_2l_2$

Every small current element of this wire is perpendicular to the position vector of O, with respect to it.

$\therefore$ Biot-Savart's Law gives, magnetic field at O due to ACB, as

$$B_1 = \frac{\mu_0}{4\pi}\frac{I_1l_1 sin90^o}{r^2}$$

and that due to ADB,

$$B_2 = \frac{\mu_0}{4\pi}\frac{I_2l_2 sin90^o}{r^2}$$

Since, $I_1l_1 = I_2l_2$

we get, $B_1 = B_2$

According to right hand rule the directions of B_1 and B_2 are opposite to each other. Hence the resultant magnetic field at O will be zero.

4.5 Ampere's Circuital Law

We have obtained line integration in the case of electric field. Same can be done for magnetic field. Consider electric currents I_1, I_2, I_3, I_4, I_5 and I_6 as shown in figure 4.5. All these currents produce magnetic field in the region arround electric currents. A plane which is not necessarily horizontal is shown in the Figure. An arbitrary closed curve is also shown on it. Now let us take a line integration of magnetic field on this loop.

You must be remembering that we have taken a sign convention for electric charges ($\pm$) while considering surface integral in case of Gauss' theorem for electric field. In the same way we will have to decide a sign convention for the electric currents enclosed by the loop. One of the methods used in practice is as under.

Arrange a right hand screw perpendicular to the plane containing closed loop and rotate it in direction of vector line elements taken for line integration. Electric currents in the direction of advancement of the screw are considered positive and the currents in the opposite direction are considered negative.

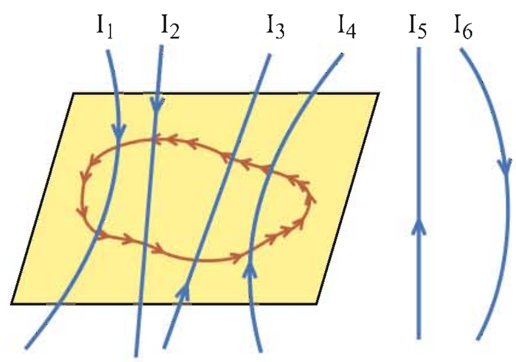


Figure 4.5 Ampere's Circuital Law

Now, using the above mentioned sign convention in figure 4.5, we have I_1 and I_2 negative and I_3 and I_4 positive.

Hence the algebraic sum of all these current will be

$I_3 + I_4 - I_1 - I_2 = \Sigma I$

Here do not worry about the currents which are not enclosed by the closed loop selected.

The statement of Ampere's circuital Law is as under :

"The line integral of magnetic induction over a closed loop in a magnetic field is equal to the product of algebraic sum of electric currents enclosed by the loop and the magnetic permeability."

The Law can be represented mathematically as

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I \tag{4.5.1}$$

The magnetic induction in the above equation is due to all the currents (I_1, I_2, I_3, I_4, I_5, I_6 in our case). Whereas the algebraic sum of currents on the right hand side is only of those currents which are enclosed by the closed loop. It is important to note that Ampere's Law is true only for steady currents.

Just as in case of static electricity, the electric field due to a symmetric charge distribution can be determined using Gauss' Law, the intensity of magnetic field due to symmetric current distributions can be determined in the same manner using Ampere's Law.

Gauss' Law for the electric field and Ampere's Law for the magnetic field have their own importance in physics. Gauss' Law and Ampere's Law form two basic pillars out of four pillars of Maxwell's electromagnetic theory. Third pillar is the fact that magnetic field lines form closed loops and the fourth is the concept of displacement current.

Here note that Ampere's Law is the integral form of Biot–Savart's Law and Gauss' Law is the integral form of Coulomb's Law. These representations have become very fruitful in physics.

4.5.1 Uses of Ampere's Circuital Law

(1) To Find Magnetic Field Due to a Very Long Straight Conductor Carrying Electric Current, Using Ampere's Law :

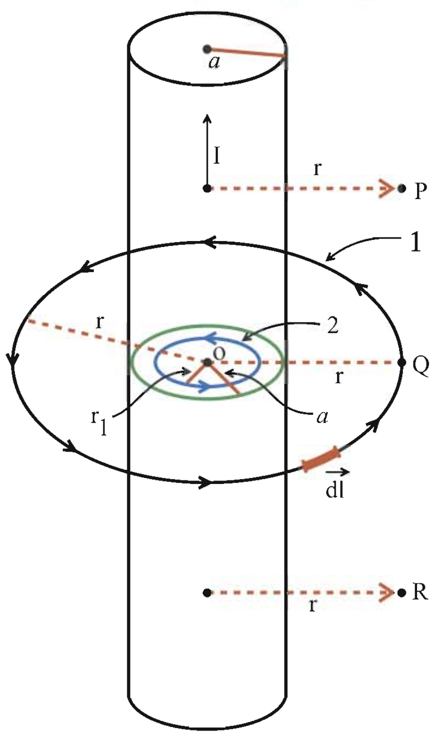


Figure 4.6 Magnetic Field Produced by Straight Conductor Carrying Electric Current

We have seen that magnetic field produced due to symmetric distribution of electric currents can easily be determined by Ampere's Law. Consider a very long (in principle infinitely long) straight conductor carrying electric current I as shown in figure 4.6.

Where is the symmetry in this case ? This can be understood as follows.

First of all see that uniform electric current I is flowing through the whole conductor. Now keep the wire between your two palms and rotate like a churn. This does not make any change in the magnetic field produced by the wire (electric current).

Now consider points like P, Q and R located at same perpendicular distance r from the wire. Both the ends of the wire are at infinite distance. Since the two ends of the wire are at infinite distance, these points P, Q and R can be considered at equal distance from the ends of the wire and in this sense they are equivalent.

This discussion of symmetry shows that the magnetic field at points like P, Q and R must be same. Moreover it is also clear from the fact of rotating the wire like churn that the magnetic field at all the points on the circumference of a circle of radius OQ = r with O at centre must also be the same. In this case we have to find magnetic field at point Q using

Ampere's Law. For this consider circle of radius OQ = r (amperean loop 1) as shown figure 4.6 which is perpendicular to the wire as a closed loop. Such a circle and line elements ($\vec{dl}$) over its circumference are shown in Figure 4.6.

Suppose the magnetic field of all such element is $\vec{B}$. Using this fact in the equation representing Ampere's Law.

$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I$, we get

$\oint B.dl \cos\theta = \mu_0 I$

As $\vec{B}$ and $\vec{dl}$ are in the same direction at every element,
$\cos\theta = \cos 0 = 1$

$\therefore \oint B.dl = \mu_0 I$

As B is constant

$B \oint dl = \mu_0 I$

Here $\oint dl = dl$ circumference of the circle with radius $r = 2r$

$\therefore B(2\pi r) = \mu_0 I$

$$\therefore B = \frac{\mu_0}{2\pi} \frac{I}{R} \quad (4.5.2)$$

Here current is positive as per our sign convention.
from equation (4.5.2)

$B \propto \frac{1}{r}$ (outside the conductor)

Magnetic Field Inside the conductor : Now as shown in the figure 4.6 radius of the wire is a and we want to find magnetic field at a perpendicular distance r_1 from its axis inside the wire that is $r_1 < a$. Consider circle with radius r_1 as amperean loop 2 as shown in figure 4.6 (which is around the axis inside the wire). If current enclosed by this loop is I_e then

$$I_e = \left(\frac{I}{\pi a^2}\right)\pi r_1^2 = I\frac{r_1^2}{a^2}$$

Using Ampere's Law

$$B(2\pi r_1) = \mu_0 \frac{r_1^2}{a^2} I$$

$$\therefore B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r_1 \quad (4.5.3)$$

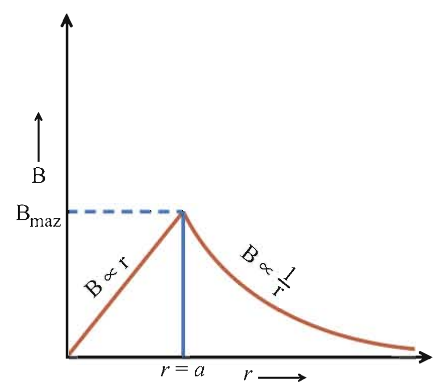


Figure 4.6 Magnetic Field B at distance r from the Centre of the Wire

Now representing r_1 by r that is for $r < a$ (for magnetic field inside the conductor)
$B \propto r$

Hence in the form of common symbol r the above facts can be represented as follows

(i) If $r > a$, then $B \propto \frac{1}{r}$

(ii) If $r < a$, then $B \propto r$

(iii) At $r = a$ B is maximum.

These facts are shown in the from of plot of $B \rightarrow r$ in figure 4.7.

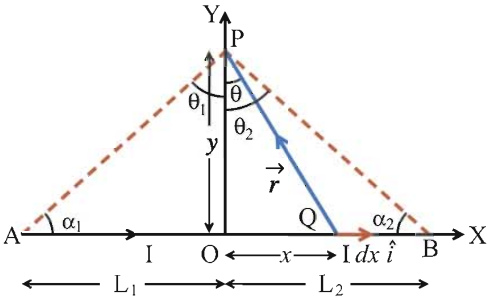


Figure 4.8 Magnetic Field Due to Current Carrying Conductor of Finite Length

For Conductor of Finite Length : To find the magnetic field $\vec{B}$ produced due to the conductor with finite length carrying current consider figure 4.8.

In this case following formula for $\vec{B}$ can be obtained using Biot–Savart Law.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{y}[sin\theta_1 + sin\theta_2]\hat{k} \qquad (4.5.4)$$

Where y is perpendicular distance of the given point P from the wire, θ_1 and θ_2 are the angles subtended with the perpendicular drawn on the wire from the given point by the lines joining given point and the ends of the wire (See Figure 4.8)

(2) Solenoid : As shown in the Figure 4.9 two identical rings carrying same current are placed closed to each other co-axially.

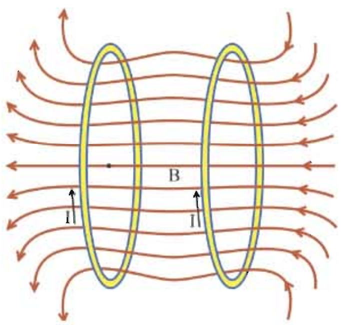


Figure 4.9

It is obvious from the Figure that the magnetic field produce due to the rings are in the same direction on their common axes. Moreover the lines close to the axis are almost parallel to the axis and in the same direction. Thus if a number of such rings (in principle of infinite number) are kept very close to each other and current is passed in the same direction, it is found that inside the region covered by the rings, the field lines are arranged at equal distance from each other obout the axis i.e. magnetic field is uniform. But the magnetic field due to two consecutive rings are in mutually opposite directions outside the rings, so they multiply each other. Hence, magnetic field in the outer region near the rings is zero. Solenoid is a device in which this situation is realized.

A helical coil consisting of closely wound turns of insulated conducting wire is called a solenoid

In practice long and short solenoids are used. **When length of a solenoid is very large as compared to its radius, the solenoid is called long solenoid.**

To find magnetic field inside a long solenoid using Ampere's Circuital Law.

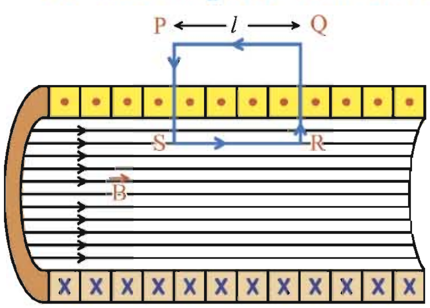


Figure 4.10 Solenoid

Figure 4.10 shows a cross-section of a long solenoid taken with plane of the page of the book. Symbol (X) shows the direction of currents going inside the plane of the page and symbol (·) shows the directions of the current coming out of the plane of the page.

Suppose we want to find the magnetic field at point S lying inside the solenoid. Considering a rectangular loop of length l, PQRS as shown in the Figure 4.10 as Amperean loop, we will take line integral $\vec{B}$ over the loop.

$$\therefore \oint \vec{B}.\vec{dl} = \int_P^S \vec{B}\cdot\vec{dl} + \int_S^R \vec{B}\cdot\vec{dl} + \int_R^Q \vec{B}\cdot\vec{dl} + \int_Q^P \vec{B}\cdot\vec{dl}$$

From the figure 4.10 it is clear that the magnetic field on part PQ of the loop will be zero as it is lying outside the solenoid and hence $\int_{Q}^{P} \vec{B} \cdot \vec{dl} = 0$

Moreover, some part of sides QR and SP is outside the solenoid and the part which is inside is perpendicular to the magnetic field, therefore $\int_{R}^{Q} \vec{B} \cdot \vec{dl} = \int_{P}^{S} \vec{B} \cdot \vec{dl} = 0.$

$$\therefore \oint \vec{B} \cdot \vec{dl} = \int_{S}^{R} B dl \cos 0^{\circ} = B \int_{S}^{R} dl = Bl \qquad (4.5.5)$$

Now suppose that the number turns per unit length of the solenoid is n. Therefore, the number of turns passing through the Amperean loop is nl. Current passing through each turn is I, so total current passing through the loop is $\Sigma I = nlI$.

From Ampere's Circuital Law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 nlI$$

$$\therefore Bl = \mu_0 nlI \quad \text{(from equation 4.5.5)}$$

$$\therefore B = \mu_0 nI \qquad (4.5.6)$$

This method can be used only for a long solenoid because only in case of a long solenoid, all the points inside the solenoid can be considered equivalent and magnetic field inside the solenoid as uniform. In the region outside the solenoid in the vicinity of it is zero. This method should not be used for a solenoid of finite length.

For Solenoid of Finite Length : For solenoid of finite length magnetic field inside of it can be determined using Biot–Savart's Law. For this consider figure 4.11. Formula for the magnetic field inside the solenoid of finite length is as under.

$$B = \frac{\mu_0 nI}{2}(\sin\alpha_1 + \sin\alpha_2) \qquad (4.5.7)$$

Here α_1 and α_2 are the angles subtended by two ends of the solenoid with normal drawn at point P respectively.

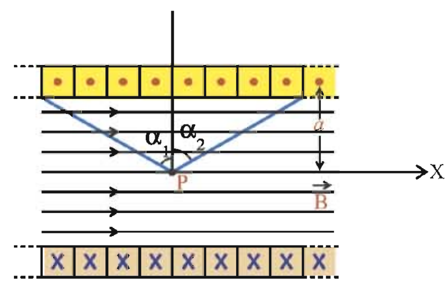


Figure 4.11 Solenoid of Finite Length

Toroid : If a solenoid is bent in the form of a circle and its two ends are joined with each other then the device is called a toroid.

A toroid can also be prepared by closely winding an insulated conducting wire arround non-conducting hollow ring. (In short, the shape of a toroid is the same as that of an inflated tube, also called doughnut shape.) The magnetic field produced inside the toroid carrying electric current can be obtained using Ampere's Circuital Law.

Suppose we want to find the magnetic field at a point P inside the toroid which is at a distance r from its centre as shown in the figure 4.12. If we consider a circle of radius r with its centre at O as an Amperean loop from the symmetry it is clear that the magnitude of the magnetic field at every point on the loop is same and directed towards the tangent to the circle. Therefore,

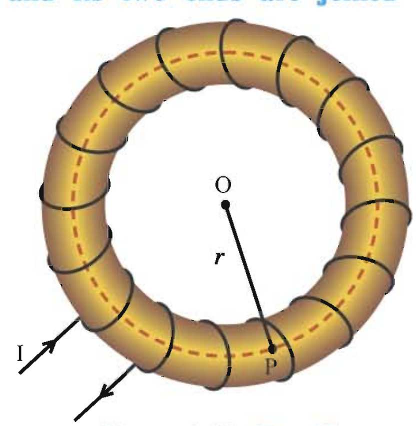


Figure 4.12 Toroid

$$\oint \vec{B}.\vec{dl} = \oint B dl = B \oint dl = B(2\pi r) \quad (4.5.8)$$

If the total number of turns is N and current passing is I, the total current passing through the said loop must be NI. From Ampere's Circuital Law,

$$\oint \vec{B}.\vec{dl} = \mu_0 NI \quad (4.5.9)$$

Comparing equations (4.5.8) and (4.5.9)

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 nI}{2\pi r} = \mu_0 NI \quad (4.5.10)$$

Here, $n = \frac{N}{2\pi r}$ the number of turns per unit length of the toroid. This is the equation of magnetic field produced inside the toroid. This magnetic field is uniform at each point inside the toroid.

In an ideal toroid, the turns are completely circular. In such a toroid magnetic field the inside the toroid is uniform and outside the toroid is zero. But in the toroid used in practice, the will is helical and hence, a small magnetic field also exist outside the toroid.

For nuclear fusion, the device Tokamak is used for the confinement of plasma. Toroid is an important component of Tokamak.

4.6 Force on a Current Carrying Wire Placed in a Magnetic Field

Within week of the publicity of the news of Oersted's observation scientist Ampere made another observation. In this observation he showed that **"Two parallel wires placed near each other exert an attractive force if they are carrying currents in the same direction, and exert a repulsive force if they are carrying currents in the opposite directions."**

We have seen that magnetic field is created around the wire carrying electric current. Now, if another wire carrying current is placed in its neighbourhood (i.e. second wire carrying current is placed in the magnetic field produced by the current in the first wire) then the force acts on the other wire due to magnetic field produced by current in the first wire. In the same manner the first wire is lying in the magnetic field produced by the current in the other wire. Hence the force acts on the first wire due to the magnetic field produced by the current in the other wire. This is the magnetic force between two wires.

This interaction can schematically be represented as follows.

$$\left.\begin{matrix}\text{Current in the}\\ \text{first wire}\end{matrix}\right\} \underset{\leftarrow}{\rightarrow} \text{Magneticfield} \left\{\begin{matrix}\text{Current in the}\\ \text{second wire}\end{matrix}\right.$$

Thus in other words the force acts between the two wires (carrying current) is due to magnetic field.

To find this force acting between two wires, one must know, the force acting on a wire currying a current placed in magnetic field. The Law giving this force was established by Ampere through the experimental studies is as under :

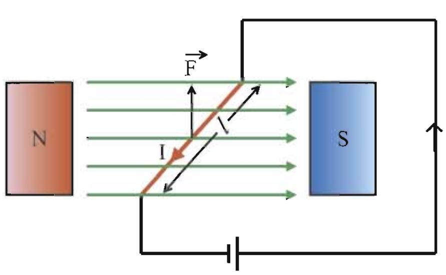


Figure 4.13

The force acting on a current element $I\vec{dl}$ due to the magnetic induction $\vec{B}$ is given by

$$\vec{dF} = I\vec{dl} \times \vec{B} \quad (4.6.1)$$

If a straight wire of length l carrying current I is placed in uniform magnetic field $\vec{B}$, the force acting on the wire can be given by

$$\vec{F} = I\vec{l} \times \vec{B} \quad (4.6.2)$$

Such an arrangement is shown in the Figure 4.13.

The direction of force can be determined using the right hand rule for vector product.

4.6.1 The Force between Two parallel Current Carrying Wires

Consider two very long conducting wires placed parallel to each other along X-axis, separated by a distance y and carrying currents I_1 and I_2 in the same direction (See figure 4.14)

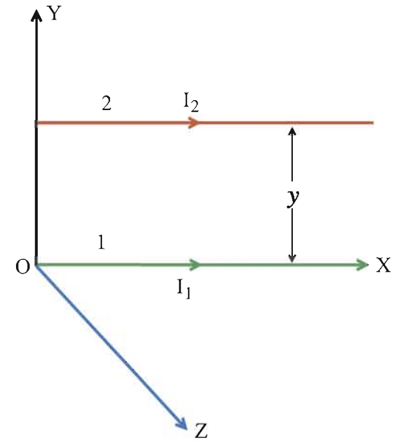


Figure 4.14

Magnetic field at a distance y from first conductor carrying current I_1 is

$$\vec{B}_1 = \frac{\mu_0}{2\pi} \cdot \frac{I_1}{y}\hat{k} \quad (4.6.3)$$

The strength of this field is same at all points on the second wire carrying current I_2 and directed along Z-axis. Therefore, the force acting on the second wire over its length l will be

$$\vec{F}_2 = I_2\vec{l} \times \vec{B}_1 \quad \text{(From equation 4.6.2)}$$

substituting value of B_1 from equation (4.4.3) in the above equation

$$\vec{F}_2 = I_1 I_2 \frac{\mu_0}{2\pi y} l\hat{i} \times \hat{k} \quad \text{(As current } I_2 \text{ being along the X-axis)}$$

$$\therefore \vec{F}_2 = -\frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{y}\hat{j} \quad (4.6.4)$$

Above equation shows that the force $\vec{F}_2$ acts along negative Y-direction.

Now the force $\vec{F}_1$ acting on the first wire carrying current I_1 can be obtained in the same manner which is as under :

$$\vec{F}_1 = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{y}\hat{j} \quad (4.6.5)$$

The above equation shows that the force F_1 acting on the first wire is in positive y direction.

$$\text{Thus } \vec{F}_1 = -\vec{F}_2 \quad (4.6.6)$$

This fact shows that force acting between indicates attraction takes place between them.

If the currents are flowing in the mutually opposite directions in the two wires then repulsion is produced between them.

From equation (4.6.6) it is obvious that here also Newton's third Law is obeyed.

Definition of Ampere :

In equation (4.6.4) if we take

$I_1 = I_2 = 1A$, $y = 1$ m and $l = 1$m

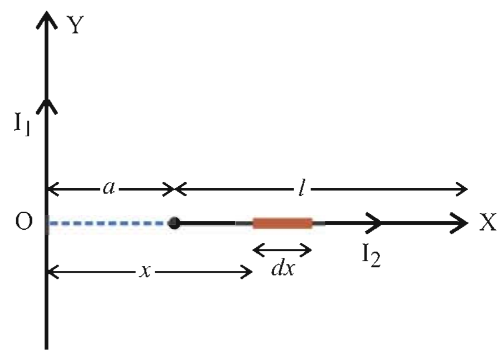
$$|\vec{F}_2| = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N} \quad (4.6.7)$$

Using this fact definition of SI unit of 1 ampere current is given as under :

"When the magnetic force acting per metre length in two infinitely long wires placed parallel to each other at a distance of 1 meter in vacuum, carrying identical current is 2×10^{-7} N, the current passing through each wire is 1 ampere."

Illustration 5 : As shown in the figure very long conducting wire carrying current I_1 is arranged in y direction. Another conducting wire of length l carrying current I_2 is placed on X-axis at a distance from this wire. Find the torque acting on this wire with respect to point O.

Solution : The force acting on a current element $I_2 dx$ located at a distance x from O is,



$$d\vec{F} = I_2 dx\,\hat{i} \times \vec{B}$$

where, $\vec{B} = \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$

(the magnetic field due to a very long conductor)

$$\therefore\ d\vec{F} = I_2 dx\,\hat{i} \times \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi x}\hat{j}$$

$\therefore$ The torque acting on this element with respect to O is,

$$d\vec{\tau} = x\hat{i} \times d\vec{F} = x\hat{i} \times \frac{\mu_0 I_1 I_2 dx}{2\pi x}\hat{j} = \frac{\mu_0 I_1 I_2}{2\pi} dx\,\hat{k}$$

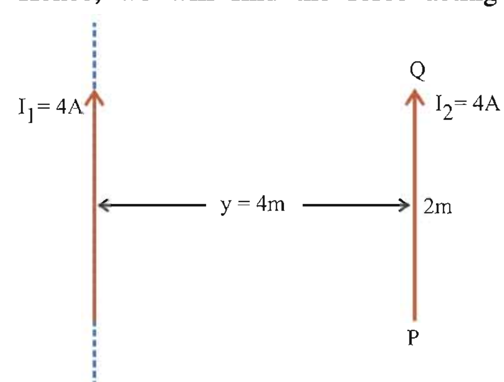
Total torque acting on this coil can be obtained by taking integration of this equation between $x = a$ to $x = a + l$,

$$\therefore\ \vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi}\int_a^{a+l} dx\,\hat{k} = \frac{\mu_0 I_1 I_2}{2\pi}[x]_a^{a+l}\,\hat{k} = \frac{\mu_0 I_1 I_2}{2\pi}[a + l - a]\,\hat{k}$$

$$\therefore\ \vec{\tau} = \frac{\mu_0 I_1 I_2 l}{2\pi}\hat{k}$$

Illustration 6 : As shown in the figure, a straight wire PQ of length 2 m carrying 2A current is placed parallel to a very long wire at a distance of 2m. Find the force acting on wire PQ if the current passing through the long wire is also 2A.

Solution : According to Newton's 3rd Law of motion, the force exerted by the smaller wire on the longer wire is the same as the force exerted by the long wire on the smaller one. Hence, we will find the force acting on the smaller wire.



Suppose magnetic field on the smaller wire due to the longer wire is $\vec{B}$

$$\therefore\ \vec{B} = \frac{\mu_0 I_1}{2\pi y}\hat{n} \qquad (1)$$

where $\hat{n}$ is the unit vector in the direction of $\vec{B}$. (1)

Now force on the longer wire is,

$$\vec{F} = I_2 \vec{l} \times \vec{B}$$

$\therefore\ |\vec{F}| = I_2 l B\ (\because\ \vec{l} \perp \vec{B})$

Using the equation (1),

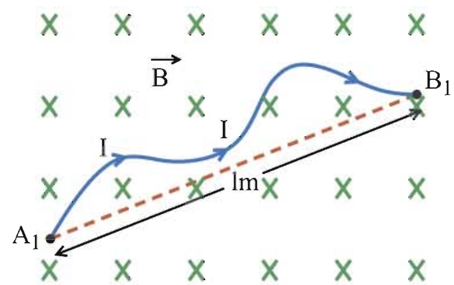
$$\therefore\ |\vec{F}| = \frac{I_1 l \mu_0 I_1}{2\pi y}$$

$$= \frac{2\times2\times4\times3.14\times10^{-7}\times2}{2\times3.14\times2}$$

$\therefore\ |\vec{F}| = 8 \times 10^{-7}$ N

This force produced here is attractive.

Illustration 7 : A wire carrying electric current I is placed on the plane of paper. A magnetic field of induction $\vec{B}$ is applied in a direction going into the plane of paper normally. Find the force acting on the wire.



A straight line joining A_1 and B_1, which is not a part of the wire, of length 1 m is shown in the figure.

Solution : The force acting on a current element $I\vec{dl}$ due to the magnetic field $\vec{B}$ is,

$$\vec{dF} = I\vec{dl} \times \vec{B}$$

$\therefore$ The total force acting on the wire is,

$\vec{F} = \int I\cdot\vec{dl} \times \vec{B}$ (Here integration is taken over the whole length of the wire.) Here, n is the number of (free) charge carriers per unit volume of the conductor.

$$\therefore\ \vec{F} = I[\int_l \vec{dl}\,] \times \vec{B}$$

But, $\int \vec{dl} = \vec{A_1B_1} = 1\hat{n}\ (\because\ A_1B_1 = 1\text{m})$

where, $\hat{n} = \vec{A_1B_1}$ the unit vector in the dirction of

$$\therefore\ \vec{F} = I\hat{n} \times \vec{B} \Rightarrow |\vec{F}| = IB$$

4.7 Force on an Electric Charge Moving in a Magnetic Field and Lorentz Force

In Chapter-3 we studied that the current I flowing through a cross section A of a conductor is

$I = nAv_d q$

Here q = Charge on the positively charged particle.

n = number of (free) charge carrier per unit volume of the conductor

v_d = drift velocity

$\therefore\ I\vec{dl} = qnAv_d\vec{dl} = qnA\vec{v_d}\,dl\ (\because\ v_d$ and dl are in the same direction)

When this conductor is placed in a magnetic field of intensity $\vec{B}$, the force acting on current element $I\vec{dl}$ is given by

$$\vec{dF} = I\vec{dl} \times \vec{B}$$

$$\therefore \vec{dF} = qnAdl(\vec{v_d} \times \vec{B}) \quad (4.7.1)$$

But $nAdl$ = total number of charged particle in current element

∴ the magnetic force acting on a single particle of charge q will be given by

$$\vec{F}_m = \frac{\vec{dF}}{nAdl} = \frac{qnAdl(\vec{v_d}\times\vec{B})}{nAdl}$$

$$\therefore \vec{F}_m = q(\vec{v_d} \times \vec{B}) \quad (4.7.2)$$

$|\vec{F}_m| = Bqv_d \sin\theta$. This shows (i) if charge is stationary this force is zero (ii) moreover if charge is moving paralel or anti-parallel to the magnetic field then also this force is zero.

Now, if this electric charge q is moving in the electric field of intensity $\vec{E}$ over and above the magnetic field $\vec{B}$, the force $\vec{F}_e = \vec{E}\cdot q$ due to electric field acts on the charge q. In this circumstanes total force acting on the charge.

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\therefore \vec{F} = q[\vec{E} + (\vec{v_d} \times \vec{B})] \quad (4.7.3)$$

$$\therefore |\vec{F}_m| = Bqv_d \sin\theta$$

the force obtained by this equation is called Lorentz Force.

The magnetic force acting on a charge moving through the magnetic field is perpendicular to the velocity of the particle, work done by the force is zero and hence its kinetic energy remains constant. Only direction of velocity goes on changing at every instant.

The magnitude of the magnetic force depends on the velocity of the particle, hence such a force is called velocity dependent force.

Illustration 8 : A particle having 2 C charge passes through magnetic field of $4\hat{k}$ T and some uniform electric field with velocity $25\hat{j}$. If the Lorentz force acting on it is $400\hat{i}$ N find the electric field in this region.

Solution : Lorentz force

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

Here, $q = 2$ C, $\vec{v} = 25\hat{j}$ m s^{-1}, B $= 4\hat{k}$T, $\vec{F} = 400\hat{i}$

$$\therefore 400\hat{i} = 2 \quad [\vec{E} + (25)(4)(\hat{j} \times \hat{k})]$$

$$= 2\vec{E} + 200\hat{i}$$

$$\therefore 2\vec{E} = 200\hat{i}$$

$$\therefore \vec{E} = 100\hat{i} \text{ V m}^{-1}$$

Illustration 9 : In copper there are 8×10^{28} free (conducting) electrons per cubic meter. A current copper wire having length 1 m and cross-sectional area 8×10^{-6} m^2 is placed perpendicularly in the magnetic field of 4×10^{-3} T. The force acting on this wire is $8\text{-}0 \times 10^{-2}$ N. Find the drift velocity of the free electron.

Solution : Magnetic force acting on the wire is given by the formula $\vec{F} = I\vec{l} \times \vec{B}$. Here wire perpendicular to the magnetic field. $|\vec{F}| = IlB$ where $F = 8.0 \times 10^{-2}$, $B = 4.0 \times 10^{-3}$T and $l = 1$m

$$\therefore I = \frac{F}{Bl} = \frac{8\times10^{-2}}{4\times10^{-3}\times1} = 20 \text{ A.}$$

Now $I = Av_d n.e$

n = No. of electrons in the unit volume = 8×10^{28}

$A = 8 \times 10^{-6}$ m^2 and $e = 1.6 \times 10^{-19}$ C

$$\therefore v_d = \frac{I}{n.A.e}$$

$$= \frac{20}{8\times10^{28}\times8\times10^{-6}\times1.6\times10^{-19}} = 1.953 \times 10^{-4}$$

$$\approx 2 \times 10^{-4} \text{ m s}^{-1}$$

Illustration 10 : Write the equation of magnetic force acting on a particle moving through a magnetic field. Using it obtain Newton's equation of motion and show that kinetic energy of the particle remains constant with time.

Solution : $\vec{F}_m = q(\vec{v} \times \vec{B})$

$$\therefore m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B})$$

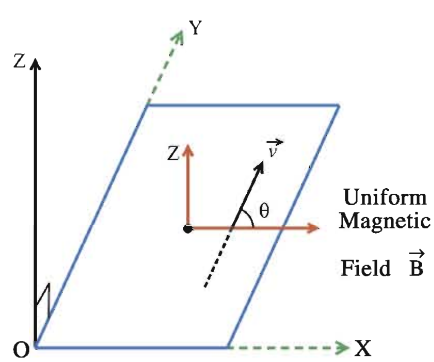
Taking dot product $\vec{v}$ with on both the sides,

$$m\vec{v} \cdot \frac{d\vec{v}}{dt} = q\vec{v}(\vec{v} \times \vec{B})$$

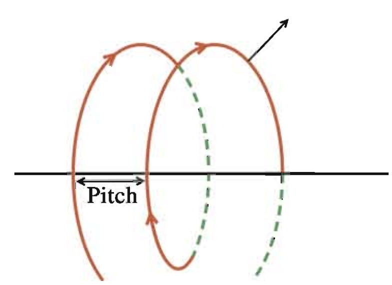
$$\therefore m\frac{1}{2}\frac{d}{dt}(\vec{v} \cdot \vec{v}) = 0 \ (\because .\vec{v} \text{ and } \vec{v} \times \vec{B} \text{ are mutually perpendicular})$$

$$\therefore \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0 \Rightarrow \frac{1}{2}mv^2 = \text{constant}$$

Illustration 11 : Suppose a particle of mass m and charge q is incident on XZ plane with velocity v in a direction making angle θ with a uniform magnetic field applied along X-axis according to figure (a). Show that motion of this particle is helical and find the pitch of the path.



(a)



(b)

Solution : Considering two components of velocity in XZ plane,

$v_z = v\ sin\theta$ and $v_x = v\ cos\theta$

As v_x component is in the direction of magnetic field, $qv_x\hat{i} \times B\hat{i} = 0$. Since this force is zero, the particle will continue to move with constant velocity $v_x = v\ cos\theta$ along X axis.

Now the force due to v_z component $= qv_z\hat{k} \times B\hat{i} = qv_zB\,\hat{j}$. This force acts perpendicularly to v_z hence the particle will perform circular motion on YZ plane with linear velocity v_z.

Now the centripetal force needed for circular motion is,

$$\frac{mv_z^2}{r} = qv_zB$$

$$\therefore\ r = \frac{mv_z}{qB} = \frac{mv\ sin\theta}{qB}$$

Radius of the circular path of the particle can be determined using above equation, period, $T = \frac{2\pi r}{v_z}$.

$$\therefore\ T = \frac{2\pi r}{v\ sin\theta} = \frac{2\pi m}{qB}$$

The particle covers a distance of v_xT during the time interval equal to its period along X axis.

$$\therefore \text{ distance travelled along X direction } = \frac{2\pi mv_x}{qB} = \frac{2\pi mv\ cos\theta}{qB}$$

It is clear from this discussion that the particle moves on a helical path whose axis is along X direction. Here, distance v_xT is called the pitch of the helix (See figure (b)).

4.8 Cyclotron

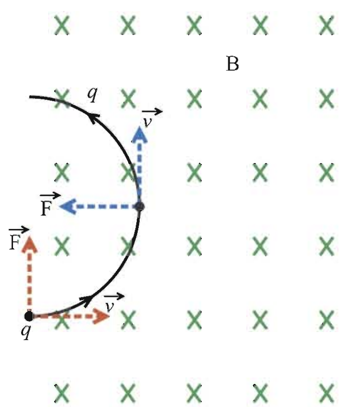


Figure 4.15
Motion of Charged Particle Entering Normally in the Magnetic Field

In the study of nuclear structure very high energy particles are required to be Bombarded on the Nucleus. For this purpose the charged particles are to be accelerated. To do so E.O. Lawrence and M. S. Livingston developed an instrument called cyclotron.

In this instrument the force on a charged particle moving perpendicularly inside a magnetic field is being used. Hence to understand its working we have to study the motion of a charged particle moving perpendicularly inside a magnetic field.

Consider a particle with charge q, moving with velocity $\vec{v}$ in the magnetic field of induction $\vec{B}$ as shown in the figure 4.15.

Here the magnetic field $\vec{B}$ perpendicularly entering into the plane of paper and the electron is maving in the plane of paper.

According to equation (4.7.2), the magnetic force on this particle is $\vec{F} = q(\vec{v} \times \vec{B})$

The value of this force is $qvBsin\theta$ and the direction is normal to the plane formed by $\vec{v}$ and $\vec{B}$. Here, since the particle is moving perpendicular to the magnetic field the value of this force is qvB. It is clear that in this condition the path of the particle will be circular. Since this force is normal to its velocity at every moment, the value of velocity will not change, only its direction will be continuously changing. As a result it will perform circular motion. The necessary centripetal force for this motion is the magnetic force Bqv.

$$\therefore\ qvB = \frac{mv^2}{r}$$

Where m = mass of particle and r = radius of circular path.

$$\therefore\ r = \frac{mv}{qB} = \frac{p}{qB} \qquad (4.8.1)$$

This equation shows that the radius of the circular path of the particle is proportional to the momentum of particle $p = mv$. If the momentum increases the radius of the circular path of the particle also increases.

Here for the circular motion we can write $v = rw_c$. w_c is the angular frequency of the particle which is called the cyclotron frequency. Substituting this value in equation (4.8.1), we get

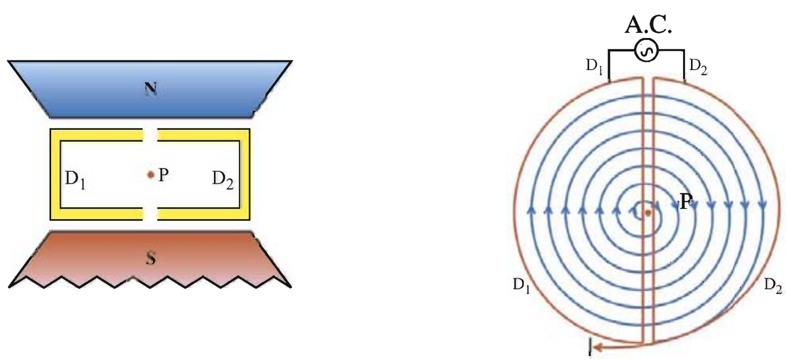
$$r = \frac{m(w_C r)}{qB}$$

$$\therefore\ w_C = \frac{qB}{m} \qquad (4.8.2)$$

$$\therefore\ f_C = \frac{qB}{2\pi m} \qquad (4.8.3)$$

This f_C is called cyclotron frequency.

Here, it is clear that the angular frequency of the particle w_C does not depend on its momentum. Hence on increasing the linear momentum of the particle, the radius of its cercular path definitely increases but the frequency w_C does not change. This fact is used in the design of a cyclotron.



(a) Side View (b) Top View

Figure 4.16 Schematic Diagram of Cyclotorn

Construction : Two hollow matallic boxes of D-shape are kept in front of each other with their diameters facing each other and with a small gap between them as shown in the figure 4.16. Two strong electromagnets are kept in such a way that a uniform magnetic field is developed in the space enveloped by the two boxes. These two boxes are called Dees as they are D-shaped. An A.C. of high frequency is applied between the two Dees. This device is then kept in an evacuated chamber in order to avoid the possible collision of charged particle with the air molecules.

Working : Suppose a charged particle is released from the centre P of the gap between the Dees of time $t = 0$. Exactly at the same time suppose one of the Dees is at negative potential. **If the particle is positively charged,** it gets attracted towards this Dee. Now as a uniform magnetic field is existing in the space between the Dees, the charged particle performs circular motion in the gap and enters the magnetic field in the Dees perpendicularly with a certain momentum. Now there is no electric field in the Dees, hence the particle moves on a circular path of radius depending on its momentum and comes out of the Dee after completing a half circle.

Now, if the opposite Dee becomes negative at the moment at which the particle emerges from one Dee the particle gains momentum due to electric field while passing through the gap before entering the other Dee. It moves in the other Dee on a circular path of larger radius. When this particle emerges out from the second Dee, if the opposit Dee acquires negative potential, the particle gets even more momentum and moves on a circular path of even greater radius in this Dee.

If this process is repeated the radius of circular path goes on increasing but the frequency w_c remains constant. To make this possible the frequency of A.C. voltage (f_{AC}) should be equal to the frequency of revolution f_C. (Here $w_C = 2\pi f_C$). This is nothing but resonance.

In this manner the charged particle goes on gaining energy which becomes maximum on reaching the circumference of the Dee.

For bambarding this charged particle on some target it should be brought out of the Dee. For this when the particle is on the edge, it is brought out of the Dee by deflecting with the help of another magnetic field and allowed to hit the nuclei of the atoms of target.

Here, we have discussed about accelerating positively charged particle (e.g. proton, positive ions), such accelerated particles are used in the study of nuclear reactions, preparation of artificial radioactive substances, treatment of cancer and ion implantation in solids.

Limitations : According to the theory of relativity as the velocity of particle approaches that of light, its mass goes on increasing. In this situation the condition of resonance ($f_{AC} = f_C$) is not satisfied.

To accelerate very light particles like electron, the frequency of A.C. is required to be very high (of the order of GHz)

Moreover, the size of Dees is also large. It is difficult to maintain a uniform magnetic field over a large region. Hence accelerators like synchrotron are developed.

4.9 Torque Acting on a Rectangular Current Carrying Coil Kept in Uniform Magnetic Field

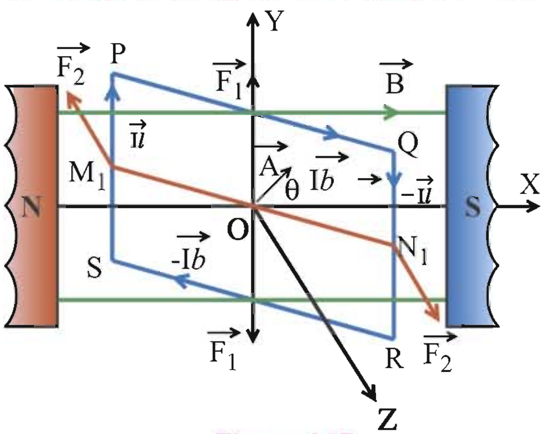


Figure 4.17

Consider a rectangular coil of length QR = l and width PQ = b carrying Current I as shown in figure 4.11. Here, direction of the magnetic field $\vec{B}$ is taken along X-axis.

$\therefore \vec{B} = B\hat{i}$

The force acting on the element constituted by side PQ of the coil $= I\vec{b}$.

Therefore force acting on this element $\vec{F_1} = I\vec{b} \times \vec{B}$. (Positive Y-direction). Similarly the force acting on the element formed by side RS is $\vec{F_1}' = I\vec{b} \times \vec{B}$ (negative Y-direction).

Here, forces $\vec{F_1}$ and $\vec{F_1}'$ are equal in magnitude, opposite in direction and collinear hence, they cancel each other.

Now consider the element (QR)I $= -Il\hat{j}$. The force acting on it

$$\vec{F_2} = -Il\hat{j} \times \vec{B}\hat{i} = -IlB(\hat{j} \times \hat{i}) = IlB\hat{k} \qquad (4.9.1)$$

is along positive Z-direction.

Similarly the force acting on the element (SP) I $= Il\hat{j}$ is

$$\vec{F_2}' = Il\hat{j} \times \vec{B}\hat{i} = -IlB\hat{k} \qquad (4.9.2)$$

is in negative Z-direction.

Equations (4.9.1) and (4.9.2) show that $|\vec{F_2}| = |\vec{F_2}'|$

It is also clear from the figure 4.17 that they are opposite in direction. But they are non-collinear. So they constitute a torque (couple)

Viewing the coil from above (in negative Y–direction), $\vec{F_2}$, $\vec{F_2}'$, X-axis and vector $\vec{A}$ appear as shown in figure 4.18. Here $\vec{A}$ is the vector representing the area of the plane of the coil which makes an angle θ with X-axis.

Thus,

Torque acting on coil = (magnitude of a force) (Perpendicular distance between two forces)

The perpendicular distance between two forces is (See Figure 4.18)

$$M'N' = 2\frac{b}{2}\cos\left(\frac{\pi}{2} - \theta\right) = b\sin\theta \qquad (4.9.3)$$

$$\therefore \text{ Torque } |\vec{\tau}| = |\vec{F_2}| \,(M'N') = (IlB)(b\sin\theta) \qquad (4.9.4)$$

$$\therefore |\vec{\tau}| = IAB\sin\theta$$

Where lb = A is the area of the coil.

For coil having N turns,

$$|\vec{\tau}| = NIAB \sin\theta \qquad (4.9.5)$$

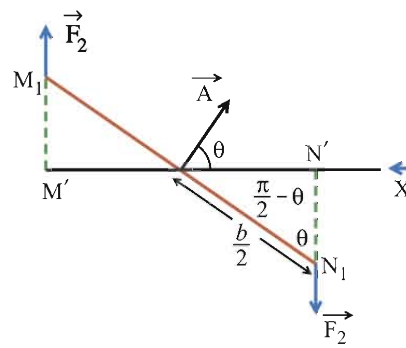


Figure 4.18 Torque Acting on Rectangular Coil

Taking area A of the coil in the vector form, equation (4.9.5) can be written in the vector form as

$$\vec{\tau} = NI\vec{A} \times \vec{B} \qquad (4.9.6)$$

The vector quantity $NI\vec{A}$ is called "magnetic moment" linked with the coil and denoted by ($\vec{\mu}$)

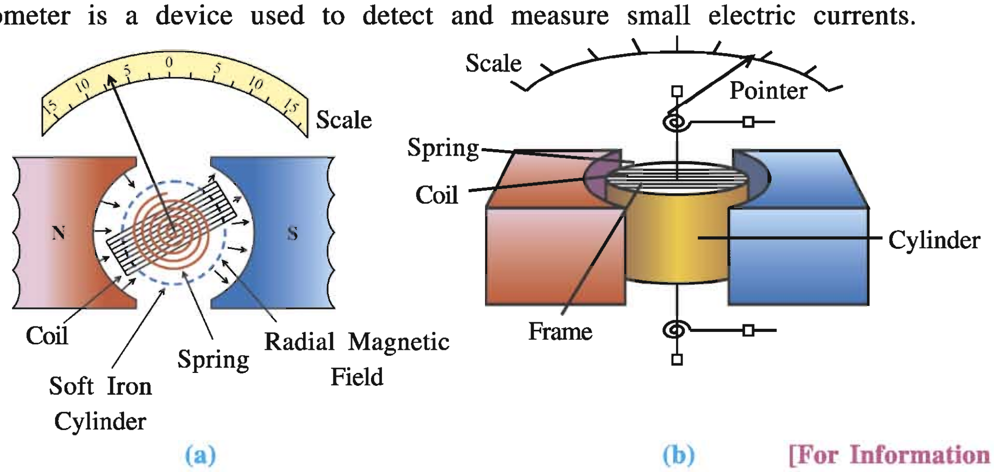
$$\therefore \vec{\tau} = \vec{\mu} \times \vec{B} \qquad (4.9.7)$$

equation (4.9.7) is valid for any shape of the coil.

Direction of $\vec{\mu}$ can be determined using right hand screw rule. Keep a right hand screw perpendicular to the plane of the coil and rotate it in the direction of current, the direction in which screw advances shifts gives the direction of $\vec{\mu}$.

4.10 Galvanometer

Galvanometer is a device used to detect and measure small electric currents.



[For Information Only]

Figure 4.19 Construction of Galvanometer

In galvanometer, a coil of thin insulated copper wire is wound on a light rectangular (non-magnetic) frame. The frame is pivoted between two almost frictionless pivots and placed between two cylindrical poles of a permanent magnet so that it can freely move in the region between the poles. A small soft iron cylindrical core is placed at the axis of the coil (free from coil) so that uniform radial magnetic field is produced. When current is passed through the coil a torque acts on it and deflected. The steady deflection coil is indicated by a pointer attached with it. Knowing the position of the pointer on the scale current can be known.

Principle and Working : If the area vector of the coil marks an angle θ with the magnetic field, from equation (4.9.5) torque acting on the coil.

$\tau = NIAB\sin\theta$ (where N = number of turns in the coil) (4.10.1)

(For Information Only : In the present case magnetic field is radial)

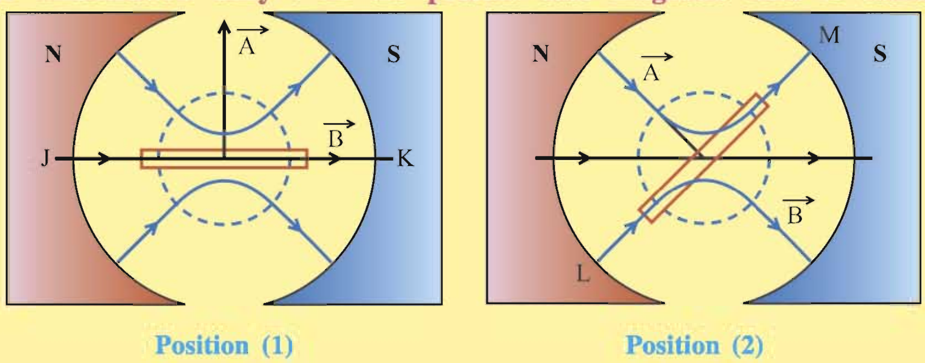


Figure 4.20

Figure 4.10 represent figure 4.20 the radially uniform magnetic field obtained in presence of a cylinder of soft iron. For convenience only a few magnetic field lines are shown here. When the coil is in position 1, the line JK is the only effective line. In this case the angle between $\vec{A}$ and $\vec{B}$ is 90°.

Similarly for position 2 of the coil, the line LM becomes effective. In this case also the angle between $\vec{A}$ and $\vec{B}$ is 90°. Thus for any position of the coil the angle between $\vec{A}$ and $\vec{B}$ is 90°.

Due to the raidal field, the angle between $\vec{A}$ and $\vec{B}$ will always be 90°.

$\therefore \tau = NIAB$ (4.10.2)

which is called deflecting torque. (The torque due to which the coil is deflected.)

Due to the deflection of the coil, the restoring torque is produced in the springs which is directly proportional to the deflection of the coil.

$\therefore \tau$ (restoring) $= k\phi$ (4.10.3)

Here k = effective torsional constant of the springs.

If the coil becomes steady after a deflection ϕ,

Deflecting torque = Restoring torque. $NIAB = k\phi$

$$\therefore I = \left[\frac{k}{NAB}\right]\phi \quad (4.10.4)$$

$$\therefore I \propto \phi \quad (4.10.5)$$

The scale of a galvanometer can be appropriately calibrated to measure I by knowing ϕ.

From equation (4.10.5)

$$\frac{\phi}{I} = \frac{NAB}{k} \quad (4.10.6)$$

Where $\frac{\phi}{I}$ is called current sensitivety(s) of the galvanometer.

Thus, deflection produced per unit current is called current sensitivity of the galvanometer one of the ways to increase the current sensitivity of the galvanometer is to use stronger magnetic field $\vec{B}$.

To measure very weak currents of the order of 10^{-11} A, the galvanometer with coil suspended by an elastic fibre between magnetic poles are used.

4.10.1 Measurement of Electric Current and Potential Difference

We often need to measure the parameters related to a circuit component like the electric current passing through it and the potential difference across its two ends. The instruments to measure these quantities are called an ammeter and a voltmeter respectively. The basic instrument to measure electric current or the voltage is the galvanometer.

4.10.1 (a) Ammeter : A galvanometer has to be joined in series with the component through which the electric current is to be measured. If the potential difference between the two ends of a component is to be measured, the galvanometer has to be joined in parallel between these two ends.

In practice if a galvanometer is directly used as a current–meter, two difficulties arise.

(1) To measure the electric current passing through a component of a circuit, the current–meter is to be joined in series with that component. As for example, we want to measure current passing through the resistance R in a circuit shown in the figure 4.21(a). For this purpose, current meter is joined in series with resistance R, as shown in the figure 4.21(b). In such a connection the resistance G of the galvanometer is added in the circuit. As the total resistance of the circuit is changed the value of current to be measured itself is changed. Thus the true value of current is not obtained. This fact indicates that the resistance of current meter should be as small as possible (in principle zero)

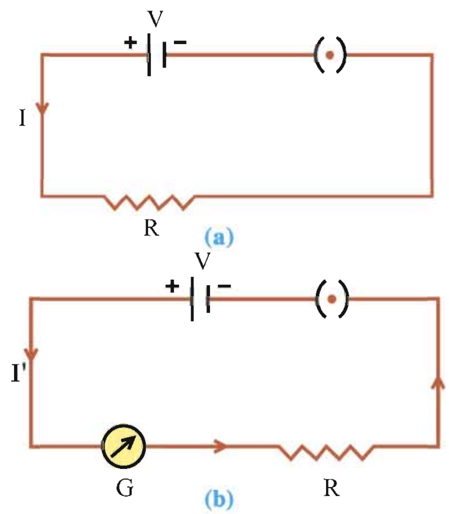


Figure 4.21

(2) Moreover, the moving coil galvanometers are very sensitive. Even when a small fraction of one ampere current (of the order of 10^{-6} A) passes through it, it shows full scale deflection. The electric current, for which the galvanometer shows full scale deflection, is called the current capacity of galvanometer (I_G). If the galvanometer is used to measure a current greater than its range (current capacity), it is likely to be damaged.

Moreover due to larger current passing through thin copper wire of its coil, large quantity of heat is produced according to I^2Rt and hence it is likely to be burnt.

In order to remove the above mentioned difficulties a resistance of proper small value is joined in parallel to the coil of galvanometer. This resistance is called a **Shunt**. As the value of shunt is very much smaller than the resistance of galvanometer (G), most of the current passes through the shunt and the galvanometer is protected against the damage.

Moreover the shunt and the resistance of galvanometer being in parallel their equivalent resistance becomes even smaller than the value of shunt. Thus after joining the shunt the resistance of the current meter becomes very small. Hence both of the above mentioned difficulties are removed.

Known currents are passed through the instrument prepared after joining the shunt and its scale is calibrated in ampere, milliampere or microampere.

The instrument thus prepared is called ammeter, milliammetre or microammeter respectively. For this purpose the proper value of shunt is obtained as follows :

Formula for shunt : Suppose a galvanometer having resistance G and current capacity I_G is to be converted into an ammeter which can measure a maximum current I. For this the value of required shunt is suppose S. Here the shunt should be so chosen that out of current I, only I_G current passes through the galvanometer and the remaining $I_S = I - I_G$ current passes through the shunt. This situation is shown in the figure 4.22.

Using Krichoff's first Law, at junction A,

$$I = I_G + I_S \quad (4.10.7)$$

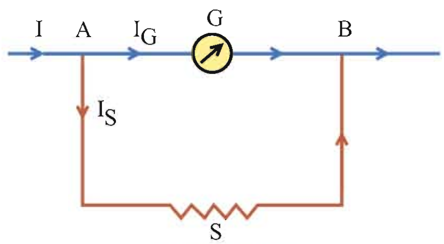


Figure 4.22

Using Kirchoff's second Law on ASBGA path,

$$-I_G G + I_S S = 0$$

$$\therefore S = \frac{GI_G}{I_S}$$

From equation 4.10.7, $I_S = I - I_G$

$$\therefore S = \frac{GI_G}{I - I_G} \quad (4.10.8)$$

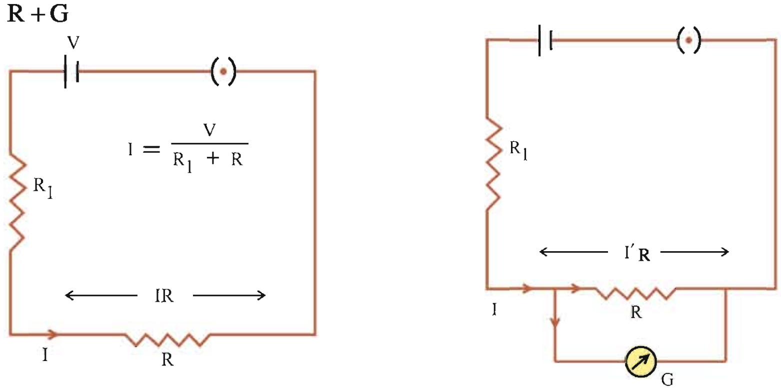
This is the formula for the required shunt. It is clear from this that in order to make the range of ammeter higher and higher the value of the required shunt is smaller and smaller.

To make the range of ammeter n times, the required shunt will be $S = \frac{G}{n-1}$, which you may varify for yourself.

4.10.1 (b) Voltmeter : The instrument to measure the potential difference (also called valtage) between the two ends of component in a circuit, is called voltmeter. For this purpose the voltmeter is joined in parallel to that component.

Suppose the voltage across the two ends of the resistance R shown in the figure 4.23(a) is to be measured. For this if a galvanometer with resistance G and current capacity I_G is used, we find the following difficulties. On joining the galvanometer as shown in the figure 4.23(b), the total resistance of circuit becomes

$$R' = R_1 + \frac{RG}{R+G} \quad (4.10.9)$$



(a) Figure 4.23 (b)

As a result, after joining the galvanometer, the resistance of circuit change and the current passing through R also changes. Thus value of potential difference = IR (which is to be measured), between two ends of the resistance R, also changes.

If the value of G is very high, then in R + G; neglecting R as compared to G,

$$R' = R_1 + \frac{RG}{R+G} \approx R_1 + R \quad (4.10.10)$$

In this condition the resistance of the circuit is not appreciably changed and since value of G is greater, most of the current passes through R and hence the value of IR is almost maintained.

The above discussion shows that the resistance of the instrument measuring the electric potential difference should be as great as possible (in principle infinite). Thus by joining a proper greater resistance in series with the galvanometer, it can be converted into a voltmeter. Here since the resistane is very large, the current passing through the galvanometer is very small and it is not likely to be damaged.

The maximum voltage that can be measured with a galvanometer ($I_G G$) is called its (voltage capacity).

Formula for Series Resistance : Suppose the resistance of a galvanometer is G and its current capacity is I_G. Hence its voltage capacity will be $I_G G$. This galvanometer is to be converted into a voltmeter which can measure a maximum potential difference of V volt. For this the required series resistance is suppose R_S. In figure 4.24 if the potential difference between A and B is V, then by joining the galvanometer and R_S between these points, the galvanometer shows full scale deflection that is the current passing through it will be I_G. From the Figure,

$$I_G G + I_G R_S = V$$

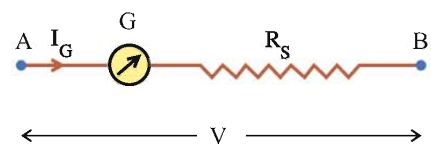


Figure 4.24

$$\therefore\ R_S = \frac{V}{I_G} - G \qquad (4.10.11)$$

By joining a resistance given by the above formula in series with the given galvanometer, and then by properly calibrating the scale of galvanometer, the voltmeter is prepared. From equation 4.10.11, it is clear that in order to make the range of voltmeter greater and greater the larger and larger value of series resistance (R_S) should be taken.

In order to make the voltage capacity of voltmeter, n times, the required series resistance will be $R_S = (n - 1)G$; which you may varify.

By dividing both the sides of equation 4.10.6 by the resistance of voltmeter R.

$$\frac{\phi}{IR} = \frac{NAB}{k}\frac{1}{R}$$

$$\therefore\ \frac{\phi}{V} = \frac{NAB}{kR} \qquad (4.10.12)$$

Here, $\frac{\phi}{V}$ is called the voltage sensitivity (S_V) of voltmeter.

Illustration 12 : There are 21 marks (zero to 20) on the dial of a galvanometer, that is there are 20 divisious. On passing 10 μA current through it, it shows a deflection of 1 division. Its resistance is 20 Ω (a). How can it be converted into an ammeter which can measure 1 A current ? (b) How can the original galvanometer be converted into a voltmeter which can measure a potential difference of 1 V ? Also find the effective resistance of both of the above mentioned meters.

Solution : (a) When a current of 10 μA passes through the galvanometer, its pointer shows a deflection of 1 division. There are 20 divisions in this galvanometer.

∴ The maximum current which can be measured by it (current capacity) $I_G = 10 \times 10^{-6} \times 20 = 200 \times 10^{-6}$A.

For ammeter, the required shunt to be joined in parallel to galvanometer is

$$S = \frac{GI_G}{I - I_G} \qquad I_G = 200 \times 10^{-6}A = 2 \times 10^{-4}A$$

$$= \frac{20 \times 200 \times 10^{-6}}{(10000 \times 10^{-4}) - (2 \times 10^{-4})} \qquad G = 20\ \Omega$$

$$= \frac{20 \times 2 \times 10^{-4}}{(10000 \times 10^{-4}) - (2 \times 10^{-4})} \qquad I = 1\ A = 10000 \times 10^{-4}A$$

$$= \frac{40}{9998} \approx 0.004\ \Omega$$

Thus to convert this galvanometer into an ammeter which can measure 1 A current, a shunt of 0.004 Ω should be joined.

The effective resistance of this ammeter will be $G' = \frac{GS}{G+S} = \frac{20 \times 0.004}{20 + 0.004} \approx 0.004\ \Omega$.

(b) For Voltmeter : In order to convert the galvanometer into a voltmeter, the required series resistance is

$$R_S = \frac{V}{I_G} - G \qquad \text{Here, } V = 1 \text{ volt}$$

$$I^G = 2 \times 10^{-4}A$$

$$G = 20\ \Omega$$

$$= \frac{1}{2 \times 10^{-4}} - 20$$

$$= 0.5 \times 10^4 - 20$$

$$= 5000 - 20$$

$$= 4980\ \Omega$$

In order to convert this galvanometer into a voltmeter which can measure 1 volt, a series resistance of 4920 should be joined with it.

The effective resistance of this voltmeter will be $R'_S = R_S + G = 4980 + 20 = 5000\ \Omega$. ($\therefore$ R_S and G are in series)

SUMMARY

1. **Oersted's Observation :** "When electric current is passed through a conducting wire kept parallel to and below the magnetic needle, the magnetic needle is deflected."

2. **Biot-Savart's Law :** The magnetic field due to a current element $I\vec{dl}$ at a point with position vector $\vec{r}$ with respect to it, is given by

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \hat{r}}{r^2}$$

Since such elements are continuously distributed in the entire conducting wire, the magnetic field due to such a wire can be written in the form of a line integral as

$$\vec{B} = \int \vec{dB} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$\text{or } \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \vec{r}}{r^3}$$

Here, the line integral is on the entire circuit made up with the conducting wire.

3. The magnetic field due to a circular coil (ring) of N turns, radius a and carrying current I at a point on its axis at a distance x from its centre is

$$B(x) = \frac{\mu_0 N I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

For magnetic field at the centre of the coil (ring),

taking $x = 0$, $B(\text{centre}) = \frac{\mu_0 N I}{2a}$

For a point very much away from the centre,

taking $x >> a$;

$$B(x) = \frac{\mu_0 N I a^2}{2x^3}$$

4. **Ampere's Circuital Law :** "The line integral of magnetic field on a closed curve (loop) in a magnetic field, is equal to the product of the algebraic sum of the electric currents enclosed by that closed curve and the permeability of vacuum."

In the form of an equation this Law can be written as under :

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I.$$

5. If current I is passed through a very long straight wire, the magnetic field at a point at normal distance r from the wire is,

$$B = \frac{\mu_0 I}{2\pi r}$$

6. The magnetic field at a point on the axis of a very long solenoid carrying current is $B = \mu_0 n I$

Where n = number of turns per unit length of solenoid.

7. The force on a conducting wire of length l and carrying current I placed in a magnetic field $\vec{B}$, is $\vec{F} = I\vec{l} \times \vec{B}$

The direction of this force can be found by the right hand screw rule for the vector product.

8. The force between two very long parallel current carrying conductors is $F = \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{y}$,

Where y = perpendicular distance between two wires. If the currents in the wires are in mutually opposite directions, the force is repulsive and if the currents are in the same direction, the force is attractive.

9. The magnetic force on a charge q, moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is $\vec{F}_m = q(\vec{v} \times \vec{B})$

The force on the charge q in an electric field $\vec{E}$ is $\vec{F}_e = q\vec{E}$

The force on the charge in the region where both the fields are present simultaneously, is $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$, which is called the Lorentz force.

10. Cyclotron is the instrument to accelerate the charged particles. The radius of the circular path of the charged particle moving in it, is

$r = \frac{mv}{Bq}$ which is dependent on its momentum.

The angular frequency w of this particle is called the cyclotron frequency (w_e)

$w_C = \frac{qB}{m}$ or $f_C = \frac{qB}{2\pi m}$ ... $(\because\ w_C = 2\pi f_C)$

11. The torque acting on a current carrying coil suspended in a uniform magnetic field is $\vec{\tau} = NI\vec{A} \times \vec{B}$

$\vec{\mu} = NI\vec{A}$ is called the magnetic moment of the coil.

$\therefore\ \vec{\tau} = \vec{\mu} \times \vec{B}$

12. For measuring very small electric currents galvanometer is used. In a moving and pivoted coil galvanometer, $\tau = NIAB$. Due to this the coil is diflected and springs attached with it are twisted. Hence restoring torque is produced. The restoring torque is $\tau = k\phi$.
In equilibrium condition.
$k\phi = NIBA$

$\therefore\ I = \frac{k}{NBA}\phi \qquad \therefore\ I \propto \phi$

13. The small resistance joined in parallel to a galvanometer to convert it into an ammeter is called a shunt. Its formula is $S = \frac{G\,I_G}{I - I_G}$.

To convert a galvanometer into a voltmeter a resistance of a high value is joined in series with it. The formula to find this series resistance Rs is $R_S = \frac{V}{I_G} - G$.

EXERCISE

For the following statements choose the correct option from the given options :

1. Two concentric rings are kept in the same plane. Number of turns in both the rings is 20. Their radii are 40 cm and 80 cm and they carry electric currents of 0.4 A and 0.6 A respectively, in mutually opposite directions. The magnitude of the magnetic field produced at their centre is ...... T.

(A) $4\mu_0$ (B) $2\mu_0$ (C) $\frac{10}{4}\mu_0$ (D) $\frac{5}{4}\mu_0$

2. A particle of mass m has an electric charge q. This particle is accelerated through a potential difference V and then entered normally in a uniform magnetic field B. It performs a circular motion of radius R. The ratio of its charge to the mass $\left(\frac{q}{m}\right)$ is = ... [$\left(\frac{q}{m}\right)$ is also called specific charge.]

(A) $\frac{2V}{B^2R^2}$ (B) $\frac{V}{2BR}$ (C) $\frac{VB}{2R}$ (D) $\frac{mV}{BR}$

3. A proton, a deuteron ion and an α-particle of equal kinetic energy perform circular motion normal to a uniform magnetic field B. If the radii of their paths are r_p, r_d and r_α respectively then..... [Here, $q_d = q_p$, $m_d = 2m_p$]

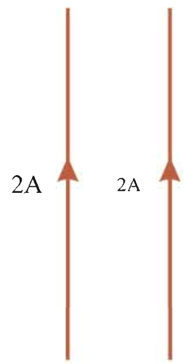
(A) $r_\alpha = r_p > r_d$ (B) $r_\alpha = r_d > r_p$

(C) $r_\alpha > r_d > r_p$ (D) $r_\alpha = r_d = r_p$

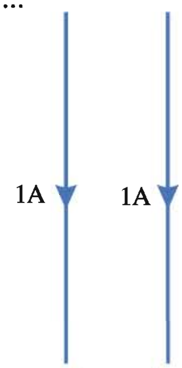
4. An electron performs circular motion of radius r, perpendicular to a uniform magnetic field B. The kinetic energy gained by this electron in half the revolution is ....

(A) $\frac{1}{2}mv^2$ (B) $\frac{1}{4}mv^2$ (C) zero (D) $\pi rBev$

5. As shown in the figure two very long straight wires are kept parallel to each other and 2A current is passed through them in the same direction. In this condition the force between them is F. Now if the current in both of them is made 1 A and directions are reversed in both, then the force between them ....



(a)

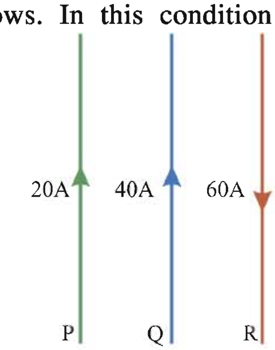


(b)

(A) will be $\frac{F}{4}$ and attractive (B) will be $\frac{F}{2}$ and repulsive

(C) will be $\frac{F}{2}$ and attractive (D) will be $\frac{F}{4}$ and repulsive.

6. As shown in the figure 20A, 40A and 60A currents are passing through very long straight wires P, Q and R respectively in the directions shown by the arrows. In this condition the direction of the resultant force on wire Q is



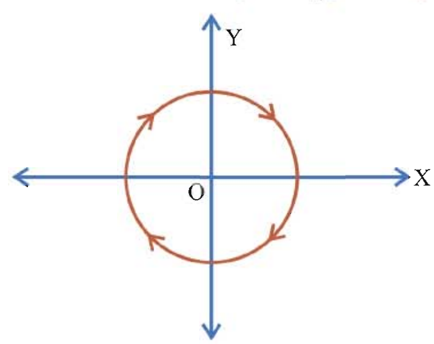
(A) towards left of wire Q

(B) towards right of wire Q

(C) normal to the plane of paper

(D) in the direction of current passing through Q.

7. As shown in the figure a circular conducting wire carries current I. It lies in XY-plane with centre at O. The tendency of this circular loop is to



(A) contract

(B) expand

(C) move towards positive X-direction

(D) move towards negative X-direction.

8. At a place an electric field and a magnetic field are in the downward direction. There an electron moves in the downward direction. Hence this electron ....

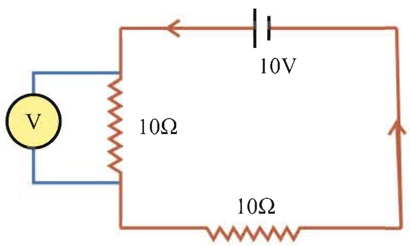
(A) will bend towards left (B) will bend towards right

(C) will gain velocity (D) will lose velocity.

9. Two parallel thin wires, each carrying current I are kept at a separation r from each other. Hence the magnitude of force per unit length of one wire due to the other wire is ....

(A) $\frac{\mu_0 I^2}{r^2}$ (B) $\frac{\mu_0 I^2}{2\pi r}$ (C) $\frac{\mu_0 I}{2\pi r}$ (D) $\frac{\mu_0 I}{2\pi r^2}$

10. A voltmeter of a very high resistance is joined in the circuit as shown in the figure. The voltage shown by this voltmeter will be ....



(A) 5 V (B) 10 V

(C) 2.5 V (D) 7.5 V

11. A particle of charge q and mass m moves on a circular path of radius r in a plane inside and normal to a uniform magnetic field B. The time taken by this particle to complete one revolution is ....

(A) $\frac{2\pi mq}{B}$ (B) $\frac{2\pi q^2 B}{m}$ (C) $\frac{2\pi qB}{m}$ (D) $\frac{2\pi m}{Bq}$

12. A long wire carries a steady current. When it is bent in a circular form, the magnetic field at its centre is B. Now if this wire is bent in a circular loop of n turns, what is the magnetic field at its centre ?

(A) nB (B) n^2B (C) $2nB$ (D) $2n^2B$

13. A conducting wire of 1 m length is used to form a circular loop. If it carries a current of 1 ampere, its magnetic moment will be .... Am^2.

(A) 2π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4\pi}$

14. When a charged particle moves in a magnetic field its kinetic energy ....

(A) remains constant (B) can increase

(C) can decrease (D) can increase or decrease

15. At each of the two ends of a rod of length $2r$, a particle of mass m and charge q is attached. If this rod is rotated about its centre with angular speed ω, the ratio of its magnetic dipole moment to the total angular momentum of this particle is ....

(A) $\frac{q}{2m}$ (B) $\frac{q}{m}$ (C) $\frac{2q}{m}$ (D) $\frac{q}{\pi m}$

16. There are 100 turns per cm length in a very long solenoid. It carries a current of 5 A. The magnetic field at its centre on the axis is .... T.

(A) 3.14×10^{-2} (B) 6.28×10^{-2} (C) 9.42×10^{-2} (D) 12.56×10^{-2}

17. Two very long conducting parallel wires are separated by a distance d from each other and equal currents are passed through them in mutually opposite directions. A particle of charge q passes through a point, at a distance $\frac{d}{2}$ from both wires, with velocity v perpendicularly to the plane formed by the wires. The resultant magnetic force acting on this particle is ....

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$ (C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) zero

18. A very long solenoid of length L has n layers. There are N turns in each layer. Diameter of the solenoid is D and it carries current I. The magnetic field at the centre of the solenoid is ....

(A) directly propotional to D (B) inversely proportional to D.
(C) independent of D (D) directly proportional to L.

19. The angular speed of the charged particle is independent of ....

(A) its mass (B) its linear speed
(C) charge of particle (D) magnetic field.

20. A charged particle gains energy due to ....

(A) electric field (B) magnetic field
(C) both these fields (D) none of these fields.

21. A charged particle is moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The magnetic force acting on it will be maximum when ....

(A) $\vec{v}$ and $\vec{B}$ are in same direction

(B) $\vec{v}$ and $\vec{B}$ are in opposite direction

(C) $\vec{v}$ and $\vec{B}$ are mutually perpendicular

(D) $\vec{v}$ and $\vec{B}$ moke an angle of 45^o with each other

22. Equal currents are passing through two very long and straight parallel wires in mutually opposite directions. They will ....

(A) attract each other (B) repel each other
(C) lean towards each other (D) neither attract nor repel each other.

23. A charged particle is moving in a uniform magnetic field. Then ....

(A) its momentum changes but kinetic energy does not change
(B) its momentum and kinetic energy both change
(C) neither the momentum nor kinetic energy changes.
(D) Kinetic energy charges but the momentum does not change.

24. If the speed of a changed particle moving through a magnetic field is increased, then the radius of curvature of its trajectory will ....

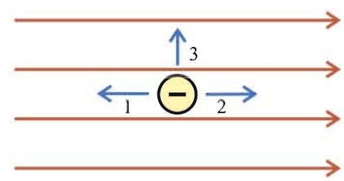
(A) decrease (B) increase (C) not change (D) become half

ANSWERS

1. (C)	2. (A)	3. (A)	4. (C)	5. (A)	6. (A)
7. (B)	8. (D)	9. (B)	10. (A)	11. (D)	12. (B)
13. (D)	14. (A)	15. (A)	16. (B)	17. (D)	18. (C)
19. (B)	20. (A)	21. (C)	22. (A)	23. (A)	24. (B)

Answer the following questions in brief :

1. State the observation made by Oersted.
2. Write the statement of Biot–Savart's Law.
3. Give the formula showing Ampere's Circuital Law.
4. State the Law giving the direction of magnetic field due to a straight conductor carrying current.
5. What is the magnitude of the magnetic field in the region near the outside of the solenoid.
6. State the direction of magnetic field due to current in a toroid.
7. State Ampere's observation after the observation made by Oersted.
8. Does the angular frequency of particle depend on its momentum in cyclotron ? Yes or No ?
9. Can a neutron be accelerated using cyclotron ? Why ?
10. State the functions of electric field and magnetic field in a cyclotron.
11. State two limitations of cyclotorn.
12. What should be the resistances of an ideal ammeter and an ideal voltmeter ?
13. What is meant by current sensitivity of a galvanometer ?
14. What should be done to increase the voltage capacity of a voltmeter.
15. If the radius of the ring and the current through it both are doubled, what change would occur in the magnetic field at its centre ?
16. Give the magnitude of the magnetic force on the electron for the three cases of its motion shown in the Figure.



Answer the following questions :

1. Write Biot–Savart's Law and explain it.
2. Write the formula for the magnetic field at a point on the axis of a current carrying circular ring and explain with a suitable diagram the right hand rule to find the direction of this magnetic field.
3. State and explain Ampere's Circuital Law.
4. Using Ampere's Circuital Law, obtain the magnitude of magnitic field at a perpendicular disance r due to very long staright conductor carrying current I.
5. Using Ampere's circuital Law obtain the formula for the magnitude of magnetic field due to current in a toroid.
6. Obtain the formula for the force of attraction between two parallal wires carrying currents in the same direction.
7. Obtain the formula for the Lorentz force on a moving electric charge
8. Explain the working of cyclotron and obtain the formula for the cyclotron frequency w_C.
9. With a suitable diagram explain the construction of galvanometer.
10. What should be d one to convert a galvanometer into an ammeter. Obtain the formula for the shunt.
11. Derive an expression for the magnetic field at a point on the axis of a current carrying circular ring.
12. Obtain the formula for the magnetic field produced inside a very long current carrying solenoid uising Ampere's Circuital Law.
13. Obtain the formula for the torque acting on a rectangular coil carrying current, suspended in a uniform magnetic field.

Solve the following examples :

1. Distance between two very long parallel wires is 0.2 m. Electric currents of 4 A in one wire and 6A in the other wire are passing in the same direction. Find the position of a point on the perpendicular line joining the two wires at which the magnetic field intensity is zero.

[Ans : 80 mm away from the wire with 4A current and between the two wires]

2. A very long wire is held vertical in a direction perpendicular to the horizontal component of Earth's magnetic field. Find the value of current to be passed through this wire so that the resultant magnetic field at a point 10 cm away from this wire becomes zero. What will be the magnetic induction at a point 10 cm away from the wire on the opposite side of this point ? Horizontal component of Earth's magnetic field H = 0.36×10^{-4}T, $\mu_0 = 4\pi \times 10^{-7}$T m/A. [**Ans. :** 18 A, 0.72×10^{-4} T]

3. When a galvanometer with a shunt is joined in an electrical circuit 2% of the total current passes through the galvanometer. Resistance of galvanometer is G. Find the value of shunt. [**Ans. :** $\frac{G}{49}$]

4. Two particles of masses M_1 and M_2 and having the equal electric charge are accelerated through equal potential difference and then move inside a uniform magnetic field, normal to it. If the radii of their circular paths are R_1 and R_2 respectively find the ratio of their masses. [**Ans :** $\frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^2$]

5. A circular coil having N turns is made from a wire L meter long. If a current of I ampere is passed through this coil suspended in a uniform magnetic field of B tesla, find the maximum torque that can act on this coil. [**Ans. :** $\frac{IL^2B}{4\pi N}$ N m]

6. A proton and a deuteronion having the same kinetic energies enter a region of uniform magnetic field perpendicularly. Deuteron's mass is twice that of proton. Calculate the ratio of the radii of their circular paths. [**Ans. :** $\frac{r_d}{r_p} = \sqrt{2}$]

7. A rectangular coil of 120 turns and an area of 10×10^{-4} m^2 is suspended in a radial magnetic field of 45×10^{-4} T. If a current of 0.2 mA through the coil gives it a deflection of 36° find the effective torsional constant for the spring system holding the coil. [**Ans. :** 17.2×10^{-8} N m/rad]

8. Two rings X and Y are placed in such a way that their axes are along the X and the Y axes respectively and their centres are at the origin. Both the rings X and Y have the same radii of 3.14 cm. If the current through X and Y rings are 0.6 A and 0.8 A respecively, find the value of the resultant magnetic field at the origin. $\mu_0 = 4\pi \times 10^{-7}$SI. [**Ans. :** 2×10^{-5}T]

9. Two parallel very long straight wires carrying currents of 20 A and 30 A respectively are at a separation of 3 m between them. If the currents are in the same direction, find the attractive force between them per unit length. [**Ans. :** 4×10^{-5}N m^{-1}]

10. A very long straight wire carries a current of 5 A. An electron moves with a velocity of 10^6 m s^{-1} remaining parallel to the wire at a distance of 10 cm from wire in a direction opposite to that of electric current. Find the force on this electron. (Here the mass of electron is taken as constant) $e = -1.6 \times 10^{-19}$C, $\mu_0 = 4\pi \times 10^{-7}$SI. [**Ans. :** 16×10^{-19}N]

11. A current of 6 A passes through the wire shown in the Figure. Find the magnitude of magnetic field at point C. The radius is 0.02 m $\mu_0 = 4\pi \times 10^{-7}$T m A^{-1}. [**Ans. :** 1.41×10^{-4}T]