6 Ray Optics

6.1 Introduction

Light is the agency which stimulates our sense of vision or sight. All curious questions regarding light: its nature, its generation, its interaction with matter, its speed and propagation through medium, etc., are described and explained in a branch of physics called optics. Developments in optics can be classified into three branches:

(1) Ray (Geometric) optics, (2) Wave optics and (3) Quantum optics

Since the wavelength of visible electromagnetic waves (400 nm to 800 nm) is too small compared to objects around us, light can be considered to travel from one point to another along a straight line. This is called rectilinear propagation of light. The path of the light propagation is called a ray, which is never diverging or converging. A bundle of such rays is called deam of light.

The optical phenomena like reflection, refraction and dispersion can be explained by the ray optics. The ray optics is based mainly on the following three assumptions.

(1) Rectilinear propagation of light

(2) Independence of light rays (i.e., they do not disturb one another when they intersect).

(3) Reversibility of path (i.e., they retrace exactly the same path on reversing their direction of propagation).

In the present chapter, we shall study reflection, refraction and dispersion phenomena using ray optics. Optical instruments like microscope and telescope are also studied at the end of the chapter.

6.2 Reflection by Spherical Mirrors

For studying reflection of light by spherical mirrors, we shall revise certain points as under :

The laws of reflection

(1) In the case of reflection of light, the angle of incidence and angle of reflection are equal.

(2) Incident ray, reflected ray and normal drawn at the point of incidence lie in the same plane. While the incident ray and the reflected ray are on either side of the normal.

These laws are valid at every point on any reflecting surface, whether plane or curved.

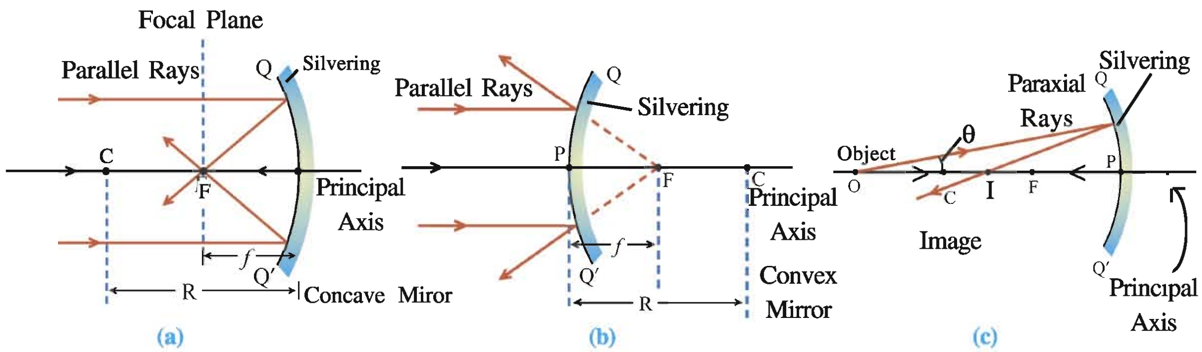


Figure 6.1 Image Formation by Curved Mirrors

Some useful terms used to study reflection of light by curved mirrors are as follows :

Pole : The centre of the reflecting surface of a curved mirror is called its pole (P)

Principal Axis : The imaginary line passing through the pole and the centre of curvature ($\overleftrightarrow{CP}$) is called the principal axis of the mirror.

Radius of Curvature : The radius of the spherical shell from which mirrors are made is called the radius of curvature (R) of the curved mirrors. It is the distance between C and P.

Centre of Curvature : The centre of the spherical shell from which mirrors are made is called the centre of curvature (C) of the mirror.

Aperture : The diameter of the reflecting surface (QQ′) is called the aperture of the mirror.

Principal Focus : The point where the rays parallel to the principal axis meet for concave mirror or appear to meet for convex mirror on reflection is called the principal focus of the mirror.

Focal Plane : A plane passing through the principal focus and normal to the principal axis is called the focal plane of the mirror.

Focal Length : The distance between the pole and the principal focus of a mirror is called its focal length (f).

Paraxial Rays : Rays close to the principal axis are called Paraxial Rays. We shall study lens and mirrors in reference to Paraxial Rays only.

Sign Convention : In order to specify the position of the object and the image, we require a reference point and sign convention. We adopt Cartesian sign convention as follows.

(1) All the distances are measured from the pole of the mirror on the principal axis.

(2) Distances measured in the direction of the incident ray are taken positive, while those measured in the opposite direction are taken negative.

(3) Height above the principal axis is taken positive, while that below the principal axis is taken negative.

6.3 Relation between Focal Length and Radius of Curvature

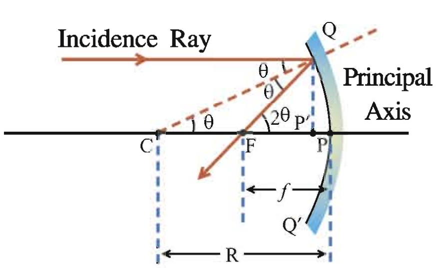


Figure 6.2 Relation between Focal Length and Radius of Curvature

In figure 6.2, a ray paraxial and parallel to the principal axis is shown to incident at point Q of a concave mirror of small aperture. The reflected ray passes through the principal focus. Normal drawn to the surface at point Q passes through centre of curvature. $\therefore$ CQ = CP. If the angle of incidence is θ, then the angle of reflection $\angle CQF = \theta = \angle QCF$.

From the geometry of the figure, exterior angle,

$\angle QFP = \theta + \theta = 2\theta$

Since the incident ray is paraxial and the aperture of the mirror is small, points P and P' are very close to each other. i.e., CP' ≈ CP = R

and FP' ≈ FP = f

In ΔQFP, $\sin 2\theta \approx 2\theta = \frac{QP'}{FP'} = \frac{QP}{FP}$

$$\therefore\ 2\theta = \frac{QP}{f} \Rightarrow \theta = \frac{QP}{2f} \qquad (6.3.1)$$

Similarly, from ΔCQP', $\sin\theta \approx \theta = \frac{QP'}{CP'} \approx \frac{QP}{CP}$

$$\therefore\ \theta = \frac{QP}{R} \qquad (6.3.2)$$

From equations (6.3.1) and (6.3.2) $R = 2f$ or $f = \frac{R}{2}$ (6.3.3)

Equation (6.3.3) is also true for a convex mirror. In the case of plane mirror, R is infinite, and therefore its focal length is also infinite.

6.4 Spherical Mirror Formula

Now we shall derive the relation between the object distance (u) image distance (v) and focal length (f) for a concave mirror. As shown in figure 6.3, consider a point object O on the principal axis at a distance u from the pole. Let the aperture of the mirror be small. Let the incident ray OQ makes a small angle (α) with the principal axis and gets reflected as QI. Another ray from object O moving along the axis is incident at P, and gets reflected in the direction PC. Both reflected rays meet at point I and forms the point like image.

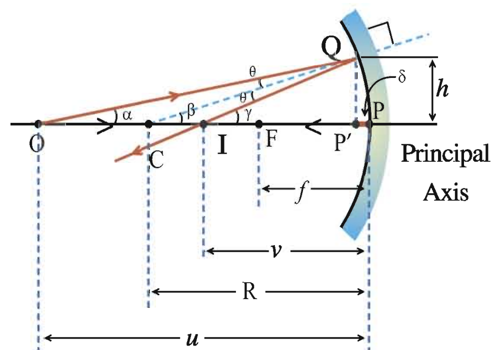


Figure 6.3 Image of a Point Object Due to Concave Mirror

Since the aperture of the mirror is small, distance PP' = δ is very small and can be neglected. Hence regions OPQ and IQP can be approximated by ΔOQP' and ΔIQP', respectively.

According to the laws of reflection, angle of incidence, ∠OQC = angle of reflection, ∠CQI = θ. Let CQ and IQ make angle β and γ, respectively, with the principal axis.

In ΔOCQ, exterior angle $\beta = \alpha + \theta$

In ΔCQI, exterior angle $\gamma = \beta + \theta$

Eliminating θ from above equations,

$$\alpha + \gamma = 2\beta \qquad (6.4.1)$$

Using the figure, in ΔOQP',

$$\alpha \text{ (rad)} = \frac{\text{arc QP}}{\text{OP}},$$

$$\beta \text{ (rad)} = \frac{\text{arc QP}}{\text{CP}} \text{ and}$$

$$\gamma \text{ (rad)} = \frac{\text{arc QP}}{\text{IP}}$$

Using these values in equation (6.4.1) we have,

$$\frac{\text{arc QP}}{\text{OP}} + \frac{\text{arc QP}}{\text{IP}} = 2\frac{\text{arc QP}}{\text{CP}}$$

$$\frac{1}{\text{OP}} + \frac{1}{\text{IP}} = \frac{2}{\text{CP}}$$

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{2}{\text{R}} \text{ or } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (6.4.2)$$

Equation (6.4.2) represents the numerical relationship between object distance, image distance and focal length (or radius of curvature). While using it for calculating any of these physical quantities, we must apply sign convention. In the present case, $u \rightarrow -u$, $v \rightarrow -v$ and f (or R) $\rightarrow$ $-f$ (or $-$R)

Equation (6.4.2) is the **Gauss' equation** for curved mirrors. It is also valid for convex mirror.

6.5 Lateral Magnification

The ratio of the height of the image (h') to the height of the object (h) is called the **transverse** or **lateral magnification (m)**.

i.e., $$m = \frac{h'}{h} \quad (6.5.1)$$

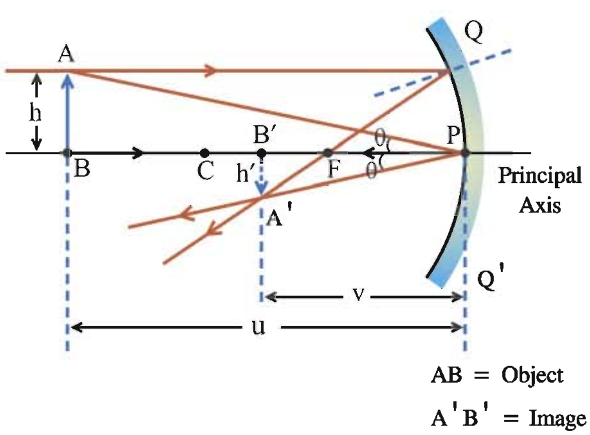


Figure 6.4 Image of an Extended Object

For right angled triangles ABP and A'B'P,

$$\tan\theta = \frac{\text{AB}}{\text{BP}} = \frac{\text{A'B'}}{\text{B'P}} \quad (6.5.2)$$

But AB = h, A'B' = $-h'$, PB = $-u$ and B'P = $-v$ (using sign convention), equation (6.5.2) becomes,

$$\frac{h}{-u} = \frac{-h'}{-v}$$

$$\therefore \frac{h'}{h} = \frac{-v}{u} \qquad (6.5.3)$$

Combining equations (6.5.1) and (6.5.3)

$$m = \frac{-v}{u} \qquad (6.5.4)$$

Equation (6.5.4) is also true for convex mirror.

Illustration 1 : An object lies on the principal axis of a concave mirror with radius of curvature 160 cm. Its vertical image appears at a distance 70 cm from it. Determine the position of the object and also the magnification.

Solution : The mirror equation is

$$\frac{2}{R} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore \frac{1}{u} = \frac{2}{R} - \frac{1}{v} = \frac{2}{-160} - \frac{1}{70} \text{ (using sign convention)}$$

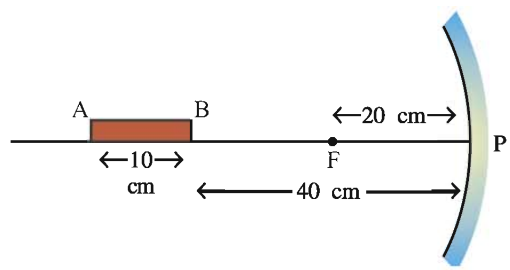
$$\therefore u = -37 \text{ cm} = \frac{-15}{560}$$

i.e., The object is at a distance 37 cm in front of the mirror.

$$\text{Lateral magnification, } m = -\frac{v}{u} = -\frac{70}{-37} = 1.89$$

Illustration 2 : As shown in the figure, a thin rod AB of length 10 cm is placed on the principal axis of a concave mirror such that it's end B is at a distance of 40 cm from the mirror. If the focal length of the mirror is 20 cm, find the length of the image of the rod.

Solution : f = 20 cm and the end B is at distance 40 cm = $2f$ = R. Thus the image of B is formed at B only.



Now for end A,

u = –50 cm, f = – 20 cm, v = ?

In $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, putting these values

$$-\frac{1}{50} + \frac{1}{v} = -\frac{1}{20}$$

$$\therefore \frac{1}{v} = \frac{1}{50} - \frac{1}{20} = \frac{20-50}{20\times50} = -\frac{30}{1000}$$

$$\therefore v = -\frac{100}{3} = -33.3 \text{ cm}$$

This image A' is on the same side as the object.

Now, length of the image = 40 – 33.3 = 6.70 cm

Illustration 3 : Derive the formula for lateral magnification, $m = \frac{f}{f-u}$ for spherical mirrors; where f = focal length and u = object distance.

Solution : $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u-f}{uf}$

$$\therefore v = \frac{fu}{u-f} \Rightarrow \frac{v}{u} = \frac{f}{u-f}$$

and $m = -\frac{v}{u} = \frac{f}{f-u}$

Note : For a plane mirror $f \rightarrow \infty \therefore m = 1$ (Magnitude)

6.6 Refraction of Light

When a ray of light enters obliquely from one transparent medium to another transparent medium its direction changes at the surface separating two media. This phenomenon is known as refraction.

For information only :

• When the characteristics of a medium are same at all points, it is said to be homogeneous. When the characteristics are same in all directions it is said to be isotropic.

• If a medium is not homogenous then a light ray continuously gets refracted and its path is curved.

• If the medium is not isotropic light ray refracts by different amount in different directions.

Laws of Refraction :

(1) The incident ray, refracted ray and the normal drawn to the point of incidence are in the same plane.

(2) "The ratio of the sine of the angle of incidence to the sine of the angle of refraction for the given two media is constant." This constant is called **relative refractive index** of the two media. This statement is known as the Snell's law.

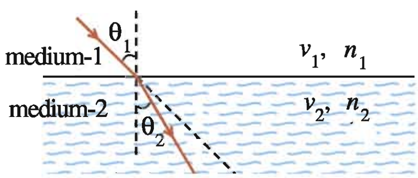


Figure 6.4 (a)

If θ_1 is the angle of incidence (in medium-1) and θ_2 is the angle of refraction (in medium-2) then,

$$\frac{\sin\theta_1}{\sin\theta_2} = n_{21}, \tag{6.6.1}$$

where n_{21} is known as the refractive index of medium-2 with respect to medium-1.

n_{21} depends on the type of media, their temperature and the wavelength of light.

Relative refractive index may also be defined in terms of speed of light in two media.

$$n_{21} = \frac{v_1}{v_2}, \tag{6.6.2}$$

where v_1 = speed of light in medium-1

and v_2 = speed of light in medium-2.

Similarly, refractive index of a medium with respect to vacuum (or in practice air) is

$$n = \frac{c}{v}. \tag{6.6.3}$$

Here, n is known as absolute refractive index. Now,

$$n_{21} = \frac{v_1}{v_2} = \frac{c}{v_2} \times \frac{v_1}{c} = \frac{n_2}{n_1} \tag{6.6.4}$$

$\therefore$ equation 6.6.1 becomes,

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

or $n_1\sin\theta_1 = n_2\sin\theta_2$ (6.6.5)

This equation (6.6.5) is known as general form of Snell's law.

For given media, if $n_2 > n_1 \Rightarrow \sin\theta_1 > \sin\theta_2$

$\therefore\ \theta_1 > \theta_2$

When a light ray enters from optically rarer medium to optically denser medium, angle of refraction is smaller than the angle of incidence, and the ray bends towards the normal.

If $n_2 < n_1 \Rightarrow \sin\theta_1 < \sin\theta_2$

$\therefore\ \theta_1 < \theta_2$

When a light ray enters from optically denser medium to optically rarer medium, angle of refraction is greater than the angle of incidence, and the ray bends away from the normal.

The medium with greater refractive index is called **optically denser** medium and the one with smaller refractive index is called optically rarer medium. This optical density is different from the mass density.

Refraction Through Compound Slab :

As shown in figure 6.5, if light passes through a compound slab, refractive index of medium-3 with respect to medium-1 can be written as

$$n_{31} = \frac{v_1}{v_3}$$

$$= \frac{v_2}{v_3} \times \frac{v_1}{v_2} = n_{32} \times n_{21} \qquad (6.6.6)$$

Also, $n_1\sin\theta_1 = n_2\sin\theta_2 = n_3\sin\theta_3$ (6.6.7)

and $n_{21} = \dfrac{v_1}{v_2} = \dfrac{1}{\left(\dfrac{v_2}{v_1}\right)} = \dfrac{1}{n_{12}}$

$\therefore\ n_{21} \times n_{12} = 1$ (6.6.8)

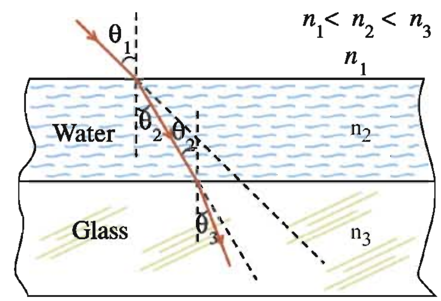


Figure 6.5 Refraction Through Compound Slab

For information only : The visibility of a transparent medium is due to the difference in its refractive index from that of the surrounding medium.

6.6.1 Lateral Shift : As shown in the figure 6.6, light rays undergo refraction twice, once from top (AB) and then from bottom (CD) surfaces of a given homogeneous medium. The emergent ray is parallel to PQR'S' ray. Here, PQR'S' is the path of light ray in absense of the other medium.

Since the emergent ray is parallel to the incident ray but shifted sidways by distance RN. This RN distance is called **lateral shift** (x). We can now calculate this lateral shift as follows :

Let n_1 and n_2 be the refractive indices of the rarer and denser medium, respectively. Also, $n_1 < n_2$. From the figure, $\angle RQN = \theta_1 - \theta_2$, RN = x.

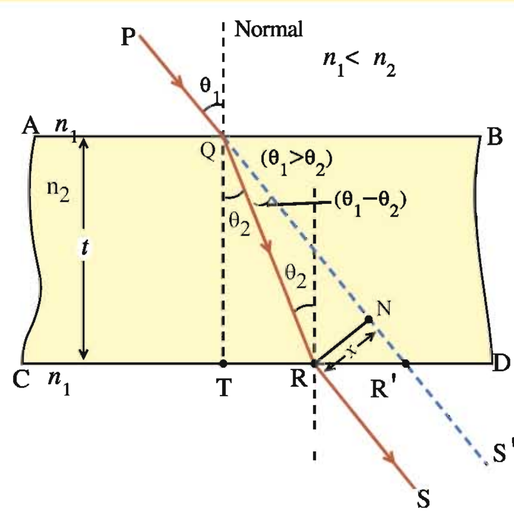


Figure 6.6 Lateral Shift Due to Rectangular Slab

From ΔQRN, $\sin(\theta_1 - \theta_2) = \frac{RN}{QR} = \frac{x}{QR}$ (6.6.9)

In ΔQTR, $\cos\theta_2 = \frac{QT}{QR}$

$\therefore QR = \frac{QT}{\cos\theta_1} = \frac{t}{\cos\theta_2}$

$\therefore$ from equation (6.6.9), $\sin(\theta_1 - \theta_2) = \frac{x}{\left(\frac{t}{\cos\theta_2}\right)}$

$$\therefore x = \frac{t \cdot \sin(\theta_1 - \theta_2)}{\cos\theta_2} \quad (6.6.10)$$

Since angle of incidence θ_1 is very small, θ_2 will also be small.

$\therefore \sin(\theta_1 - \theta_2) \approx (\theta_1 - \theta_2)$ & $\cos\theta_2 \approx 1$

$$\therefore x = \frac{t \cdot (\theta_1 - \theta_2)}{1}$$

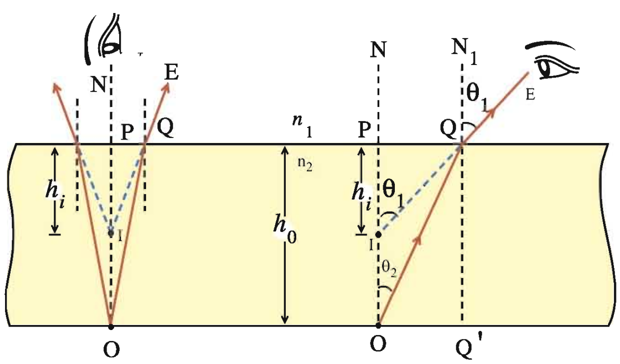
$$x = t \cdot \theta_1 \left(1 - \frac{\theta_2}{\theta_1}\right) \quad (6.6.11)$$

But according to Snell's law, $\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} \approx \frac{\theta_1}{\theta_2}$

$\therefore$ From equation (6.6.11),

$$x = t \cdot \theta_1 \left(1 - \frac{n_1}{n_2}\right)$$

6.6.2 Real Depth and Virtual Depth : Another manifestation of lateral shift is the apparent depth or hight seen through transparent medium.



(a) Normal Incidence (b) Oblique Incidence

Figure 6.7

As shown in figure 6.7, an object O is kept at depth h_0 in a denser medium (e.g. water) with refractive index n_2. In figure 6.7(b) Ray OQ on refraction moves along QE at the interface. If EQ is extended in the denser medium it meets the normal PN at point I.

So the observer sees the image of object O at position I. Here, PO = h_0 = real depth of an object.

PI = h_i = virtual depth of an image.

From figure 6.7(a) even when $\theta_1 = 0$, $h_0 \neq h_i$ (You will see this as a case of equation (6.8.10)).

But as θ_1 increases h_i becomes smaller compared to h_0. Also, the object appears curved when viewed obliquely through the refracting medium.

6.6.3 Real Height and Virtual Height :

Suppose an observer (e.g., fish) is inside a denser medium (e.g., water). It sees the eye (E) of a person at point E' instead of E. i.e., object is appeared lifted up (figure 6.8).

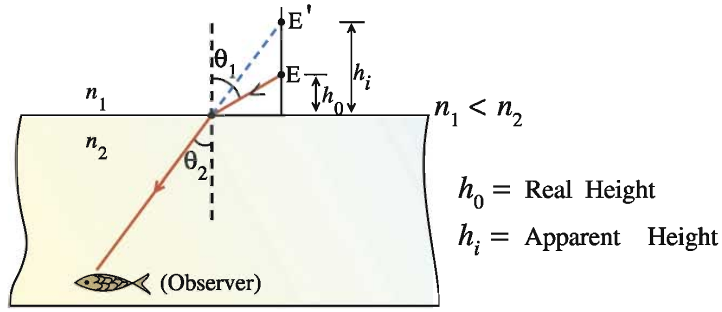


Figure 6.8 Virtual Height

Illustration 4 : Assuming that the angle of incidence at a refractive surface is sufficiently small, derive the relation between real depth, apparent depth and refractive index.

Solution : In figure 6.7, refractive index of denser medium = n_2 and refractive index of rarer medium = n_1. Real depth of the object O is PO = h_O. Depth of the image, i.e., apparent depth of the object = PI = h_i

Applying Snell's law at point Q,

$n_2 \sin\theta_2 = n_1 \sin\theta_1$

For nearly normal incidence, θ_1 and θ_2 are very small.

$\therefore \sin\theta \approx \theta \approx \tan\theta$

$n_2 \tan\theta_2 = n_1 \tan\theta_1$

But, $\tan\theta_2 = \frac{PQ}{PO} = \frac{PQ}{h_O}$ and $\tan\theta_1 = \frac{PQ}{PI} = \frac{PQ}{h_i}$

Using this in equation (1) $n_2\left(\frac{PQ}{h_O}\right) = n_1\left(\frac{PQ}{h_i}\right)$

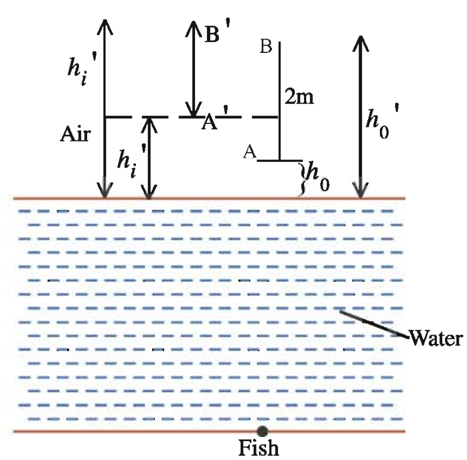
$$\therefore \frac{n_2}{n_1} = \frac{h_O}{h_i} \Rightarrow \frac{h_i}{h_O} = \frac{n_1}{n_2} = \frac{n(\text{rarer})}{n(\text{denser})}$$

Note : It can be proved that if an object kept in a rarer medium, at height h_O from the interface, is viewed normally from the denser medium and it appears to be at height h_i ($h_i > h_O$), then

$$\frac{h_i}{h_O} = \frac{n(\text{denser})}{n(\text{rarer})}$$

Illustration 5 : A swimmer is diving in a swimming pool vertically down with a velocity of 2 m s^{-1}. What will be the velocity as seen by a stationary fish at the bottom of the pool, right below the diver ? Refractive index of water is 1.33.

Solution : In the figure, vertical distance 2m is shown by AB. The height of A from the surface of water is h_O. Suppose it's apparent height is h_i ($h_i > h_O$).



$$\therefore \frac{h_i}{h_O} = \frac{n(\text{water})}{n(\text{air})}$$

$\therefore h_i = h_O \times 1.33$ (1)

Now the real height of B, $h_O' = (h_O + 2)$m

$\therefore$ if it's apparent height is h'_i, then

$$\frac{h_i'}{h_O'} = \frac{n(\text{water})}{n(\text{air})} = 1.33$$

$\therefore\ h'_i = h_O' \times 1.33$

$= (h_O + 2) \times 1.33$ (2)

From equations (1) and (2), the apparent distance, seen by the fish

$= h_i' - h_i = (h_O + 2) \times 1.33 - h_O \times 1.33$

$= 2 \times 1.33 = 2.66$ m

So the fish will see the swimmer falling with a speed of 2.66 m s^{-1}.

6.7 Total Internal Reflection

When light ray enters from one transparent medium to another, it is partially reflected and partially transmitted at an interface. This is true even if light is incident normally to a surface separating two media. In this case, intensity of reflected light is given by

$$I_r = I_0\left(\frac{n_2 - n_1}{n_1 + n_2}\right)^2 \qquad (6.7.1)$$

where, I_O = intensity of incident light

I_r = intensity of reflected light.

n_1 = refractive index of the medium–1

n_2 = refractive index of the medium–2

For air (n_2 = 1.0) and glass (n_1 = 1.5), I_r is 4% of the incident intensity. It is to be noted that equation 6.7.1 is true for normal incidence only. For other cases, I_r also depends on the angle of incidence.

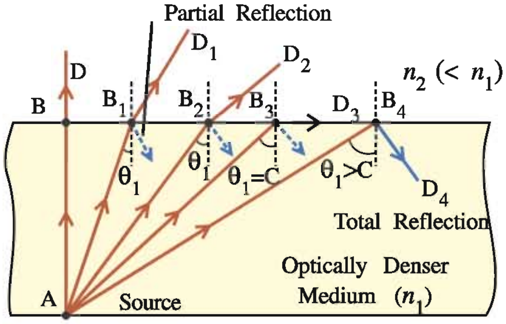


Figure 6.9 Total Internal Reflection

Here, A is a point object (or a light source) in a denser medium. Ray AB, AB_1, AB_2, ... get partially reflected and partially transmitted at points B, B_1, B_2 ... at the interface. It is observed that as the angle of incidence increases (going from B $\rightarrow B_1 \rightarrow B_2 \rightarrow$ ....) the angle of reflection ray also increases. It happens that at particular angle of incidence, refracted ray moves parallel to the surface separating two media. For this particular case, angle of refraction is 90°.

The angle of incidence for which the angle of refraction is 90° is called the critical angle (C) of the denser medium with respect to the rarer medium.

In this situation the interface appears bright. Using Snell's law for the critical angle of incidence,

$n_1\sin\theta_1 = n_2\sin\theta_2$

when $\theta_1 = C,\ \theta_2 = 90°$

$\therefore\ n_1\sin C = n_2$

$\therefore\ \sin C = \frac{n_2}{n_1}$

If rarer medium is air, i.e. $n_2 = 1$

$\therefore\ \sin C = \frac{1}{n_1} = \frac{1}{n}$ (Let $n_1 = n$)

$$\text{or } C = \sin^{-1}\left(\frac{1}{n}\right) \qquad (6.7.2)$$

At the critical angle, the reflected ray is known as the **critical ray**.

Now if the angle of incidence is increased slightly more than the critical angle, the intensity of reflected light immediately increases, and the incident ray gets completely (i.e. 100%) reflected back into the denser medium. This is called total internal reflection. It is true for any of incidence greater than the critical angle. In this situation, the surface separating the two media behaves like a perfect mirror. It is to be noted that the total internal reflection obeys the laws of reflection.

For Information Only :

When total internal reflection is studied with respect to electromagnetic waves, it is found that a very small portion of incident light enters into the rarer medium upto a distance equals few wavelengths. Though, its intensity is diminutive. This in quantum mechanics is called **tunneling** effect.

Illustration 6 : As shown in figure, a ray of light is incident at angle of 30° on a medium at $y = 0$ and proceeds ahead in the medium. The refractive index of this medium varied with distance y as given by,

$n(y) = 1.6 + \frac{0.2}{(y+1)^2}$ where y is in cm. What is the angle formed by the ray with the normal at a very large depth ?

Solution : Suppose the angle is θ at distance y in the medium.

Appyling Snell's law at this point,

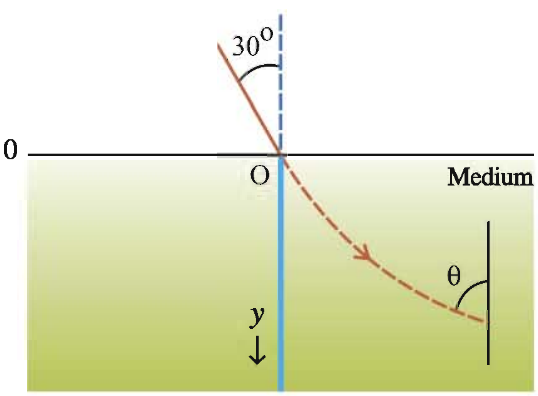
$n(y)\sin\theta = C$, where C = constant (1)

This formula is true for all the points.

Applying it to point O,

$n(0)\sin 30^\circ = C$

But, $n(0) = 1.6 + \frac{0.2}{(0+1)^2} = 1.8$



$\therefore\ 1.8 \times \frac{1}{2} = C$

$\therefore\ C = 0.9$

Putting this value in (1), $n(y)\sin\theta = 0.9$

$$\therefore \left\{1.6 + \frac{0.2}{(y+1)^2}\right\} \sin\theta = 0.9 \quad \therefore\ \sin\theta = \frac{0.9}{1.6 + \frac{0.2}{(y+1)^2}}.$$

When y is very large, taking $y \rightarrow \infty$; we get $\sin\theta = \frac{0.9}{1.6}$

$\therefore\ \theta = 34^\circ\ 14'$

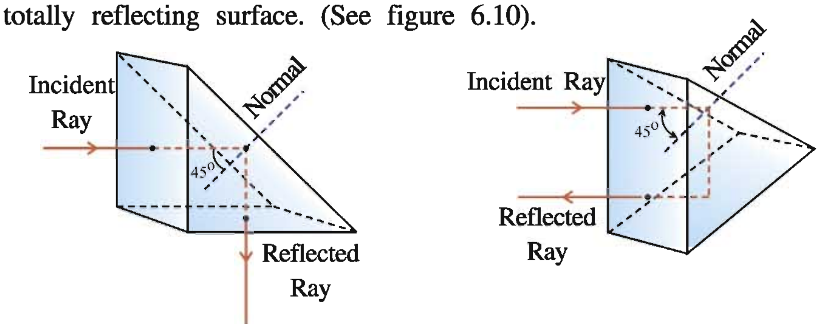
6.7.1 Uses of Total Internal Reflection :

(1) The refractive index of diamond is 2.42 and its critical angle is 24.41°. Thus, with proper cutting of its faces, whatever the angle at which light enters into the diamond, it undergoes many total internal reflections. Hence it looks bright from the inside, and we call the diamond is sparkling.

(2) For a glass with refractive index 1.50 has a critical angle for an air-glass interface,

$C = \sin^{-1}\left(\frac{1}{1.50}\right) \approx 42^\circ$

This angle is slightly less than 45°, which makes possible to use prisms with angles 45°–45°–90° as totally reflecting surface. (See figure 6.10).



(a) (b)

Figure 6.10 Totally Reflecting Prisms

The advantages of totally reflecting prisms over metallic reflectors are (1) superior reflection and (2) the reflecting properties are permanent and not affected by tarnishing.

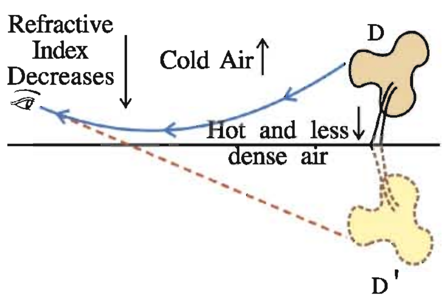


Figure 6.11 Mirage Formation

(3) Mirage : In summer, due to heat, the air in contact with the ground becomes hot while above it is cooler. Thus, air in contact with the ground is rarer and air above is denser. i.e., its refractive index increases as one moves upwards. As shown in the figure 6.11, a ray going from the top of the tree (D) to the ground is travelling continuously from a denser medium to a rarer medium. As it comes closer to the ground its angle of refraction increases and finally it undergoes total internal reflection, and enters into the eye of an observer. Thus, the image of D appears at D' to an observer, giving a feeling of image in a water surface. This phenomenon is called a mirage.

(4) Optical Fibres : The phenomenon of total internal reflection is used in optical fibres. They are made of glass or fused quartz of about 10 to 100 μm in diameter. They are in the form of long and thin fibres. The outer coating of the fibres **(cladding)** has a lower refractive index (n_1) than the core (material) of fibre (n_2). Here, $n_2 > n_1$.

In absense of the cladding layer, due to dust particles, oil or other impurities, some leakage of light may take place. In 1 m distance, in fact, light gets reflected thousands of times. Thus, if leakage occurs, light cannot travel far. Such leakage is prevented using cladding.

Fused quartz is usually used for making optical fibres because of its high transparency.

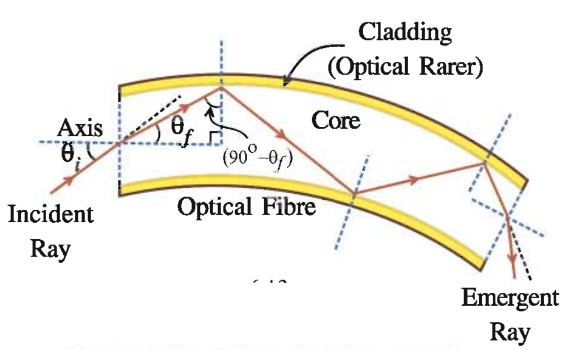


Figure 6.12 Schematic Diagram of Optical Fibre

In figure 6.12, a ray is incident at an angle θ_i to the axis of a fibre from air. θ_f is the angle of refraction. The refracted ray makes an angle θ_f with the axis of the fibre. As shown in the figure, this ray is incident on the wall of the fibre at an angle $(90^\circ - \theta_f)$. It is clear that if angle $(90^\circ - \theta_f)$ is greater than the critical angle for fibre cladding interface, the ray will undergo a total internal reflection. In short, the greater the value of (90° − θ_f) the greater is the chance for total internal reflection. That is, a small value of θ_f is preferable. This also suggests that smaller the value of θ_i the greater are the chances of total internal reflection. Thus, for a given fibre the value of θ_i should not be greater than some particular value.

The above condition for total internal reflection can also be discussed in terms of the refractive index of the material of the fibre.

We have seen that the value (90° − θ_f) should be greater than the critical angle. Thus, the smaller the value of critical angle, the more are the chances of total internal reflection.

Now, $\sin C = \frac{1}{n}$ relation shows that n should be large in order to have small value of C. Thus, the material of an optical fiber should have value of n more than some minimum value. In our discussion we have taken the medium outside the optical fiber as air.

6.8 Refraction at a Spherically Curved Surface

Images can be formed by reflection as well as by refraction. Here we study the refraction at a spherical surface, i.e., at a spherical interface between two transparent media having different refractive indices. In the following discussion, we shall study refraction of paraxial rays at a spherically curved surface. This will help us to understand the image formation by lenses, though lens has two refracting surfaces. We will follow cartesian sign convention in our discussion, and the spherical surface as a very small part of the sphere.

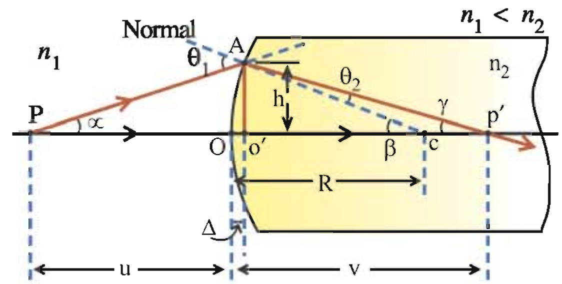


Figure 6.14 Refraction Due to Conve Curved Surface

As shown in the figure, O is the centre of the refracting surface, C is the centre of curvature, OC is the radius of curvature. A point object P is kept at a distance u from O on the principal axis.

To form image after refraction, consider two rays PO and PA from point object P.

For ray PO, angle of incidence is zero. Therefore, according to Snell's law this ray will move along OCP' without bending.

Ray PA is incident at point A on the surface. AC is the normal to the surface at point A. θ_1 is the angle of incidence. Suppose the refractive index (n_1) of the medium-1 is less than the refractive index (n_2) of the medium-2. As a result, the refracted ray bends towards the normal and moves along AP'. Let α, β and γ be the angles made by PA, CA and P'A respectively with principal axis.

Both refracted rays OP' and AP' meet at point P', and forms point like image of an object P. Here, θ_2 is the angle of refraction.

Applying Snell's law at point A,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (6.8.1)$$

Since we are considering paraxial rays, θ_1 and θ_2 are small (measured in radian)

$$\therefore\ n_1\theta_1 = n_2\theta_2 \qquad (6.8.2)$$

From figure, θ_1 is the exterior angle of ΔPAC.

$$\therefore\ \theta_1 = \alpha + \beta \qquad (6.8.3)$$

Similarly, angle β is exterior to ΔCP'A.

$$\therefore \ \beta = \theta_2 + \gamma$$

$$\therefore \ \theta_2 = \beta - \gamma \qquad (6.8.4)$$

Using (6.8.3) and (6.8.4) in equation (6.8.2),

$$n_1(\alpha + \beta) = n_2(\beta - \gamma)$$

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta \qquad (6.8.5)$$

From right angled triangle O'P'A, $\tan\gamma \approx \gamma = \frac{h}{v-\Delta}$ (6.8.6)

where v = image distance

From right angled ΔO'CA, $\tan\beta \approx \beta = \frac{h}{R-\Delta}$ (6.8.7)

And from ΔPAO', $\tan\alpha \approx \alpha = \frac{h}{-u+\Delta}$, (6.8.8)

where $u \rightarrow -u$, object distance, as per the sign convention.

The curved surface considered here is a very small part of the sphere from which it is cut. Thus, Δ is negligible compared to R, u and v.

$$\therefore \ \gamma \approx \frac{h}{v}, \ \beta = \frac{h}{R} \text{ and } \alpha = \frac{h}{-u} \qquad (6.8.9)$$

Combining equations (6.8.5) and (6.8.9), $n_1\left(\frac{h}{-u}\right) + n_2\left(\frac{h}{v}\right) = (n_2 - n_1).\frac{h}{R}$

$$\therefore \ \frac{-n_1}{u} + \frac{n_2}{v} = \frac{(n_2-n_1)}{R} \qquad (6.8.10)$$

Equation (6.8.10) is valid for concave surface also. Equation (6.8.10) is the general equation which relates object distance, image distance and radius of curvature of the curved surface. This equation is derived for the ray travelling **from rarer medium (with refrective index n_1) to the denser medium (with refrective index n_2)**. In the similar way when the ray travels **from denser medium (with refrective index n_2) to the rarer medium (with refrective index n_1)**, we can derive the following equation using Snell's law.

$$\frac{-n_2}{u} + \frac{n_1}{v} = \frac{(n_1-n_2)}{R} \qquad (6.8.11)$$

Case : If surface is plane (plane glass slab).

i.e. R = ∞. Therefore equation (6.8.10) becomes $\frac{+n_1}{u} = \frac{n_2}{v}$

or $\frac{v}{u} = \frac{n_2}{n_1} = \frac{h'}{h}$ (see the topic of magnification).

Whether the image is real or virtual is decided by the sign convention. If image distance is positive, i.e., image is formed on the right of point O, it is real or otherwise.

6.9 Spherical Lenses

In general, a lens is an image forming device, having two bounded refracting surfaces. Of the two surfaces at least one surface is curved. For example, the following figure depicts different types of lenses.

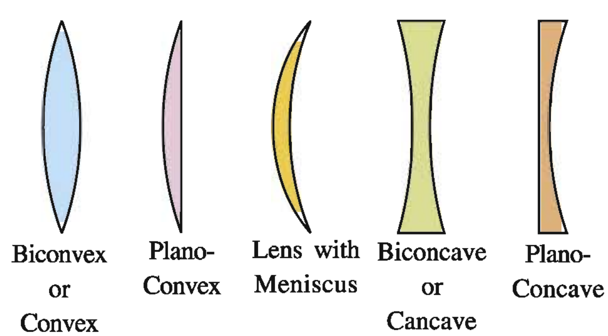
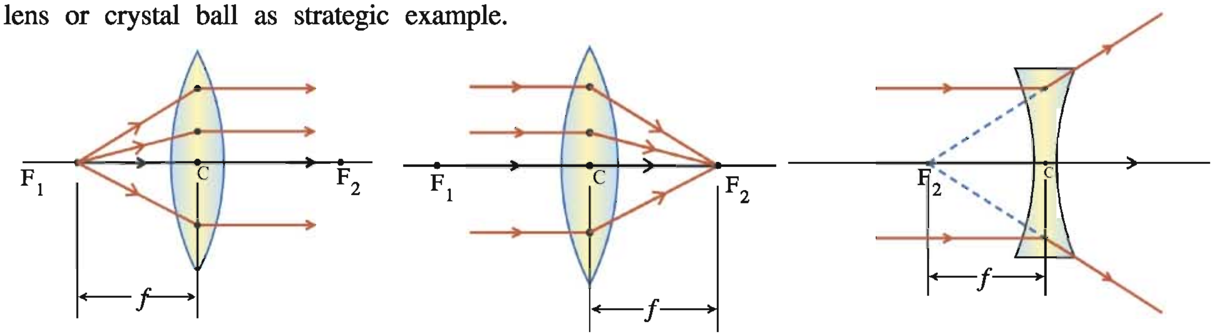


Figure 6.15 Different Type of Lenses

Since spherical surfaces are easy to construct, we first consider image formation by a spherical lens or crystal ball as strategic example.



(a) (b) (c)

Figure 6.16 Focus of Thin Lens

If a point object is placed on the principal axis of a convex lens such that the rays refracted are parallel to the axis figure (a), then the position of the point object is called the **first principal focus** (F_1) of the lens.

If the object is situated at infinite (figures (b) and (c)), refracted rays meet (or appear to meet) for convex (or concave) lens to a point (F_2), then the position of this point is known as **second principal focus** (F_2).

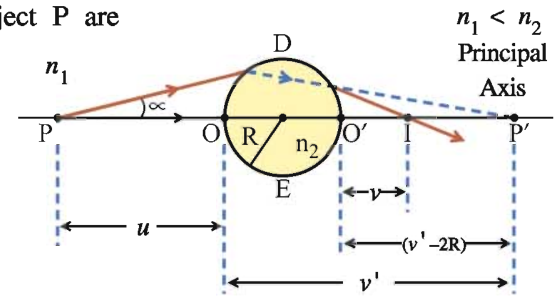
The geometrical centre of the medium of the lens is called its **optical centre** (C).

Distance of principal focus from the optical centre (C) is known as focal length (f) of the lens.

As per the sign convention, f is positive for convex lens and negative for concave lens.

Illustration 8 : Obtain the expression for image distance in terms of the radius of curvature for crystal.

Solution : Here, the rays coming from point object P are refracted twice at surfaces DOE and DO'E, respectively, before forming the final image. But for the sake of understanding, we consider both the refraction separately. Using the formula for spherical surface (equation 6.7.10) at both the surfaces we can determine the position of the (final) image.



At the surface DOE,

$$\frac{-n_1}{(-u)} + \frac{n_2}{v'} = \frac{(n_2 - n_1)}{R} \qquad (1)$$

(We have used Cartesian sign convention.)

Let $u > R$. In this case, v' will be large and positive. That is, image of P due to spherical surface DOE will form at point P' on the right and far from the ball.

Now, for surface DO'E image P' will behave as virtual object. Therefore at the surface DO'E,

$$-\frac{n_1}{(v'-2R)} + \frac{n_2}{v} = \left(\frac{n_1 - n_2}{R}\right) \qquad (2)$$

Since v' is very large, $(v' - 2R)$ is positive. This gives v to be positive, i.e., the final point image will form on the right of the surface DO'E.

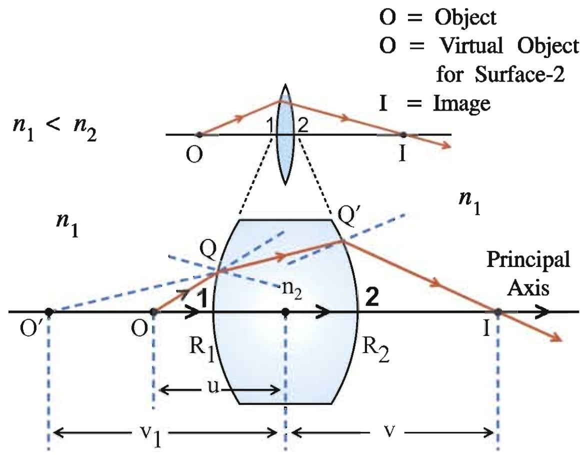


Figure 6.17 Image Formation Due to Thin Lens

6.9.1 Thin Lens : The lens for which the distance between the two refracting surfaces is negligible as compared to the object distance, the image distance and radius of curvature is called a thin lens. In general, radii of curvatures of the two refracting surfaces need not be equal. Being thin lens, the distance can be measured from either surface or even from the centre of the lens.

To obtain the relation between object distance, image distance and radii of curvatures for thin lens consider the following case as shown in the figure 6.17.

To understand, how final image due to thin lens is formed, assume that the two refracting surfaces are separated. Thus, the final image (I) is assumed to be formed due to two refractions at curved surface-1 and then due to surface-2, respectively.

The object O is in the medium having refractive index n_1. The incident ray OQ is refracted at surface-1 into the denser medium with refractive index n_2. (Here $n_2 > n_1$). The image is formed at O′. For the refraction at surface-1 using equation (6.8.10), we can write,

$$\frac{-n_1}{u} + \frac{n_2}{v_1} = \frac{(n_2 - n_1)}{R_1} \qquad (6.9.1)$$

Here, u = object distance and v_1 = image distance.

This image O′ serves as virtual object for surface-2. For surface-2 the ray QQ′ travelling from denser medium is refracted into rarer medium and meets the axial ray from O at I. Thus I is the final image. For refraction at surface-2, using equation (6.8.11), we can write,

$$\frac{-n_2}{v_1} + \frac{n_1}{v} = \frac{(n_1 - n_2)}{R_2} = \frac{(n_2 - n_1)}{-R_2} \qquad (6.9.2)$$

Here, v_1 = object distance for surface-2 and v = image distance.

Adding equations (6.9.1) and (6.9.2), we have

$$\frac{-n_1}{u} + \frac{n_1}{v} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore\ -\frac{1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6.9.3)$$

Equation (6.9.3) is the disired equation. While using it in practice, proper sign convention should be employed.

6.9.2 Lens-Maker's Formula :

If medium on both sides of a lens is same, and object is at infinite (i.e., $u = \infty$) then $v = f$. From equation (6.9.4)

$$\frac{1}{f} - \frac{1}{\infty} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore\ \frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6.9.4)$$

Equation (6.9.4) is known as **lens-maker's** formula. It is named so because it enables one to calculate focal length and radii of curvatures of the lens.

When equations (6.9.3) and (6.9.4) are compared, we have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (6.9.5)

This equation is known as **Gauss'** formula for a lens.

From equation (6.9.4), if the lens turns around, i.e. R_1 and R_2 get interchanged, then also for proper change in sign, f will be found to be same. Therefore, for a thin lens the focal length is independent of the order of the surfaces. If medium-1 is air (i.e., $n_1 = 1$) and let refractive index of medium-2 be $n_2 = n$, equation (6.9.4) becomes

$$\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (6.9.6)$$

For information only : Most general form of lens–maker's formula is,

$$\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \left(\frac{n-1}{n}\right)\cdot\frac{t}{R_1 \cdot R_2};$$

where 't' is the thickness of the lens. For thin lens t is negligible and equation (6.9.6) can be recovered. Above equation also suggests that for thick lens, i.e., t is large, and R_1 and R_2 are small, second term contributes significantly. Thus, for thick lens f is small, i.e., **thick lens converges or diverges strongly.**

6.9.3 Newton's Formula : As we have observed that lens-maker's formula relates radii of curvatures and refractive index of the lens to its focal length. We can also derive an expression relating focal length to image and object distances, which we call **lens user's formula** or **Newton's formula.**

On the left of the lens, ΔABF_1 and $\Delta CF_1P'$ are similar triangles. Therefore,

$$\frac{h_1}{x_1} = \frac{h_2}{f_1} \text{ (writing only magnitude)} \quad (6.9.7)$$

Similarly, for right of the lens,

$$\frac{h_1}{f_2} = \frac{h_2}{x_2} \quad (6.9.8)$$

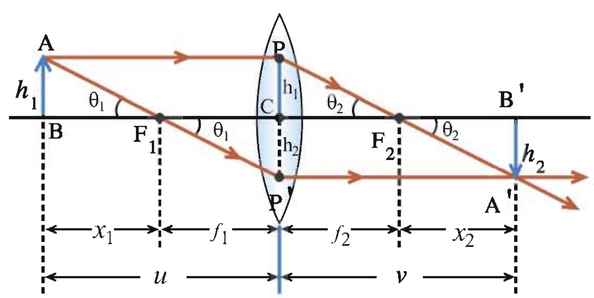


Figure 6.18 Extra Focal Distances of a Convex Lens

Writing combinedly for the ratio $\frac{h_1}{h_2}$,

$$\frac{h_1}{h_2} = \frac{x_1}{f_1} = \frac{f_2}{x_2} \quad (6.9.9)$$

$$\therefore\ x_1 \cdot x_2 = f_1 \cdot f_2 \quad (6.9.10)$$

Equation (6.9.10) is known as the Newton's lens formula. **Here, x_1 and x_2 are known as extra focal object distance and extra focal image distance.** Since these distances are measured from focii rather than from the lens, Newton's formula can be used equally for thin and thick lenses.

When $f_1 = f_2 = f$ (say), equation (6.9.10) becomes

$$x_1 \cdot x_2 = f^2 \quad (6.9.11)$$

6.9.4 Conjugate Points and Conjugate Distances

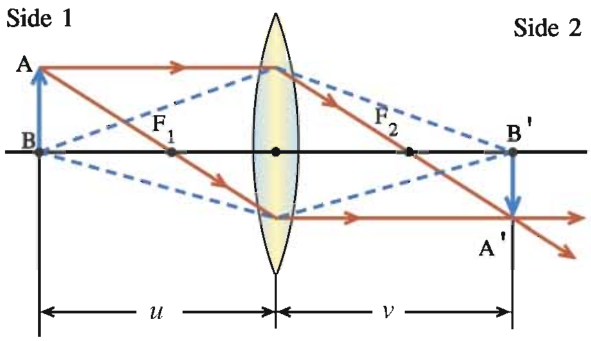


Figure 6.19 Conjugate Points and Distances

As shown in the figure 6.19, all the rays from point A and B are brought to focus at poins A' and B', respectively. Thus, A'B' is the image of an object AB. The principle of reversibility for light rays permits interchange in positions of image and an object. That is, if A'B' is an object, AB becomes the image. Thus, object and image are **conjugate.** Points A and A', and B and B' are called **conjugate points.**

Now, by keeping image distance as the object distance image will form at the object distance. That is, image and object distances are **conjugate distances.**

6.10 Magnification

Convex lenses are used properly to form a magnified image.

$$\text{Magnification, } m = \frac{\text{size of the image}}{\text{size of the object}} \qquad (6.10.1)$$

Since for three-dimensional object the image will also three dimensional, correspondingly we have three types of magnifications. Lateral magnification, longitudinal magnification and angular magnification. We discuss only the lateral magnification below.

Lateral Magnification

Lateral magnification is also called as transverse magnification. It is defined as the ratio of height of an image (h_2) to that of the object (h_1) from the figure 6.18,

$$|m| = \frac{h_2}{h_1} \qquad (6.10.2)$$

According to Cartesian sign convention, height measured above the principal axis is taken positive and below the principal axis it is negative. Hence, the lateral magnification is positive for erect image and negative for a inverted image. Also, from the figure 6.18,

$$\frac{h_1}{u} = \frac{h_2}{v} \text{ (only magnitude)}$$

$$\therefore m = \frac{h_2}{h_1} = \frac{v}{u} \qquad (6.10.3)$$

From equation (6.9.10),

$$m = \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \qquad (6.10.4)$$

6.11 Power of a Lens

It is defined as the converging or diverging capacity of a lens. General form of lens-maker's formula suggests that the thicker the lens, smaller is the focal length and higher is the convergence or divergence. Thus, converging or diverging ability of a lens is inversely proportional to its focal length.

$$\therefore \text{Power of a lens, } P = \frac{1}{f} \qquad (6.11.1)$$

For convex lens power is positive, while for the concave lens it is negative.

Its SI unit is m^{-1} or diopter (D).

i.e., 1D = 1 m^{-1}

When an optician prescribes lens of + 2.0 D, it means a convex lens of focal length = $\frac{1}{2}$ = 0.5 m.

6.12 Combination of Thin Lenses in Contact

Consider a simple optical system that consists of two thin lenses L_1 and L_2 in contact and placed on a common axis. Their focal lengths are f_1 and f_2 respectively. For such an optical system, we assume that the image formed by the first lens becomes the object for a second lens, and we get final image due to the system. We now derive formula for focal length of this equivalent lens as follows.

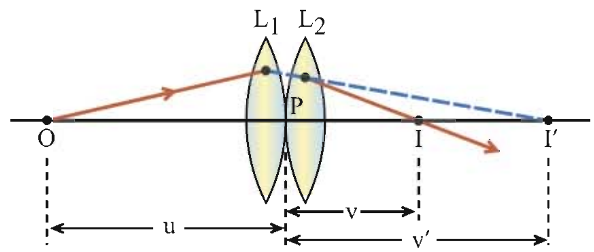


Figure 6.20 Combination of Thin Lenses

From the figure 6.20, consider a case of point like object (O) whose final image (I) is formed due to two thin lenses in contact.

Using Gauss' formula for lens L_1, $-\frac{1}{u} + \frac{1}{v'} = \frac{1}{f_1}$ (6.12.1)

For lens L_2, $-\frac{1}{v'} + \frac{1}{v} = \frac{1}{f_2}$ (6.12.2)

Adding these equations, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2}$ (6.12.3)

If we assume that the final image is formed by a single equivalent lens of the focal length f, then

$$\frac{1}{f} = \frac{-1}{u} + \frac{1}{v} \quad (6.12.4)$$

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \quad (6.12.5)$$

$$\text{or } f = \frac{f_1 \cdot f_2}{(f_1 + f_2)} \quad (6.12.6)$$

Equation (10.12.5) or (10.12.6) is the algebraic relation between f_1, f_2 and f. While using them to find equivalent focal length for different combinations of lenses, proper sign convention should be adopted.

If there are n number of thin lenses in contact, equivalent focal length of them is given by,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots + \frac{1}{f_n} \quad (6.12.7)$$

Lenses with Separation : If two thin lenses are not in contact, but having some separation d, then equivalent focal length can be written as,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2} \quad (6.12.8)$$

Also, $d - (f_1 + f_2)$ is known as the **optical interval** between the two lenses.

Power :

But $\frac{1}{f_1} = P_1$ = power of lens L_1

$\frac{1}{f_2} = P_2$ = power of lens L_2

Therefore, from equation (6.12.8), equivalent power of the combitation is

$$P = P_1 + P_2 + ... + P_n \quad (6.12.9)$$

Lateral Magnification :

For two-lens system lateral magnification due to lens L_1 is $m_1 = \frac{v'}{u}$.

That due to lens L_2 is $m_2 = \frac{v}{v'}$.

If resultant magnification is m, then

$$m = \frac{v}{u} = \frac{v'}{u} \times \frac{v}{v'}$$

$$m = m_1 \times m_2$$

For n number of lenses, $m = m_1 \times m_2 \times .... \times m_n$ (6.12.10)

Equation (6.12.10) suggestes that in order to improve magnification one may use combination of lenses (e.g., compound microscope).

6.13 Combination of Lens and Mirror

The combination of lenses are important for achieving proper magnification, focussing of image at a desired point, etc. Similarly combinations of lenses and mirrors are also useful. We consider one such combination of convex mirror and convex lens.

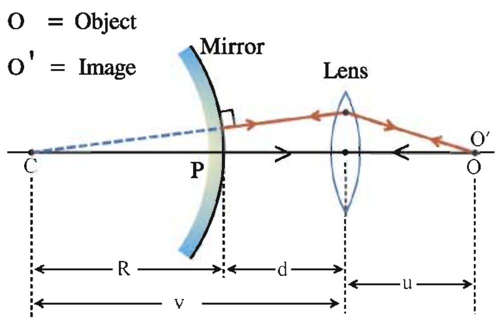


Figure 6.21 Focal Length of a Convex Mirror Using a Convex Lens

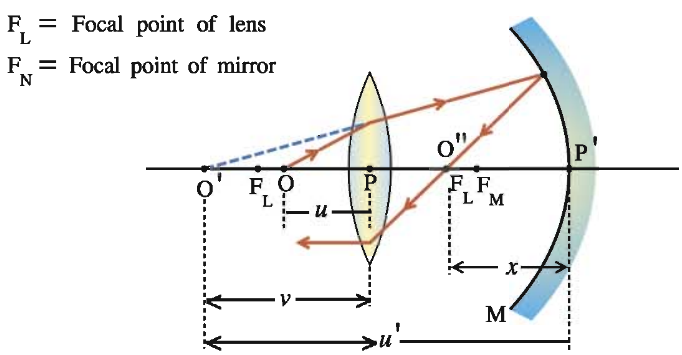
As shown in the figure 6.21, image (O') is formed on the same side of the object. For a given object distance (u), we adjust the mirror distance (d) from the lens in such a way that the image is formed at the object position itself (i.e., parallax between an object and image is removed). In this case, rays incident on to the mirror will be normal to the mirror. In absense of the mirror the image would have been formed at C. Its distance from the lens is v. Since rays falling on the mirror are normal, point C is the centre of curvature for the mirror.

Thus, by measuring v and d, we can find focal length of the mirror as,

$$f = \frac{R}{2} = \frac{1}{2}(v - d).$$

Illustration 9 : A converging lens of focal length 15 cm and a converging mirror of focal length 20 cm are placed with their principal axes coinciding. Point object is placed at a distance 12 cm from the lens. Refracted ray from the lens gets reflected from the mirror, and again refracted by the lens. It is found that the final ray coming out of the lens is parallel to the principal axis. Find the distance between the mirror and the lens.

Solution :



Applying Gauss' formula to lens,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\therefore v = \frac{u \cdot f}{u + f} = \frac{(-12)\times(15)}{-12+15}$$

(Using cartesian sign convention)

$= -60$ cm.

Negative sign indicates that image (O') is virtual. This image works as an object for the mirror.

For mirror, object distance,

$$u' = O'O'' + O''P' = (PO' + PO'') + O''P'$$

$$= (60 + 15) + x = (75 + x) \text{ cm (for mirror, PO}' = v \text{ is taken positive)}$$

Since image due to mirror is obtained at O'', its distance from the mirror is x.

Applying Gauss' formula to the mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{1}{-(75+x)} + \frac{1}{-x} = \frac{1}{-f}$$

Simplifying , $\frac{(75+2x)}{(75+x)\cdot x} = \frac{1}{20}$

$$\therefore x^2 + 35 \cdot x - 1500 = 0$$

$$\therefore x = 25 \text{ cm or } x = -60 \text{ cm.}$$

Thus, physically acceptable solution is 25 cm. Therefore, distance of the mirror from the lens is = 25 + 15 = 40 cm.

Illustration 10 : Distance between an object and a screen is d. Prove that for a thin convex lens, there are two positions for the object at which image can be obtained on the screen, and under certain condition only. Derive the condition for the same. When will the image not be formed ?

Solution : Suppose the object distance is u,

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For a convex lens u is negative. So,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

But, $u + v = d$ (given)

$$\therefore v = d - u$$

$$\therefore \frac{1}{d-u} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{u+d-u}{u(d-u)} = \frac{1}{f}$$

$$\therefore u^2 - ud + fd = 0$$

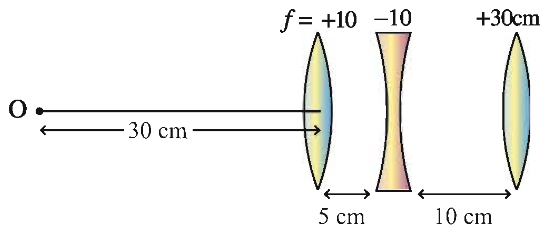
This is the quadratic equation for variable u. It's roots are as given below :

$$u = \frac{d \pm \sqrt{d^2 - 4fd}}{2}$$

Thus, if $d > 4f$, two values of u are possible and if $d < 4f$, u will be a complex number and hence the image will not be formed.

Illustration 11 : Decide the position of the image formed by the given combination of lenses.

Solution : For the image formed by first lens.



$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$$

$$\therefore \frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

$$\therefore v_1 = 15 \text{ cm}$$

Thus image formed by the first lens is formed at 15 cm distance on the right-hand side. This image is on the right-hand side of the second lens at 15 − 5 = 10 cm distance and so it acts as a virtual object for the second lens.

Now for the second lens,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\therefore \frac{1}{v_2} - \frac{1}{10} = -\frac{1}{10}$$

$$\therefore v_2 = \infty$$

This distance v_2 (= ∞) is the object distance for the third lens. So, the third image formed due to it should be on the principal focus of the third lens. Thus, as the focal length of the third lens is 30 cm, the final image is formed at 30 cm distance on the right side of the third lens.

Illustration 12 : For a thin lens prove that when the heights of the object and the image are equal, object distance and image distance are equal to $2f$.

Solution : Here, $|h| = |h'|$

$\therefore |v| = |u|$

Using the equation for lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-u} = \frac{1}{f}$$

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{2}{v} = \frac{1}{f}$$

$$\therefore v = 2f$$

$$\therefore u = v = 2f$$

Here, the points at $2f$ distance on both the sides of the lens are called **conjugate focl.**

Illustration 13 : Two converging lenses of powers 5D and 4D are placed 5 cm apart. Find the focal length and power of this combination.

Solution : Focal length of first lens, $f_2 = \frac{1}{5} = 0.2$ m = 20 cm

Focal length of second lens, $f_2 = \frac{1}{4} = 0.25$ m $= 25$ cm

Distance between two lenses, $d = 5$ cm

Now, equivalent focal length of this combination is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$f = \frac{f_1 \cdot f_2}{(f_1 + f_2) - d} = \frac{20 \times 25}{(20+25) - 5} = 12.5 \text{ cm}$$

And equivalent power is given by,

$P = (P_1 + P_2) - d.P_1P_2$

$= (5 + 4) - (0.05) \times (5)(4)$ (d is written in meter)

$\therefore$ P = 8D or $P = \frac{1}{f} = \frac{1}{0.125} = 8D$

6.14 Refraction and Dispersion of Light due to a Prism

As shown in the figure 6.22, the cross-section perpendicular to the rectangular surface of a prism is shown. A ray PQ of monochromatic light is incident at point Q on the surface AB. According to Snell's law, it is refracted and travels along the path QR. Thus, it deviates from the incident direction by an amount δ_1. This ray QR is incident on the surface AC at point R, and emerging out as a ray RS. It suffers a deviation δ_2. By extending the incident ray PQ to PQE, total deviation between the incident and the emergent ray is found. When the emergent ray RS is extended backword it meets PE at D. Angle between the incident ray and the emergent ray is called the angle of deviation, δ.

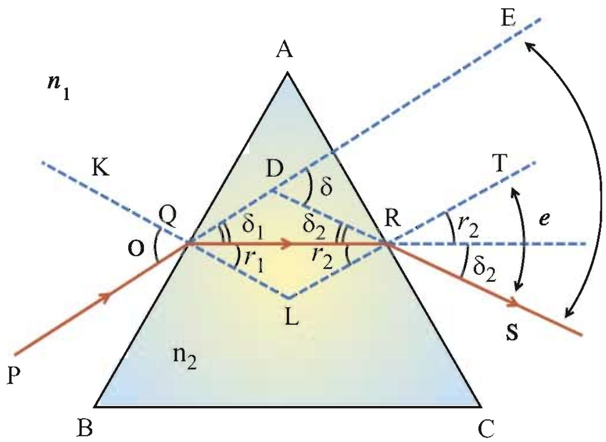


Figure 6.22 Refraction Due to Prism

From figure 6.22, in □AQLR, ∠AQL and ∠ARL are right angles.

$\therefore m\angle A + m\angle QLR = 180°$ (6.14.1)

and for ΔQLR, $r_1 + r_2 + m\angle QLR = 180°$ (6.14.2)

Comparing above equations,

$r_1 + r_2 + m\angle QLR = m\angle A + m\angle QLR$

$\therefore r_1 + r_2 = A$ (6.14.3)

For ΔDQR, $\angle EDR \equiv \angle EDS = \delta$ is the exterior angle. Therefore,

$\delta = \angle DQE + \angle DRQ$

$\therefore \delta = \delta_1 + \delta_2$ (6.14.4)

But $\delta_1 + r_1 = i$ ($\because$ vertically opposite angles)

$\therefore \delta_1 = i - r_1$ (6.14.5)

Similarly, $\delta_2 = e - r_2$ (6.14.6)

$\therefore \delta = (i - r_1) + (e - r_2) = (i + e) - (r_1 + r_2)$

Using equation (6.14.3)

$\delta = i + e - A$ or $i + e = A + \delta$ (6.14.7)

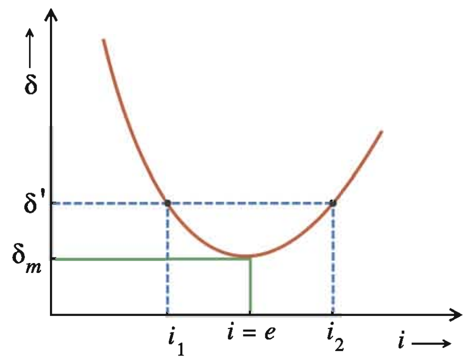


Figure 6.23 Variation of Deviation with Angle of Incidence

Equation (6.14.7) gives the relation between angle of deviation, angle of incidence and angle of emergence and the prism angle. It is known as an **equation for prism.**

It is clear from the above equation that the angle of deviation depends on the angle of incidence. For the sake of understanding, the graph of the measured values of angle of deviation against corresponding angle of incidence for an equilateral prism is shown in the figure 6.23.

We can see from the graph that for two values of angle of incidence (i_1 and i_2) angle of deviation δ is same. This can be understood from the reversibility of the rays.

If the incident ray is SR instead of PQ, then the refracted ray will follow exactly the reverse path, i.e., SRQP, and the emergent ray becomes PQ. In this case also, however, the angle of deviation remains the same. But for a particular value of angle of deviation there exists only one value of angle of incidence. And experimentally, it is found that this angle of deviation is minimum (δ_m). In the condition of minimum deviation of the incident ray the angle of deviation is called the angle of minimum deviation (δ_m). In this situation it is found that $i = e$.

From equation (6.14.7),

$$\delta_m = i + i - A = 2i - A$$

$$i = \frac{A+\delta_m}{2} \qquad (6.14.8)$$

Applying Snell's law at pont Q,

$$n_1 \sin i = n_2 \sin r_1 \qquad (6.14.9)$$

At point R, considering SR as the incident ray,

$$n_1 \sin e = n_2 \sin r_2$$

As $i = e$

$$\therefore\ n_1 \sin i = n_2 \sin r_2 \qquad (6.14.10)$$

From equations (6.14.9) and (6.14.10)

$$\therefore\ r_1 = r_2 \qquad (6.14.11)$$

From equation (6.14.3), and let $r_1 = r_2 = r$,

$$r + r = A$$

$$\therefore\ r = \frac{A}{2} \qquad (6.14.12)$$

Substituting the values of (6.14.8) and (6.14.12) in either in (6.14.9) or (6.14.10),
This gives,

$$\therefore\ n_1 \sin\left(\frac{A+\delta_m}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

$$\text{or}\quad \frac{n_2}{n_1} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (6.14.14)$$

If the prism is kept in air, i.e. $n_1 = 1$ and $n_2 = n$,

$$\therefore\ n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (6.14.15)$$

Equation (6.14.15) shows that value of δ_m depends on the angle of prism. The refractive index of the material of the prism and the medium in which prism is kept.

For equilateral prism, when δ is minimum, refracted ray (QR) through the prism is **parallel to the base BC** of the prism. Equation (6.14.15) is of practical importance to measure refractive index of the material of the prism.

Case : The prisms with small angle of prism are called thin prisms. For such prisms, angle of deviation is also small. In this case equation (6.14.15) gives

$$\delta_m = A(n - 1) \quad (6.14.16)$$

Dispersion :

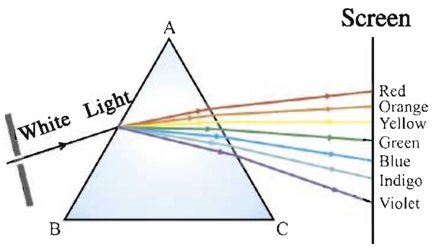


Figure 6.24 Dispersion of White Light

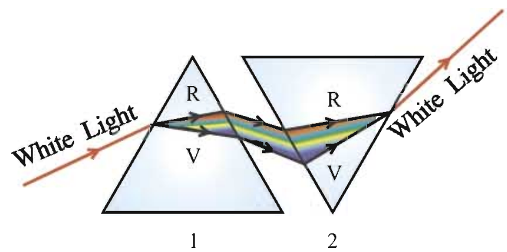


Figure 6.25 Dispersion and Recombination of White Light

As shown in figure 6.24, when a beam of white light or sun light passes through a prism the emergent light is made up of different colours. To understand this phenomenon, Newton has arranged two identical prisms as shown in figure 6.25. A ray of white light is incident on the prism-1, and emergent ray from the prism-2 is observed. It is found that this emergent ray is also white. This experiment explains that the first prism disperses the colours of white light, while the second prism brings them together.

The phenomenon in which light gets divided into its constituent colours is known as dispersion of light.

It is found that for the visible part of the electromagnetic spectrum violet colour has the maximum refractive index and red colour has the lowest. From equation (6.14.16), corresponding minimum angle of deviation through the same prism is

$$\delta_v = A(n_v - 1)$$

and $\delta_r = A(n_r - 1)$

It is now clear that as $n_v > n_r$, $\delta_v > \delta_r$.

Thus, deviation of violet colour is more compared to the deviation of red colour.

The total angle through which the spectrum is spread is called as the angular dispersion. It is defined as,

$$\theta = \delta_v - \delta_r = (n_v - n_r) \cdot A \quad (6.14.17)$$

For example, the spectrum obtained by a prism made up of flint glass is wider, more dispersed and more detailed as compared to the one obtained by common crown glass.

Illustration 14 : For a prism, angle of prism is 60° and it's refractive index is 1.5, find (1) angle of incidence corresponding to the angle of minimum deviation and (2) angle of emergence for angle of maximum deviation.

Solution : (1) For minimum deviation,

$r_1 = r_2$ and $A = r_1 + r_2$

$\therefore$ A = $2r_1$

or $r_1 = \frac{A}{2} = \frac{60}{2} = 30°$

Now $n = 1.5$ and

$$n = \frac{\sin i}{\sin r_1}$$

$\therefore$ $n \sin r_1 = \sin i$

$\therefore$ $1.5 \times \sin 30° = \sin i$

$\therefore$ $1.5 \times 0.5 = \sin i$

$\therefore$ $i_1 = 48°\ 35'$

(2) For maximum deviation, $i = 90°$

$\therefore$ $1.5 = \frac{\sin 90°}{\sin r_1}$ $\therefore$ $r_1 = 41°\ 48'$

$\therefore$ $r_2 = A - r_1 = 60 - 41°\ 48' = 18°\ 12'$ ($\because$ $r_1 + r_2 = A$)

$1.5 \sin r_2 = \sin e$ ($\because$ $n \sin r_2 = \sin e$)

$\therefore$ $1.5 \times \sin 18°\ 12' = \sin e$

$\therefore$ $\sin e = 0.4685$

$\therefore$ $e = 27°\ 56'$

Illustration 15 : An equilateral prism is kept in air and for a particular ray, angle of minimum deviation is 38°. Calculate the angle of minimum deviation if the prism is immersed in water. Refractive index of water is 1.33.

Solution : $\frac{n_g}{n_a} = \frac{\sin\left(\frac{60+38}{2}\right)^\circ}{\sin 30^\circ}$

Taking $n_a = 1$,

$$n_g = \frac{\sin 49^\circ}{\sin 30^\circ} = 1.509$$

When prism is immersed in water,

$$\frac{n_g}{n_\omega} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)^\circ}{\sin 30^\circ}$$

But $n_\omega = 1.33$

$$\therefore \frac{1.509}{1.33} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)^\circ}{0.5}$$

$$\therefore \sin\left(\frac{60+\delta_m}{2}\right)^\circ = 0.5679$$

$$\therefore \frac{60+\delta_m}{2} = 34°\ 36'$$

$\therefore$ $\delta_m = 9°\ 12'$

6.15 Scattering of Light

Light scattering is one of the two major physical processes that contribute to the visible appearance of most of routine objects, the other being absorption. Broadly, scattering can be classified either as elastic or inelastic. Natural occurrence like, colour of sky during sunrise or sunset and during day time, colour of clouds can be understood by elastic scattering of light due to atmospheric atoms, molecules, water droplets, etc. Light falling on such particles is absorbed by them and immediately radiated in different amount in different directions. As a result, part of the intensity of light ray is diverted to different directions in different proportions.

It is found that the intensity of scattered light depends on the ratio (α) of the size of the particle (i.e. its diameter, for spherical particles) and wavelength of the light.

If $\alpha << 1$: Scattering is known as Rayleigh scattering

$\therefore$ $\alpha \approx 1$: Scattering is known as Mie-scattering.

$\therefore$ $\alpha >> 1$: Geometric scattering.

6.15.1 Rayleigh Scattering : If the size of the particle which scatters the light is smaller than the wavelength of the incident light, the scattering is known as **Rayleigh scattering.**

Lord **Rayleigh** showed theoretically that the intensity of scattering is inversely proportional to the fourth power of the wavelenght of light. Since the wavelength of blue light is 1.7 times smaller than the red light. So, the intensity of scattered blue light is 8 to 9 times more than the intensity of scattered red light. Thus, intense scattered-blue light is responsible for the sky to be bluish.

Another consequence of Rayleigh scattering is the appearance of reddish colour of the sun either at the sunrise or at the sunset

As shown in figure 6.26, at the sunrise or sunset, light from the sun has to travel relatively more distance to reach the observer on the earth as compared to the noon-time. During the passage of light in the atmospheric light of smaller wavelengths scatter more. Hence, only light with high wavelengths (i.e., reddish or yellowish-red) can reach to the observer substantially. Thus, the sun appears reddish. However, if we see vertically upward, sky appears blue. This effect is maximum in the direction perpendicular to the incident light. The same is the reason for reddish full-moon while rising or setting.

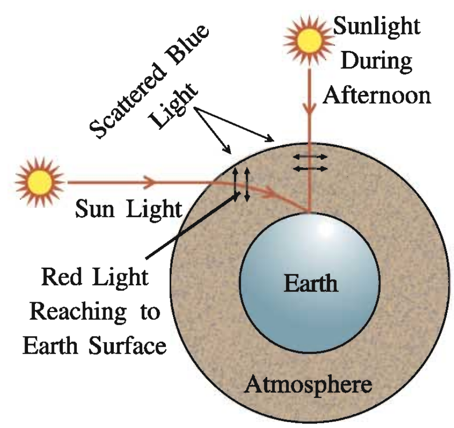


Figure 6.26 Scattering of Sun Light Due to Atmosphere

It is found that the intensity of the Rayleigh scattered light increases rapidly as the ratio α increases. Further, the intensity of Rayleigh scattered light is identical in the forward and reverse directions.

6.15.2 Mie-Scattering : If the size of scatterer particles are slightly larger than the wave length of the light, scattering is known as Mie-scattering. It was studied by Gustav Mie in 1908. It is found that as the size of the particle increases, the proportion of diffused scattering also increases. Since water droplets in the cloud have size comparable to wavelength of light, scattering of sun light through clouds is diffused scattering. It is independent of incident wavelengths. Hence, all colours scatter equally, and the clouds appear white. Unlike Rayleigh scattering, Mie-scattering is observed in larger amount in the forward direction than in the reverse direction. Also, as the particle size increases, more amount of the light is scattered in the forward direction.

For information only : The Mie-scattering shows that if the size of the particle lies between two wavelengths of light, then the light having more wavelength is scattered more than the light with smaller wavelength. If dust clouds have such size then the rising sun and moon or setting sun and moon would be seen blue or greeen !

However, such a situation rarely occurs. In the 19th century when the Volcano Krakotoa in Indonesia erupted and in 1950 when there were extensive forest fires in East-Canada and North-East USA, such situation took place.

If earth had no atmosphere, the sky would have been blackish, and stars would have been visible even during day time ! This becomes reality at or above 20 km from the earth surface.

In presense of high pollution in the atmosphere, the sky appears greyish and hazy instead of blue.

6.15.3 Raman-Scattering : The Raman effect was first reported by Indian Nobel laureate C. V. Raman. This inelastic scattering of light was also predicted by Adolf Smekal in 1923. Hence, this effect is also known as **Smekal-Raman effect.**

When a strong beam of visible or ultraviolet light is incident on gas, liquid or transparent solid, a small fraction of light is scattered in all directions. It is found that the scattered light spectrum is made up of lines of incident wavelength (Rayleigh lines) and weak additional lines of changed wave lenghts. These additional lines due to inealstic scattering are called **Raman lines.** Raman lines are found symmetrically on both sides of the central Rayleigh lines. Raman lines with low frequencies (or higher wavelengths) are known as **Stokes lines,** and the one on higher frequency (or low wavelength) sides are known as **Antistokes lines.**

Raman lines are the characteristics of the material.

Raman scattering is the most versatile technique to study characteristics of the material, different excitations in the materials, in optical amplifiers, to study biological organisms and human tissues, etc.

6.16 Optical Instruments : The purpose of most optical instruments is to enable us to see the object better. They are made up of combination of refracting and/or reflecting devices such as lenses, mirrors and prisms. They can be divided into two groups : instruments forming **real images** (e.g., projectors) and instruments forming imaginary images (e.g., microscopes and telescopes).

We first study simple microscope.

6.16.1 Simple Microscope : Suppose we want to see a microscopic object clearly and magnified.

The least distance at which a small object can be seen clearly with comfort is known as **near point** (D) or **distance of most distinct vision.** For normal eye this distance is 25 cm.

Suppose a linear object with height h_0 is kept at near point (i.e., $u \equiv D = 25$ cm) from eye. Let it subtend an angle θ_0 with the eye (See figure 6.27 (a)).

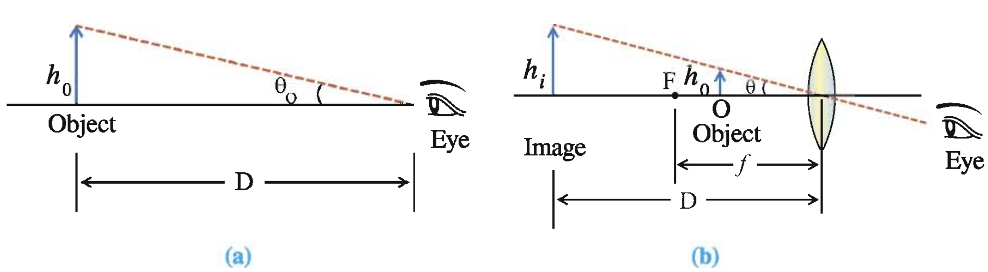


Figure 6.27 Simple Magnifier

Now, if object is kept within the focal length (f) of a convex lens such that its virtual, erect and magnified image is formed at a distance equal to the near point. Since the lens is very close to eye, angle (θ) subtended by the object with lens and eye are almost identical.

The angular magnification is defined as

$$m' = \frac{\tan\theta}{\tan\theta_0} \approx \frac{\theta}{\theta_0} \text{ (for small } \theta \text{ and } \theta_0) \qquad (6.16.1)$$

Also, from figures 6.27 (a) and (b),

$$\tan\theta_0 \approx \theta_0 = \frac{h_0}{D}$$

$$\text{and } \tan\theta \approx \theta_0 = \frac{h_i}{D}$$

$$\therefore\ m' = \frac{h_i}{h_0} \qquad (6.16.2)$$

But for convex lens, linear magnification,

$$|\ m\ | = \frac{v}{u}$$

$$|\ m\ | = \frac{D}{u} \qquad (6.16.3)$$

Using Gauss' formula,

$$-\frac{1}{(-u)} + \frac{1}{(-D)} = \frac{1}{f} \text{ (for virtual image } v = D \text{ is taken negative)}$$

$$\therefore\ \frac{1}{u} = \frac{1}{f} + \frac{1}{D} = \frac{D+f}{D \cdot f}$$

$$\therefore\ u = \frac{Df}{D+f} \qquad (6.16.4)$$

Using (6.16.3) in equation (6.16.4),

$$|\ m\ | = 1 + \frac{D}{f} \qquad (6.16.5)$$

When the image is at a very large distance

$$|\ m\ | \approx \frac{D}{f} \qquad (6.16.6)$$

Combinedly equations (6.16.5) and (6.16.6) suggest that the value of m should be between $\frac{D}{f}$ and $\left(1+\frac{D}{f}\right)$

6.16.2 Compound Microscope : We have seen that in a simple microscope magnifying power depends on $\frac{D}{f}$. Thus, we tempted to use a convex lens with small focal length in order to improve magnification. It is found, however, that by reducing the value of focal length, image becomes distorted. Thus, very large and clear image is not possible with a simple microscope. But if magnified image due to one simple microscope is used as an object for another simple microscope, then we get very enlarged image. This is the basic principle of a compound microscope.

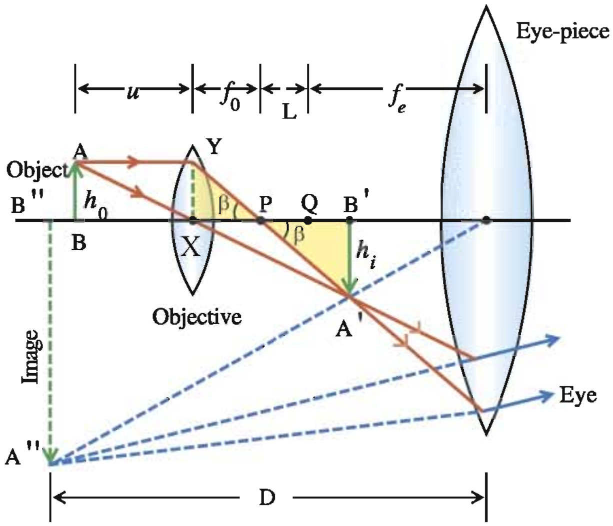


Figure 6.28 Compound Microscope

The lens kept near the object is known as **objective,** while the one nearer to eye is known as **eye-piece.** Distance between the second focal point (P) of the objective and the first focal point (Q) of the eye-piece is known as **tube-length (L)** of the microscope.

It is clear from the figure that the image obtained by the objective is real, inverted and magnified. This image acts as an object for the eye-piece. Eye-piece works as a simple microscope and gives a virtual and highly magnified final image (A''B'').

The image due to objective is observed close to the focal point of an eye-piece. Due to this reason final image is formed at a considerable large distance.

Magnification : Magnification due to the objective,

$$m_0 = \frac{h_i}{h_0} \tag{6.16.7}$$

From ΔXYP and ΔPA'B', respectively,

$$\tan\beta = \frac{XY}{PX} = \frac{h_0}{f_0} \Rightarrow h_0 = f_0 \cdot \tan\beta$$

$$\text{and } \tan\beta = \frac{A'B'}{PB'} \approx \frac{h_i}{PQ} \quad (\because \text{ Q and B' are very close to each other})$$

$$\therefore\ h_i = PQ \cdot \tan\beta = L \cdot \tan\beta$$

$$\therefore\ m_0 = \frac{L}{f_0} \tag{6.16.8}$$

Magnification due to eye-piece,

$$m_e = \left(\frac{D}{f_e} + 1\right) \quad \text{(See Equation (6.16.5)} \tag{6.16.9}$$

Resultant magnification of a compound microscope is (Equation (6.12.10)),

$$m = m_0 \times m_e$$

$$= \frac{L}{f_0} \times \left(\frac{D}{f_e} + 1\right) \tag{6.16.10}$$

In practice, eye-piece is so adjusted that image A'B' falls very close to its focus Q. Thus, image obtained by eye-piece will be at very large distance (D). Thus, above equation can be written as,

$$m \approx \frac{L}{f_0} \times \frac{D}{f_e} \tag{6.16.11}$$

In order to have large magnification, tube length (L) of the microscope should be kept large.

Illustration 16 : An object is 10 mm from the objective of a compound microscope. The lenses are 30 cm apart and the intermediate image is 50 mm from the eyepiece. What overall magnification is produced by the instrument ?

Solution : From the figure 6.28, applying Gauss's formula to the objective,

$$\frac{1}{-u} + \frac{1}{v} = \frac{1}{f_0} \qquad (1)$$

where v = image distance due to objective lens $\approx f_0 + L$ (as Q and B' are very close to each other).

Since image due to objective is formed at 50 mm from the eye-piece, and distance between two lenses is 30 cm = 300 mm (given), image distance from objective

$v = 300 - 50 = 250$ mm

From equation (1),

$$\frac{-1}{-10} + \frac{1}{250} = \frac{1}{f_0} \text{ (using sign convention)}$$

$$\therefore f_0 = \frac{250 \times 10}{(250+10)} = 9.62 \text{ mm} \approx 10 \text{ mm}$$

Since $v \simeq f_0 + L \Rightarrow L = 250 - 10 = 240$ mm

Final image is always close to the object,

D ≈ (object distance for objective) + (distance between two lenses)

= 10 + 300 = 310 mm

For eye-piece, Gauss' equation,

$$\frac{1}{-u} + \frac{1}{v} = \frac{1}{f_e}$$

$$\therefore \frac{1}{f_e} = \frac{-1}{-50} + \frac{1}{-310} \text{ (For virtual image, } v = -D)$$

$$= \frac{-310+50}{(50 \times 310)}$$

$\therefore |f_e| = 59.6 \approx 60$ mm

thus, resultant magnification is

$$m = \frac{L}{f_0} \times \frac{D}{f_e} = \frac{240}{10} \times \frac{310}{60} = 124$$

Note : Since the final image obtained at a distance 31 cm from the eye-piece is greater than the near-point distance, it can be seen comfortably.

6.16.3 Astronomical Telescope : After observing minute objects using a microscope, now it's time to observe very huge celestial bodies which are crores of kilometers away. Such bodies, in spite of being huge and very far from each other, they are seen to be small and very close to each other by our naked eyes (for example, stars). For observing such objects an Astronomical Telescope is used. It's ray diagram is shown in figure 6.29.

In this telescope two convex lenses are kept in such a way that their principal axis coincide. The lens facing the object is called objective and the lens near the eye is known as eye-piece. Here, the diameter and the focal length of the objective are greater than that of the eye-piece.

The eye-piece can move to and fro in the telescope-tube. When the telescope is focussed on a distant object, parallel rays coming from this object form a real, inverted, and small image

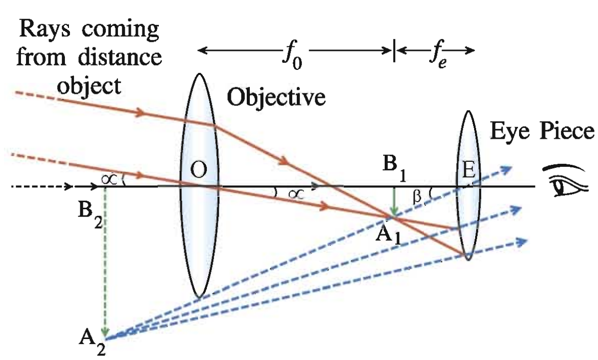


Figure 6.29 Astronomical Telescope

A_1B_1 on the second principal focus of the objective. This image is the object for the eye-piece. Eye-piece is moved to and fro to get the final and magnified inverted image A_2B_2 of the original object at a certain distance.

We obtain the expression for the magnifying power of a telescope, as follows.

Magnification of the telescope,

$$m = \frac{\text{Angle subtended by the final image with eye}}{\text{Angle subtended by the object with the objective or eye}} = \frac{\beta}{\alpha}$$

From figure 6.29

$$\text{Magnification, } m = \frac{\beta}{\alpha}$$

$$= \frac{A_1B_1}{f_e} \times \frac{f_0}{A_1B_1}$$

$$\therefore m = \frac{f_0}{f_e}$$

This equation shows that to increase the magnification of the telescope, focal length of the objective should be increased, and focal length of the eye-piece should be reduced. $f_0 + f_e$ is the **optical length** of the telescope. So, length of the tube $L \geq f_0 + f_e$.

If the focal length of the eye-piece is 1 cm and the focal length of the objective is 200 cm, magnification of the telescope would be 200. Using such a telescope, if the stars having angular distance $1'$ are observed, they would be seen at $200 \times 1' = 200' = 3.33°$ angular distance from each other.

For a telescope, light **gathering power** and **resolving power** (power to view two nearby objects distinctly) are very important.

Amount of light entering the objective of the telescope is directly proportional to the square of the diameter of the objective. Also, with increase in the diameter of the objective, resolving power also increases.

Image formed in this type of telescope is inverted. So if we see from the Earth we get an inverted view of the real scene. To get rid of this problem, an extra pair of inverting lenses in the terrestrial telescope are kept, so that the erect image of the distant object is obtained. Such a telescope is called a **terrestrial telescope.** However, Galileo had used a convex lens and a concave lens in such a telescope.

To get rid of the practical problem faced in obtaining high resolution and high magnification in refracting telescopes, mirrors are used in modern telescopes. Such a telescope is known as reflecting telescope. In such a telescope we can get rid of other problems like chromatic aberration and also spherical aberration, if a parabolic mirror is used.

(In chromatic aberration the edge of the image is seen multicoloured due to dispersion of light and in spherical aberration, image of a point like object is seen spread out).

Construction of the telescope made by Cassegrain (reflecting telescope) is shown in figure 6.30.

As shown in the figure, parallel rays coming from a distant object are incident on the reflecting surface of the primary concave mirror. The reflecting surface of the mirror is parabolic. The rays after getting reflected from this surface are focussed on the principal focus (F) of this mirror. (If the eye-piece is kept near F the image can be seen. But as F is inside the tube, it is difficult to place the eye-piece there.) Cassegrain placed a convex mirror. Rays reflected by the secondary mirror are focussed on the eye-piece after passing through the hole kept in the primary mirror. Diameter and focal length of the primary mirror are kept large in such telescope.

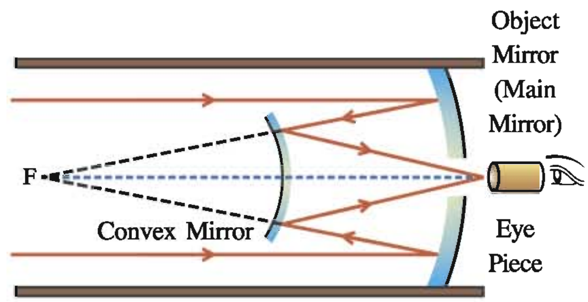


Figure 6.30 Reflecting Telescope

Binoculars used for bird watching or for viewing a cricket match are double telescopes. Here, the final image is erect. In the binoculars use of prisms reduces the size of the binoculars. Binoculars are so named because in them viewing is possible by both eyes.

6.16.4 Human Eye : Human eye is the best example of a natural optical appliance. See figure 6.31.

The ray entering the eye is first refracted in the cornea, yet the eye lens is the main factor "culprit" in this case. Due to this lens, inverted and real image is formed on the ratina. This image is processed in the human brain and as a final effect, we feel the image be erect.

Retina has two types of cells :

(1) Rods : These cells give the sensations of less intensity of light.

(2) Cones : These cells give the sensations of colour and high intensity of light.

In case of eye, distance between the retina and the lens is fixed. That is why focal length of the eye lens changes in such a way that the images of the object are always obtained on the retina. (Really, eye lens is smart lens). This becomes possible due to the ciliary muscles attached to the lens. It makes the lens thick or thin as per requirement.

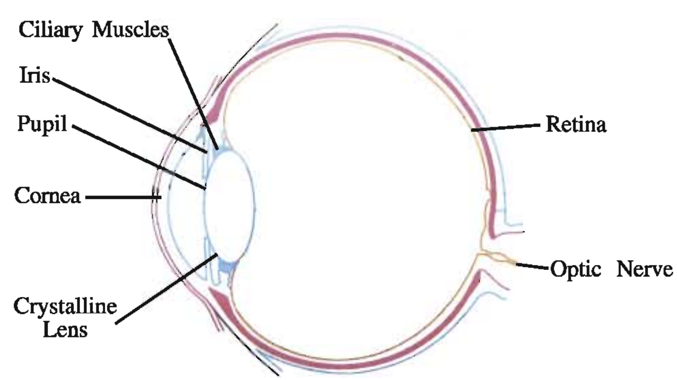


Figure 6.31 Human Eye (For Information Only)

The Iris controls the amount of light entering the eye. It does the work by controlling the size of the pupil. When we see the object kept on the side, lens of the eye rotates and brings the image on the central region of the retina, (fovea).

Defects of Vision : If the lens of eye cannot become thin as per requirement and remains thick only, then rays coming from far objects, which are parallel, undergo extra refraction as shown in figure 6.32, and get focussed in front of the retina. And therefore far off objects cannot be seen clearly. But the image of nearby objects is formed on the retina (figure 6.33). This type of defect is called **Near sightedness (myopia).**

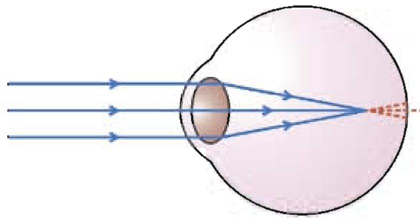


Figure 6.32 Image of Distance Object Falls in front of Retina

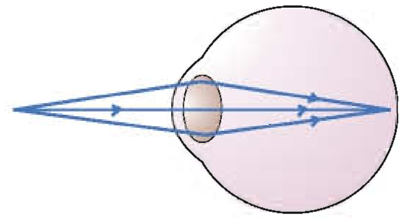


Figure 6.33 Image of Nearby Object Falls on the Retina

This defect can be corrected by using concave lens of proper focal length (figure 6.34).

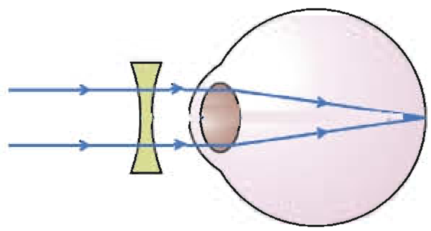


Figure 6.34 To Correct this Defect, Concave Lenses are Used

If the lens remains thin, does not become thick as per requirement, rays coming from a nearby object suffer less refraction and are focussed behind the retina. (figure 6.35). Such an image is not clear. Image of a distant object is formed on the retina only and can be seen clearly, but nearby objects cannot be seen clearly. This defect is called **far sightedness (hypermetropia)**. This type of defect is due to less convergence of rays. To correct this defect a convex lens of proper focal length is used (figure 6.36).

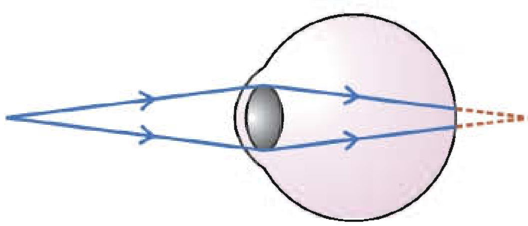


Figure 6.35 Hypermetropia

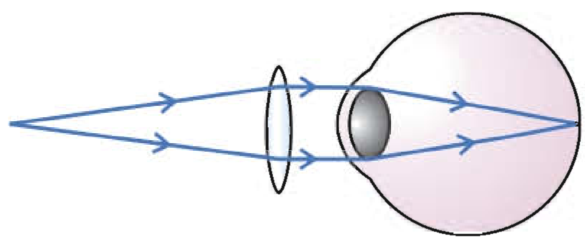


Figure 6.36 Convex Lens Between Object and Eye

Some people, if shown a wire gauge cannot see the vertical and horizontal both wires clearly, but any one is seen clearly. This defect is called **astigmatism**. If the curvature of the lens and the corena are not the same, this defect occurs. E.g., if a person can see horizontal wires but not vertical. Here, horizontal curvatures are same but vertical curvatures are not. So rays are refracted equally in the horizontal plane, but refraction in the vertical plane is not equal. As a result horizontal wires are seen clearly and vertical wires are not seen clearly. To get rid of this defect, cylindrical lens is used. In the above mentioned case a cylindrical lens of proper curvature and horizontal axis can be used to rectify the defect.

SUMMARY

1. For mirrors Gauss' equation is $\frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f}$, where u = object distance, v = image distance, R = radius of curvature and f = focal length.
2. Lateral magnification for mirrors is given by $m = \frac{h'}{h} = -\frac{v}{u}$
3. For a compound slab of different transparent media, general form of Snell's law is written as, $n_1\sin\theta_1 = n_2\sin\theta_2 = n_3\cdot\sin\theta_3 = ...$
4. Total internal reflection is used as reflectors, e.g. flint glass-prism may be used as high quality reflector. For glass–air interface, critical angle (C) is given by,

   $C = \sin^{-1}\left(\frac{1}{n}\right)$, where n = refractive index of glass.
5. Total internal reflection phenomenon is also used in optical fibres.
6. For thin lens : $\frac{-1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ and $\frac{1}{f} = \frac{-1}{u} + \frac{1}{v}$
7. Since the principle of reversibility suggests that the object and image are conjugate to each other, interchanging the positions of an object, image distance can be determined.
8. Power of lenses in contact is given by

   $P = P_1 + P_2 + ...$
9. Magnification of lenses in contact is given by

   $m = m_1 \times m_2$ .. ..
10. Focal length of lenses in contact is given by

    $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ...$
11. Prism equation is given by $\delta = i+e-A$. At minimum angle of deviation, $\delta_m = 2i - A$. For thin prisms, (A < <), $\delta_m = A(n - 1)$, where n = refractive index of the material of prism.
12. Scattering can be classified into two : elastic scattering (Rayleigh and Mie-Scattering) and inealstic scattering (e.g., Raman Scattering). If the size of the particle scattering light is smaller than the wavelength of the incident light, it is known as the Rayleigh scattering, if otherwise, it is known as the Mie-scattering.
13. Compound microscope can be thought of as made up of two cascaded simple microscopes, in which magnified image due to first simple microscope works as an object for the second.
14. For high resolution and magnification, curved mirrors are used in modern telescopes.
15. Retina has two types of cells : rods give the sensations of less intense light and cones give sensations of colour and high intense light
16. Defects of vision can be overcome by proper lenses.

EXERCISES

For the following statements choose the correct option from the given options :

1. An object is placed at a distance of 25 cm on the axis of a concave mirror, having focal length 20 cm. Find the lateral magnification of an image.

   (A) 2 (B) 4 (C) –4 (D) –2
2. A fish in a lake is at a 6.3 m distance from the edge of the lake. If it is just able to see a tree on the edge of the lake, its depth in the lake is .... *m*. Refractive index of the water is 1.33.

   (A) 6.30 (B) 5.52 (C) 7.5 (D) 1.55

3. For a thin convex lens when the heights of the object is double than its image, its object distance is equal to .... focal length of a lens is f.

(A) f (B) $2f$ (C) $3f$ (D) $4f$

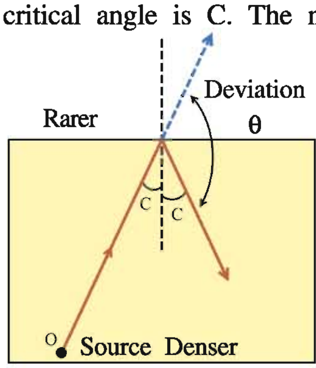
4. A liquid of refractive index n is filled in a tank. A plane mirror is kept at the bottom of the tank. A point like object (P) is kept at a height h from the mirror on the liquid surface. An observer observes the object and its image in the vertically downward direction from top. How much distance will observer note between P and its image ?

(A) $2n \cdot h$ (B) $\frac{2h}{n}$ (C) $\frac{2h}{(n-1)}$ (D) $h\left(1+\frac{1}{n}\right)$

5. Depth of a well is 5.5 m and refractive index of water is 1.33. If viewed from the bottom, by how much height would the bottom of the well appear to be shifted up ?

(A) 5.5 m (B) 2.75 m (C) 4.13 m (D) 1.37 m

6. A ray of light is travelling from a denser medium to rarer medium. For these media, the critical angle is C. The maximum possible deviation of the ray is ....

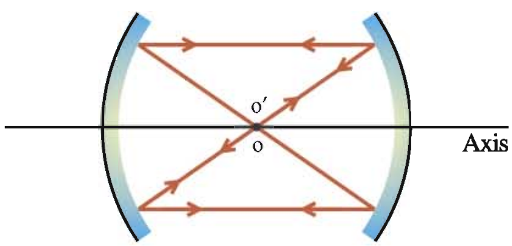


(A) $\pi - 2$ (B) $\pi - 2c$

(C) 2C (D) $\frac{\pi}{2} + C$

[Hint : The situation at total reflection is shown in the figure.]

7. A point object O is placed midway between on the common axis of two concave mirrors of equal focal length. If the final image is formed at the position of the object, the separation between two mirrors is .... Focal length of mirrors is f.



(A) f (B) $2f$

(C) $\frac{3}{2}f$ (D) $\frac{1}{2}f$.

[Hint : A situation is depicted in the figure.]

[Note : Another possible situation for which object and its image coincide is when distance between two mirrors is $4f$.]

8. The focal length of a thin lens made from the material of refractive index 1.5 is 20 cm. When it is placed in a liquid of refractive index 1.33, its focal length will be .... cm.

(A) 80.81 (B) 45.48 (C) 60.25 (D) 78.23

9. A tank contains water upto a height of 30 cm and above it an oil up to another 30 cm height. .... cm shifts in the position of bottom of the tank is observed when viewed from the above. Refractive indices of water and oil are 1.33 and 1.28, respectively.

(A) 7.44 (B) 6.46 (C) 14.02 (D) 6.95

[Hint : From $\frac{h'}{h} = \frac{n_1}{n_2}$

$$\frac{h'-h}{h} = \frac{\Delta h}{h} = \frac{n_1 - n_2}{n_2}$$

$$\therefore -\frac{\Delta h}{h} = \left(\frac{n_1}{n_2} - 1\right)$$ (Shift, Δh is negative as it represent the virtual depth.)

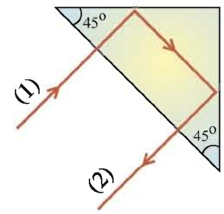
$$\therefore \text{shift, } \Delta h = h \times \left(1 - \frac{n_1}{n_2}\right) = h \times \left(1 - \frac{1}{n_{21}}\right)$$]

10. For a thin convex glass lens with radius of curvature 20 cm, focal length is .... cm. Refractive index (n) of the material of the lens is 1.5 and it is kept in air

(A) 20 (B) 40 (C) 60 (D) 80

[**Hint** : For air – glass lens, $\frac{-1}{u} + \frac{n}{v} = \frac{1}{f} = \frac{(n-1)}{R}$]

11. For right-angled prism, ray-1 is the incident ray and ray-2 is the emergent ray, as shown in the figure. Refractive index of the prism is ....



(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$

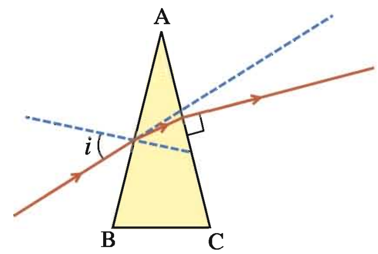
(C) $\frac{2}{\sqrt{3}}$ (D) $\sqrt{2}$

12. A ray of light is incident normally on the surface of an equileteral prism made up of material with refractive index 1.5. The angle of deviation is ....

(A) 30° (B) 45° (C) 60° (D) 75°

[**Hint** : For the present case use the formula $\sin C = \frac{1}{n}$ to understand the phenomenon.]

13. A ray is incident at an angle i on the surface of a prism with very small prism angle A, and emerges normally from the opposite surface. If the refractive index of the prism is μ the angle of incidence i is nearly equal to ....



(A) $\frac{A}{\mu}$ (B) $\frac{\mu A}{2}$

(C) $\frac{A}{2\mu}$ (D) μA

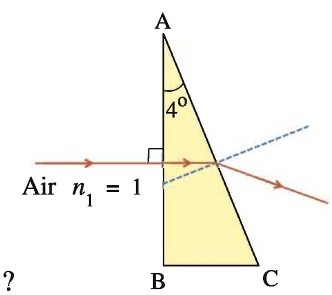
[**Hint** : Use the given figure.]

14. A small linear object of length b is placed on the axis of a concave mirror. The end of the object facing the mirror is at a distance u from the mirror. If f is the focal length of a mirror, the length of the object will be .... approximately.

(A) $b\left(\frac{u-f}{f}\right)^2$ (B) $b\left(\frac{f}{u-f}\right)$ (C) $\left(\frac{u-f}{f}\right)$ (D) $b\left(\frac{f}{u-f}\right)^2$

[**Hint** : Neglect b whenever necessary.]

15. A horizontal ray is incident on a right-angled prism with prism angle of 4°. If the refractive index of material of the prism is 1.5, angle of emergence is .... Use the given figure.



(A) 4° (B) 6°

(D) 10° (D) 0°

16. Which of the following is responsible for glittering of a diamond ?

(A) Interference (B) Diffraction (C) Total internal reflection (D) Refraction

17. The radii of curvature of both the sides of a convex lens are 15 cm and if the refractive index of the material of the lens is 1.5, then focal length of lens in air is .... cm

(A) 10 (B) 15 (C) 20 (D) 30

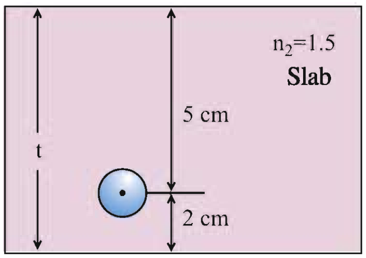
18. An image of an object obtained by a convex mirror is n times smaller than the object. If the focal length of lens is f, the object distance would be ....

(A) $\frac{f}{n}$ (B) $\frac{f}{(n-1)}$ (C) $(n - 1)f$ (D) nf

19. Time taken by the sunlight to pass through a slab of thickness 4 mm and refractive index 1.5 is .... sec.

(A) 2×10^{-8} (B) 2×10^{8} (C) 2×10^{-11} (D) 2×10^{11}

20. An air bubble in a glass slab with refractive index 1.5 is 5 cm deep when viewed from one face and 2 cm deep when viewed from the opposite face. The thickness of the slab is .... cm.



(A) 10.5 (B) 7

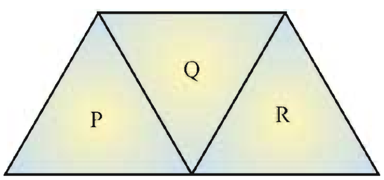
(C) 105 (D) 70

[**Hint** : Use $\frac{h'}{h} = \frac{n_2}{n_1}$]

21. The focal length of an equiconvex lens in air is equal to either of its radii of curvature. The refractive index of the material of the lens is ....

(A) $\frac{4}{3}$ (B) 1.5 (C) 2.5 (D) 0.8

22. A ray of light experiences minimum deviation by an equilateral prism P. Now two prisms Q and R made of the same material as that of P are arranged as shown in the figure. The ray of light will now experience, (The dimensions of P, Q and R are same.) ....



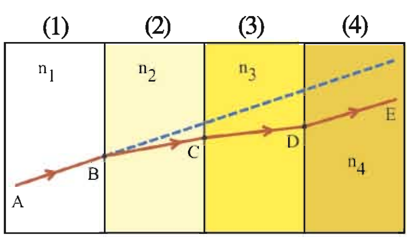
(A) larger deviation

(B) no deviation

(C) same deviation as that due to P

(D) total internal reflection

23. The refractive indices of four media, as shown in the figure, are n_1, n_2, n_3 and n_4. AB is an incident ray. DE, the emergent ray, is parallel to the incident ray AB, then ....



(A) $n_1 = n_2$ (B) $n_2 = n_3$

(C) $n_3 = n_4$ (D) $n_4 = n_1$

24. If the tube length of astronomical telescope is 105 cm and magnifying power is 20 for normal setting, then the focal length of the objective is .... cm.

(A) 10 (B) 20 (C) 25 (D) 100

[**Hint** : Optical length of astronomical telescope is given by $L \geq f_0 + f_e$]

25. The top sky looks blue in morning hours because, ...

(A) red light is absorbed (B) blue light is scattered the most

(C) sun radiates blue light only in the morning.

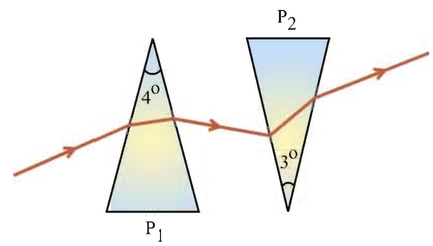
(D) blue light is absorbed by the sky

26. A defect of vision in which lines in one plane of an object appear in focus while those in another plane are out of focus is called ....
(A) astigmatism (B) distortion (C) myopia (D) hypermetropia

27. Stokes and antistokes lines observed in Raman scattering is due to .... of light.
(A) reflection (B) elastic scattering
(C) inelastic scattering (D) dispersion

28. A convex lens of focal length 10 cm is used as a simple microscope. When image of an object is obtained at infinite, magnification is .... Near point for normal vision is 25 cm.
(A) 1.0 (B) 2.5 (C) 0.4 (D) 25

29. As shown in the figure, thin prisms P_1 and P_2 are combined to produce dispersion without deviation. For prism P_1, angle of prism is 4^o and refractive index is 1.54. For prism P_2 angle of prism is 3^o. Refractive index of material of P_2 is ....



(A) 1.72 (B) 1.5
(C) 2.4 (D) 0.58

[**Hint** : For thin prism, $\delta = A(n - 1)$]

30. A spherical convex surface separates an object and image spaces of refractive index 1.0 and 1.5 respectively. If radius of curvature of the surface is 25 cm, its power is .... D.
(A) 13 (B) 33 (C) 3.3 (D) 1.3

[**Hint** : $\frac{-n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$ and $P = \frac{1}{f}$]

31. A light ray is incident at an angle 30^o with normal on a 3 cm thick plane slab of refractive index $n = 2.0$. The lateral shift of the incident ray is .... cm.
(A) 0.835 (B) 8.35 (C) 1.5 (D) 1.197

[**Hint** : Since incident angle θ_1 is not small, lateral shift, $x = \frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$]

ANSWERS

1. (C)	2. (B)	3. (C)	4. (B)	5. (D)	6. (B)
7. (B)	8. (D)	9. (C)	10. (B)	11. (D)	12. (C)
13. (D)	14. (D)	15. (B)	16. (C)	17. (B)	18. (C)
19. (C)	20. (A)	21. (B)	22. (C)	23. (D)	24. (D)
25. (B)	26. (A)	27. (C)	28. (B)	29. (A)	30. (B)
31. (A)					

Answer the following questions in brief :

1. What are paraxial rays ?
2. State Snell's Law.
3. What is total internal reflection ?
4. Light is incident normally on a glass slab with refractive index of 1.67. Find percentage reflected intenstity (I_r) compared to the incident intensity.
5. What is the use of cladding in the case of optical fibers ?

6. Define optical centre of a lens.
7. Write one advantage of using Newton's formula over lens-maker's formula.
8. Initially, two thin lenses were kept in contact. Now, if they are separated by d distance, what happens to the focal length of a combination ?
9. What are conjugate foci ?
10. Define near point or distance of most distinct vision.
11. What is the function of rods in retina ?

Answer the following questions :

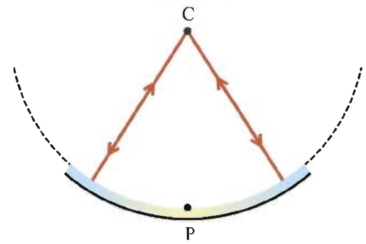
1. Obtain relation between focal length and raidus of curvature for convex mirror.
2. For concave mirror, derive the mirror formula.
3. Define lateral magnification for mirrors. Using cartesian sign convention, derive its relation with image distance and object distance.
4. Obtain an expression for lateral shift due to rectangular slab.
5. Explain the relation between real depth and the virtual depth.
6. Explain total internal reflection.
7. How right-angled prisms are useful as perfact reflecting surface ?
8. Explain how total internal reflection is useful in optical fibre.
9. For a spherically curved surface, derive the relation, $\frac{-n_1}{u} + \frac{n_2}{V} = \frac{(n_2 - n_1)}{R}$
10. Explain the image formation due to thin lens and derive $\frac{-1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ relation.
11. Derive lens-maker's formula for thin lens.
12. Derive Newton's formula for thin lens.
13. Explain conjugate points and conjugate distances.
14. Define lateral magnification for lenses. Obtain its relation to extra focal distances.
15. Derive the relation for effective focal length of an optical system made up of two thin lenses in contact.
16. Obtain the relation $f = \frac{1}{2}(v - d)$ for a convex mirror using a combination of convex mirror and convex lens.
17. Derive an equation $\delta = i + e - A$ for equilateral prism.
18. Using $\delta = i + e - A$ for equilateral prism obtain an equation for refractive index (n) of material of the prism.
19. Write note on Rayleigh scattering.
20. What is scattering ? Explian Raman Scattering.
21. Obtain an expression for magnification for simple microscope.
22. With diagram, derive an expression for magnification for compound microscope.
23. Write note on refracting telescopes.
24. What are reflecting telescopes ? What are the advantages of them over refracting telescopes ?
25. Discuss astigmatism defect of human eye.

Solve the following examples :

1. An object moves with uniform velocity (v_0) on the axis of a concave mirror. If it moves towards the mirror, show that when it is at a distance u from the mirror. The velocity of its image is given by $v_i = \left(\frac{R}{2u-R}\right)^2 v_0$, where R is radius of curvature of the mirror.

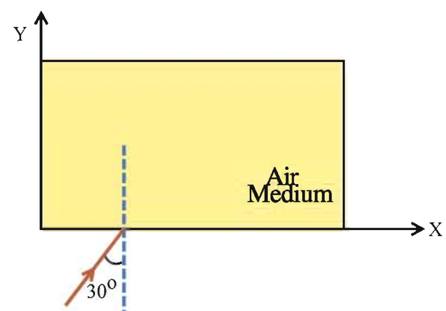
2. An image of a linear object due to a convex mirror is $\frac{1}{4}^{th}$ of the length of the object. If focal length of the mirror is 10 cm, find the distance between the object and the image. The linear object is kept perpendicular to the axis of the mirror. [Ans : 37.5 cm]

3. A concave mirror has been so placed on a table that its axis remains vertical. P and C are pole and centre of curvature respectively. When a point like object is placed at C, its real image is formed at C. If now, water is filled in mirror, obtain the image distance from the pole.



4. The diameter of the sun subtends an angle of 0.5° at the pole of the concave mirror. The radius of curvature of the mirror is 1.5 m. Find the diameter of the image of the sun. Consider the distance of sun from the mirror infinite. [Ans : 0.654 cm]

5. A ray, as shown in the figure, is incident at the angle of incidence 30° on the interface between air and a medium and travels in the medium. If the refractive index of the medium is given by $n(y) = 1.5 - ky$. Here, k is constant and it is equal to $0.25 m^{-1}$. At which value of y, will the ray becomes horizontal in the medium ? Here, y is in meter. [Ans : y = 3 m]

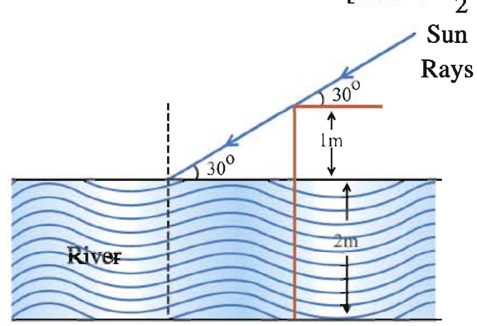


6. A narrow beam of light is incident on a glass plate of refractive index 1.6. It makes an angle 53° with normal to the interface. Find the lateral shift of the beam at the point of emergence, if thickness of the plate is 20 mm. Take sin53° = 0.8. [Ans : 9 mm]

7. A real image obtained by a concave mirror is 4 times begger than the object. If the object is displaced by 3 cm away from the mirror, the image size becomes 3 times the object size. Find the focal length of the mirror. [Ans : 36 cm]

8. The refractive index of material of a particular optical fibre is 1.75. At what maximum angle a ray can be made incident on it, so that it is totally internally reflected ? Consider air as an external medium with refractive index as 1.0. [Ans : $\frac{\pi}{2}$]

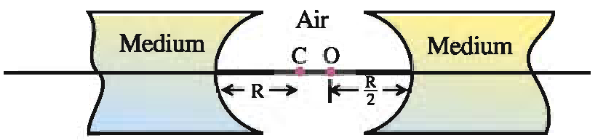
9. A level measuring post (a rod) has been kept in a river of 2 m depth vertically such that its 1 m portion remains outside the river. At this instant, the sun makes an angle of 30° with the horizontal. Find the length of the shadow of the level measuring post on the bottom of the river (see figure). The refravtive index of water is $\frac{4}{3}$.



[Ans : 3.44 m]

10. A vessel is fully filled with liquid having refractive index $\frac{5}{3}$. At the bottom of the vessel a point-like source of light is kept. An observer looks at the source of light from the top. Now, an opaque circular disc is kept on the surface of the water in such a way that its centre just rests above the light source. Now liquid is taken out from the bottom gradually. Calculate the maximum height of the liquid to be kept so that light source cannot be seen from outside. Radius of the disc is 1 cm. [**Ans :** 1.33 cm]

11. As shown in the figure, two concave refracting surfaces of equal radii of curvature (R) and refractive indices (n = 1.5) face each other in air (n = 1.0).



A point object (0) is placed midway in between the centre and one of the vertices of the refracting surfaces. Find the distance between image O' formed by one surface and image O'' formed by the other surface in terms of R.

[**Hint :** Use $\frac{-n_1}{u} + \frac{n_2}{v} = (n_2 - n_1)\frac{1}{R}$ for both the refracting surfaces.] [**Ans :** 0.114 R]

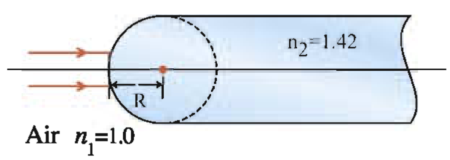
12. (1) If f = + 0.5m calculate power of a lens.

(2) The radii of curvature of a convex lens are 10 cm and 15 cm. If its focal length is 12 cm, find the refractive index of the material of the lens.

(3) The focal length of a convex lens in air is 20 cm. What will be its focal length in water. The refractive index of water is 1.33 and that of glass is 1.5.

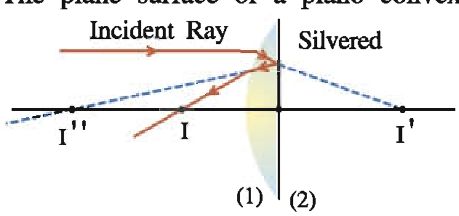
[**Ans :** (1) 12 D (2) 1.5 (3) 78.2 cm]

13. One end of a cylindrical rod made from the material of refractive index 1.42 is hemispherical.



A narrow beam of parallel rays is incident as shown in the figure. At how much distance will this beam of ray be focussed from the hemispherical surface ? [**Ans :** 3.38 R]

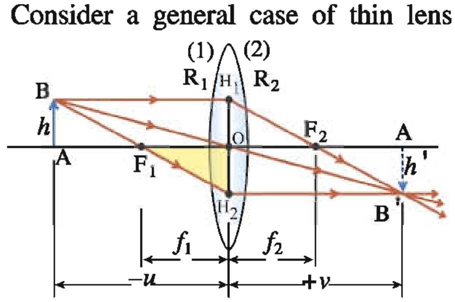
14. The plane surface of a plano convex lens of focal length 20 cm is silvered and made



reflecting, as shown in the figure. Find new focal length of the system.

[**Ans :** 10 cm]

15. Consider a general case of thin lens with first principal focal length (f_1) and second principal focal length (f_2). Obtain the expression for magnification in terms of f_1 and f_2 as $\left(\frac{v - f_2}{f_1}\right)$. Also, for a special case of $f_2 = f_1 = f$, deduce Gauss' equation from the expression for the magnification. Use cartesian sign convention.



[**Hint :** From figure ΔBH_1H_2 and ΔF_1OH_2 are similar, and $\Delta B'H_2H_2$ and ΔF_2OH_1 are similar.]