7 DUAL NATURE OF RADIATION AND MATTER

7.1 Introduction

At the end of the nineteenth century, most physicists thought that the Newtonian laws governing the motion of material particles, thermodynamics and Maxwell's theory for electromagnetic waves are complete and fundamental laws of physics. They all together constitute "Classical Mechanics". Classical physics deals primarily with macroscopic phenomena. Most of the effects with which classical theory is concerned are either directly observable or can be made observable with relatively simple instruments. Thus, there is a close link between the world of classical physics and our sense of perception. Almost all known macroscopic problems were satisfactorily solved applying the laws of classical mechanics, and therefore scientists have turned their concentration to the study of atomic and subatomic (i.e. microscopic and submicroscopic) systems. Unlike macroscopic system, since these systems are inaccessible to direct observations, the experiments which have generated interest and curiosity studying some microscopic problems are worth mentioning here.

Study of the influence of an electric field to cathode rays by Jean Perin (1895), and experimental demonstration of negatively charged particles have discovered an electron. Just later, J. J. Thomson found the ratio of charge to mass ($\frac{e}{m}$ = 1.756 × 10^{11} C/kg) for an electron, while Milikan (1909) had estimated the charge of an electron (e = 1.602 × 10^{-19} C). It was also established that the smallest basic unit of matter is an atom, and it is electrically neutral. Wilhem Rontgen (1885) accidentally discovered X-rays and just few years later, Henry Bacquerel (1896) and Madam Curie (1898) with different compounds have discovered radio activity.

These were the few experiments which provided a foundation to perform series of different experiments yielding results which could not be explained by the laws of classical mechanics. The specific heats of solids and diatomic gases at very low temperatures, large electrical conductivities of metallic solids, structure of an atom and the characteristic wave lengths emitted or absorbed by different elements, the photoelectric effect, the study of black-body radiation were the notable problems which could not be understood in terms of classical mechanics.

For the resolution of the apparent paradoxes posed by these observations and certain other experimental facts, it became necessary to introduce new ideas quite foreign to commonsense concepts regarding the nature of matter and radiation.

Historically to understand how entirely new concepts were emerged, we study the difficulties in explaining the black-body radiation.

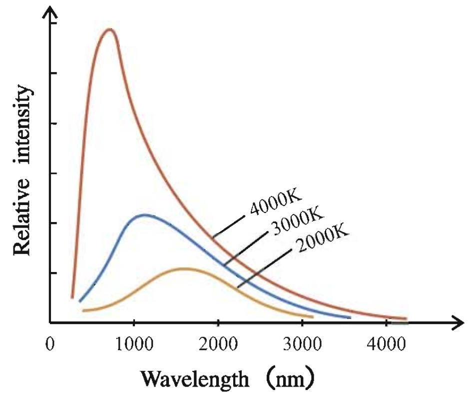


Figure 7.1 Relative Intensity as a Function of Wavelength

Black-body Radiation : In 1897, Lummer and Pringsheim measured the intensities of different wavelengths (i.e., intensity distribution) of black-body or cavity radiations, which is plotted in the figure 7.1.

Scientists were trying to explain these graphs using the laws of electromagnetic theory and thermodynamics.

On the thermodynamic grounds and by using ideas of electromagnetism, Wien gave an expression for energy density as, $u_\lambda = \frac{1}{\lambda^5} \cdot exp\left(-\frac{b}{\lambda \cdot T}\right)$; where b is cosnstant and T is absolute temperature. Such an equation can explain the experimental reuluts only for small wave lengths, but fails to explain the higher wavelengths intensity distribution.

Rayleigh and Jeans determined the number of normal modes of vibration for small intervals of wavelengths, considering the radiations as electromagnetic waves. Each normal mode corresponds to one harmonic oscillator. As the degrees of freedom for harmonic oscillator is two, according to equipartition law for energy, its kinetic energy is $k_B T$. Here, k_B is the Boltzmann constant. Based on this argument, they derived an equation for energy density as,

$$u_\lambda = \frac{8\pi k_B T}{\lambda^4} \qquad (7.1.1)$$

This equation can explain the energy distribution for large wavelengths only. Further, the total energy density (u_{tot}) covering all possible wavelengths must follow the Stefan-Boltzmann's law ($u_{tot} = \sigma \cdot T^4$; where σ = Stefan-Boltzmann's constant). But using equation (7.1.1), if we calculate the total energy density, i.e., $u_{tot} = \int_0^\infty \frac{8\pi k_B T}{\lambda^4} d\lambda$, we get infinite ($\infty$) answer ! This is called ultraviolet catastrophe. On the other hand, Wien's law requires (λ_{max}). T = constant, (b)is called wien's constant. (7.1.2.)

Here, λ_{max} is the wavelength corresponding to the peak value in the intensity distribution graph at that temperature.

Thus, all the attempts based on thermodynamics and electromagnetic theories failed to explain the entire energy distribution curves of black-body radiation.

7.2 Planck's Hypothesis for Radiation

The explanation of energy distribution curves of black-body or cavity radiation was given by Max Planck (1900) at the Academy of Science in Berlin.

He suggested – **"The walls of cavity emitting radiations are made of electric dipoles. According to their temperature, different dipoles oscillate with different frequencies and emit radiations of frequencies equal to frequencies of their oscillations."**

Now, according to the classical physics an oscillator may possess any amount of energy. That is, an oscillator may acquire continuously varying (from zero to maximum available) energy.

Planck presented a revolutionary idea that **"these microscopic oscillators may not possess any arbitrary energy as allowed by the laws of classical mechanics. If the vibrational frequency of such a microscopic oscillator is f, then it may possess energy given by,**

$$E_n = nhf, \quad (7.2.1)$$

where $n = 1, 2, 3$ ..... Here h is known as Planck's universal constant. Thus, according to Planck, energy of such microscopic oscillator depends on its vibrational frequency. This is in contrast to classical oscillator, whose energy depends on its amplitude of oscillation, as per the well known equation $\frac{1}{2}kA^2$. Here, k is the force constant and A is amplitude.

Equation (7.2.1) also suggests that the energy of an oscillator of frequency f is hf, $2hf$, $3hf$, ..., etc. It cannot possess the fractional energy like $0.1hf$, $\frac{1}{2}hf$, $0.06hf$. Thus, energy of microscopic oscillator is an integral multiple of hf. In other words, the smallest quantum of energy of an oscillator of frequency f is 'hf'.

This smallest bundle or packet or quantum of energy is known as **photon.** When an oscillator emits radiation of frequency f, its energy decreases in integral multiple of hf. And quanta of energy hf are emitted. That is, energy is not emitted continuously but in the form of quanta. This phenomenon is known as the **quantization** of energy. (You have also studied the quantization of electric charge.) If an oscillator possesses energy $5hf$, meaning 5 quanta each with energy hf.

Based on his hypothesis Planck could successfully derive the equation of spectral emissive power for a perfect black-body radiation, which is given by

$W_f = \frac{2\pi f^2}{c^2} \times \frac{hf}{\left[e^{\left(\frac{hf}{k_B T}\right)} - 1\right]}$. Here, c = speed of light in vacuum, T = absolute temperature of a perfect balck body, k_B = Boltzmann's constant. (This equation is only for information.)

Above equation gives maximum energy density at the wavelength (λ_{max}) corresponding to Wien's law. Using the experimental values of Stefan-Boltzmann constant σ, Wien's constant b (see equation (7.1.2.)) and Boltzmann's constant k_B, value of Planck's constant (h) can be determined as

$$h = 6.625 \times 10^{-34} \text{ J.s}$$

It can be proved that in the limit $hf \to 0$, above equation correctly reproduces the classical value $k_B T$, predicted by the law of equipartition of energy. It appears, therefore, that the very small but **non-zero** value of constant 'h' is a measure of the failure of classical mechanics.

Only For Information : If quantum effects are to be observed, the frequency should be high enough so that $\frac{hf}{k_B T}$ becomes comparable to unity. For example, at room temperature (T ≈ 300 K), $\frac{hf}{k_B T} \approx \frac{1}{6}$ for $f = 10^{12}$ Hz. This shows that only when oscillator of at least this frequency or higher, quantum statistical effects become noticeable at room temperature.

7.3 Photoelectric Effect

7.3.1 Emission of Electrons : We know that metals have **free electrons.** However, these free electrons normally cannot come out of the metal surface. The reason is that electrons at the surface experience strong attractive inward force due to positive metallic ions; while virtually no attractive force from the outside. In other words, very close to the surface, potential energy of electrons increase with distance as compared to inside electrons. That is, a potential-barrier exists at the surface. Thus, to bring an electron out, some minimum amount of energy must be supplied to it. This minimum energy required to get emission of an electron is known as **work function** (ϕ_0) of the metal.

The work function of a metal depends on type of the metal, nature of its surface and its temperature.

To bring an electron out of the metal, required energy may be supplied by any of the following ways.

Thermionic Emission : In this method, current is passed through a filament so that it gets heated sufficiently (normally 2500–3000 K). Hence, free electrons in it gain enough energy and get emitted from the metal. Such kind of electron emission is observed in devices like diode, triode, T.V. tube (cathode ray tube), etc.

Field Emission or Cold Emission : When a metal is subjected to strong electric field of the order of $10^8 \frac{V}{m}$, electrons are pulled out of the metal surface.

Photo Electric Emission : When an electromagnetic radiation of enough high frequency is incident on a cleaned metallic surface, electrons can be liberated from the metal surface. This phenomenon is known as the **photoelectric effect** and the electrons so emitted are known as **photo electrons.** To have photo emission, the frequency of incident light should be more than some minimum frequency. This minimum frequency is called the **threshold frequency** (f_0). It depends on the type of the metal. For most of the metals (e.g. Zn, Cd, Mg) threshold frequency lies in the ultraviolet region of electromagnetic spectrum. But for alkali metal (Li, Na, K, Rb) it lies in the visible region.

7.3.2 Hertz's Experiment : The photoelectric effect was discovered accidentally in 1887 by H. Hertz, during his study on the phenomenon of emission of electromagnetic waves by means of spark discharge. In his experiment electromagnetic waves from the transmitter (antenna) induced a potential difference across the spark-gap, as evidence from the jumping spark across it. Hertz noticed that the sparks jumped more easily when the cathode was illuminated by ultraviolet light. This observation suggested that light facilitated the escape of charges from the metallic cathode across the spark-gap. Further, Hallwachs extended this experiment for zinc plate. He connected the negatively charged zinc plate with an electroscope. When this plate was irradiated with ultraviolet light, it was observed that negative charge on the plate decreased. Not only this, even when a neutral plate is irradiated with ultraviolet light it becomes positively charged, while postively charged plates became more positively charged. Hallwachs concluded that under the effect of ultraviolet light, negatively charged electrons are emitted from the zinc plate. These electrons are known as photoelectrons.

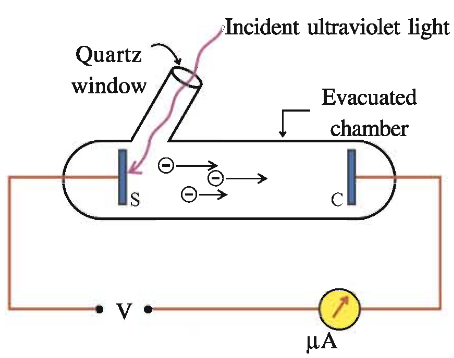


Figure 7.2 Experimental Arrangement to Study Photoelectric Effect

7.3.3 Lenard's experiment : The details of the photoelectric phenomenon were studied by P. Lenard, one of Hertz's students. The experimental arrangement to study the photoelectric effect is shown in the figure 7.2.

The ultraviolet light entering from quartz window is incident on the cleaned photosensitive surface S. C is the collector, while S is the cathode. C can be kept at different positive or negative voltages with respect to S.

The characteristies of photoelectric effect can be studied in reference to the frequency and the intensity of incident light, and also in terms of number of photoelectrons emitted and their maximum kinetic energy.

When the collector is positive with respect to S, the photo electrons are attracted to it and micro-ammeter registers a current. The amount of current passing through the ammeter gives an idea of the number of photoelectrons. At some value of positive potential difference, when all the emitted electrons are collected, increasing the potential difference further has no effect on the current.

When the collector is made negative with respect to S, the emitted electrons are repelled and only those electrons which have sufficient kinetic energy to overcome the repulsion may reach to the collector, and constitute current. So the current in ammeter falls. On making collector more negative, number of photoelectrons reaching the collector further decreases. For some specific negative potential of the collector, even the most energetic electrons are unable to reach collector, and photoelectric current becomes zero. It remains zero even if the potential is made further negative than the specific value of negative potential. This minimum specific negative potential of the collector with respect to the emitter (photo sensitive surface) at which photoelectric current becomes zero is known as the **stopping potential** (V_0) for the given surface. It is thus the measure of maximum kinetic energy ($\frac{1}{2}mv^2_{max}$) of the emitted photoelectrons. If charge and mass of an electron are e and m respectively,

$$\frac{1}{2}mv^2_{max} = eV_0 \qquad (7.3.1)$$

Lenard performed further experiments by varying the intensity (brightness) of the incident light, and measured maximum K.E. and number of photoelectrons via the photoelectric current. He found that by increasing the intensity of the incident light, photoelectric current (i.e. the number of photoelectrons) increases but do not affect the K.E. of the emitted electrons. In the contrast, when he performed the experiment with different frequencies, higher than the threshold frequency of the incident light, changes the stopping potential (V_0) and thereby the K.E. of the emitted electrons, leaving photoelectric current unaltered. It was found that by increasing frequency V_0 and therefore maximum K.E. of the photoelectrons increase, and vice versa. It was also observed that the photo-electrons are emitted within 10^{-9} s after the light is incident.

In summary,

(1) The maximum K.E. of photoelectrons depend on the frequency of incident light, and does not depend on the intensity.

(2) The number of photoelectrons depend directly on the intensity of incident light.

(3) Photoelectric effect is always observed whenever incident light has frequency either equal to or greater than the thereshold frequency for the given surface irrespetive of the intensity.

(4) The phenomenon of photoelectric effect is spontaneous (takes about 10^{-9} sec.).

Above inferences can be depicted in the graphs below : (Figure 7.3 and 7.4)

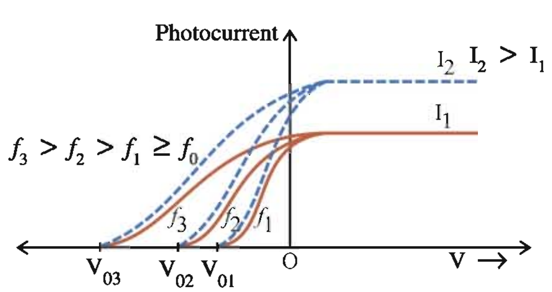


Figure 7.3 Variation of Photoelectirc Current

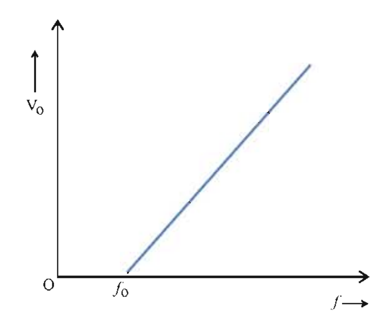


Figure 7.4 Variation of Stopping Potential with Frequency of Incident Light

7.3.4 Explanation from the wave theory of light : Above experimental results cannot be understood with the wave theory of light.

(1) According to the wave theory of light, energy and intensity of wave depend on its amplitude. Hence intense radiation has high energy and on increasing intensity, energy of photoelectrons should also increase. In contradiction to it, experimental results show that the energy of photoelectrons does not depend on the intensity of incident light.

According to the wave theory, energy of light has no relation to its frequency. Hence change in energy of photoelectrons with the change in frequency cannot be explained.

(2) Photoelectrons are emitted immediately (within the 10^{-9} s) on making light incident on the metal surface. Since the free electrons within the metal are withheld under the effect of certain forces, and to bring them out, energy must be supplied.

Now, if the incident energy is showing a wave nature, free electrons in metal get energy gradually and when accumulates energy at least equal to the work function then after they escape from the metal. Thus, electrons get emitted only sometime after the light is incident.

(3) According to the wave theory, less intense light is 'weak' in terms of energy. To liberate photoelectron with such light one has to wait long till electron gathers sufficient energy. Against that experiment shows immediate emission of electron even with diminutive intensity but of course, with sufficiently high frequency.

Thus, wave theory fails to explain the photoelectric effect.

7.3.5 Einstein's Explanation : Einstein gave a successful explanation of the photoelectric effect in 1905 for which he received the Nobel Prize in 1921.

Planck had assumed that emission of radiant energy takes place in the quantized form, the photon, but once emitted it propagates in the form of wave. Einstein further assumed that not only the emission, even the absorption of light takes place in the form of photons.

For Information Only : In the wave nature, the energy is supposed to be spread uniformly across the wave fronts, Einstein proposed that the light energy is not spread over wavefronts but is concentrated in small packets, the photons. He wrote : "According to the assumption considered here, when a light ray starting from a point is propagated, the energy is not continuously distributed over an ever increasing volume, but it consists of a finite number of energy quanta, localized in space, which move without being divided and which can be absorbed or emitted only as a whole."

Suppose frequency of incident light is f, hence energy of its photon is hf. When this photon is incident on the metal, during the interaction with an electron, it is totally absorbed if its frequency (and therefore energy) is greater than threshold frequency or otherwise does not lose energy at all.

As per the laws of classical mechanics (Newtonian mechanics and Maxwell's theory for electromagnetic waves) there is no reason to expect any sensitive frequency dependence of photon-electron interaction. (You will learn its detailed answer in advance course in physics, if you choose physics to shape your career.)

Now if f_0 is the threshold frequency the appropriate photon energy hf_0 will be equal to work function ϕ_0, and at that frequency the photoelectrons are emitted with the minimum (zero) kinetic energy. For frequency $f > f_0$, the maximum kinetic energy of emitted photoelectrons will be,

$$\frac{1}{2}mv^2_{max} = hf - \phi_0$$

From equations (7.3.1), $eV_0 = hf - hf_0$

$$\therefore \ V_0 = \frac{h}{e} \cdot f - \left(\frac{hf_0}{e}\right) \qquad (7.3.2)$$

According to this equation the graph of V_0 versus f is a straight line with a slope $\frac{h}{e}$ and intercept on the X-axis at f_0. This is in excellent agreement with the experimental results shown in the Figure 7.4.

The intensity of light incident on surface is the light energy incident per unit surface area in unit time normal to the surface. According to photon theory (particle nature) of light, if n photons are incident per unit surface area in unit time, intensity of light is $I = nhf$, where hf is the energy of the photon of frequency f. Thus, according to photon theory, more the intensity of light more is the number of photons incident per second and hence more is the photoelectric current. Again showing an experimental trend.

Also, since the interaction between photon and electron takes place as the absorption as a whole or not at all, emission of photoelectron will be instant. Unlike wave nature, where electron has to wait till it gathers enough energy for escape.

Thus, experimental observations for photoelectric effect are reproduced by considering a particle (quantized) nature (photon) of light.

Following table shows work functions and corresponding threshold freqeuncy for some metals.

Table 7.1

Workfunctions and Threshold Frequencies (For information only)

Metal	ϕ_0 (in eV)	f_0 ($\times 10^{14}$ Hz)	Metal	ϕ_0 (in eV)	f_0 ($\times 10^{14}$ Hz)
Cs	1.9	4.60	Fe	4.5	10.89
K	2.2	5.32	Ag	4.7	11.37
Ca	3.2	7.74	Au	4.9	11.86
Cd	4.1	9.92	Ni	5.0	12.10
Al	4.2	10.16	Pt	6.4	15.49

Illustration 1 : Let an electron requires 5×10^{-19} joule energy to just escape from the irradiated metal. If photoelectron is emitted after 10^{-9} s of the incident light, calculate the rate of absorption of energy. If this process is considered classically, the light energy is assumed to be continuously distributed over the wave front. Now, the electron can only absorb the light incident within a small area, say 10^{-19} m^2. Find the intensity of illumination in order to see the photoelectric effect.

Solution : The rate of absorption of energy (power) is

$$P = \frac{E}{t} = \frac{5\times10^{-19}}{10^{-9}} = 5 \times 10^{-10} \frac{J}{s}$$

From the definition of the intensity of light,

$$I = \frac{\text{Energy}}{\text{time}\times\text{area}} = \frac{5\times10^{-10}}{10^{-19}} = 5 \times 10^{9} \frac{J}{s.m^2} \text{ (i.e., 500 billion } \frac{\text{Watt}}{m^2}\text{)}$$

Since, practically it is impossibly high energy, which suggests that explanation of the photoelectric effect in classical term is not possible.

Illustration 2 : Work function of metal is 2 eV. Light of intensity 10^{-5} W m^{-2} is incident on 2 cm^2 area of it. If 10^{17} electrons of these metals absorb the light, in how much time does the photo electric effect start ? Consider the waveform of incident light.

Solution : Intensity of incident light is 10^{-5} W m^{-2}.

$\therefore$ Energy incident on 1 m^2 area in 1 s is 10^{-5} J.

$\therefore$ Energy incident on area of 2 cm^2 = 2×10^{-4} m^2

$$= 2 \times 10^{-4} \times 10^{-5} = 2 \times 10^{-9} \text{ J}$$

This energy is absorbed by 10^{17} electrons.

$\therefore$ Average energy absorbed by each electron = $\frac{2\times10^{-9}\text{J}}{10^{17}}$ = 2×10^{-26} J

Now, electron may get emitted when it absorbs energy equal to the work function of its metal. In the given problem work function is 2 eV = $2 \times 1.6 \times 10^{-19}$ J. Thus, electron requires

$(2 \times 1.6) \times 10^{-19}$ J of energy to get emitted.

To absorb 2×10^{-26} J of energy, time required is 1 s, therefore to absorb energy $2 \times 1.6 \times 10^{-19}$ J, time required is,

$$t_e = \frac{2\times1.6\times10^{-19}}{2\times10^{-26}} = 1.6 \times 10^{7} \text{ s}$$

Note : If light is considered as wave, photo electron would not be emitted instantaneously as generally seen in the experiments.

7.4 Particle Nature of Light

The photons are considered as discrete amounts of energy (packets) with smallest being the *hf*. Thus, by nature itself the concept of photon involves the essence of radiation. So, can we consider photon as a real particle ? The compton effect, in which X-rays are scattered by the free electrons, gives the answer. To explain compton effect, photon was considered as a real particle just like a material particle. The way electron collides with any other matter particles, electron may also undergo same type of collision with photon. Also, this collision was considered to follow the laws of conservation of momentum and energy. Thus, as a result of the study of photoelectric effect and compton effect, following properties were attributed to a photon.

(1) Like a material particle, photon is also a real particle.

(2) Energy of a photon of frequency f is hf.

(3) Momentum of photon of frequency f is $\frac{hf}{c}$.

According to Einstein's special theory of relativity the relation between energy (E) and momentum (p) of a particle is given by,

$$E = \sqrt{p^2c^2 + m_0^2 \cdot c^4}, \text{ where } c = \text{speed of light in vacuum and} \quad (7.4.1)$$

m_0 = rest mass.

Mass of a particle moving with speed v as obtained from equation (7.4.1) is given by,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (7.4.2)$$

Since, in vacuum, photon moves with speed equals to speed of light, its rest mass

$$m_0 = m \times \sqrt{1-\frac{c^2}{c^2}} = 0$$

From equation (7.4.1),

$$E = p.c. \tag{7.4.3}$$

$$\text{or } p = \frac{E}{c} = \frac{hf}{c} \tag{7.4.4}$$

(4) Mass of a photon, $m = \frac{E}{c^2}$ ($\because$ $E = mc^2$); where m is given by (7.4.2).

(5) Like a real particle, photon interacts with other particles obeying the laws of conservation of energy and momentum.

For Information Only : To say that electromagnetic radiations propagate as "waves" on one side, at the same time say that in their interaction with matter they exchange energy and momentum as discrete particles (photons) appears contradictory. Let us understand the situation in more details.

Because these cannot be understood in terms of our classical ideas regarding "waves" and "particles". These can be understood only if we accept that :

(1) Light is emitted from a source as described as photons.

(2) Detector records light as discrete photon.

(3) Propagation of light from the source to the detector can be described in terms of "probability waves".

(4) When a "photon" detector is placed in the radiation field of electromagnetic waves, the number of photons detected over the area of the detector is proportional to the square of the amplitude of electromagnetic waves, but the detector interacts with the field as discrete photons.

Illustration 3 : If the efficiency of an electric bulb of 1 watt is 10%, what is the number of photons emitted by it in one second ? The wavelength of light emitted by it is 500 nm. $h = 6.625 \times 10^{-34}$ J s

Solution : As the bulb is of 1 W, if its efficiency is 100 %, it may emit 1 J radiant energy in 1 s. But here the efficiency is 10%, hence it emits $\frac{1}{10}$ J = 10^{-1} J radiant energy in 1 s.

Note : The efficiency of bulb is 10 %. It means it emits 10% of energy consumed in form of light and remaining 90 % is wasted in form of heat energy (due to the resistance of filament.)

$\therefore$ Radiant energy obtained from the bulb in 1 s. = 10^{-1} J

If it consists of n photons,

$nhf = 10^{-1}$ J

$$\therefore n = \frac{10^{-1}}{hf} = \frac{0.1}{6.625 \times 10^{-34} \times \frac{c}{\lambda}} = \frac{\lambda \times 10^{-1}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} \qquad (\because f = \frac{c}{\lambda})$$

$\therefore\ n = \dfrac{0.1 \times 500 \times 10^{-9}}{6.625 \times 10^{-34} \times 3 \times 10^{8}}$ ($\because$ velocity of light, $c = 3 \times 10^8$ m s^{-1})

$\therefore\ n = 2.53 \times 10^{17}$ photons.

Illustration 4 : 11×10^{11} photons are incident on a surface in 10 s. These photons correspond to a wavelength of 10 Å. If the surface area of the given surface is 0.01 m^2, find the intensity of given radiations. Velocity of light is 3×10^8 m s^{-1}, $h = 6.625 \times 10^{-34}$ J.s.

Solution : Number of photons incident in 10 s = 11×10^{11}

$\therefore$ Number of photons incident in 1 s = 11×10^{10}

Now, these photons being incident on area 0.01 m^2

Number of photons being incident on 1 m^2 in 1 s,

$$n = \frac{11 \times 10^{10}}{0.01} = \frac{11 \times 10^{10}}{10^{-2}} = 11 \times 10^{12}$$

Energy associated with n photons,

$$= nhf = \frac{nhc}{\lambda} = \frac{11 \times 10^{10} \times 6.6 \times 10^{-34} \times 3 \times 10^{8}}{10 \times 10^{-10}} = 2.18 \times 10^{-3}$$

$\therefore$ Intensity of incident radiation = 2.18×10^{-3} W m^{-2}

Illustration 5 : A beam of photons of intensity 2.5 W m^{-2} each of energy 10.6 eV is incident on 1.0×10^{-4} m^2 area of the surface having work function 5.2 eV. If 0.5 % of incident photons emits photo-electrons, find the number of photons emitted in 1 s. Find minimum and maximum energy of these photo electrons.

1 eV = 1.6×10^{-19} J

Solution : Here, intensity of incident radiation is 2.5 W m^{-2}.

$\therefore$ Energy incident per 1 m^2 in 1 s = 2.5 J

$\therefore$ Radiant energy incident on area 1.0×10^{-4} m^2 in 1 s = $2.5 \times 1.0 \times 10^{-4} = 2.5 \times 10^{-4}$ J

Suppose there are n number of photons in this energy.

$\therefore\ nhf = 2.5 \times 10^{-4}$ (1)

but hf = energy of photon = 10.6 eV = $10.6 \times 1.6 \times 10^{-19}$ J

($\because$ 1 eV = 1.6×10^{-19} J)

Replacing it in equation (1) and making n the subject of equation,

$$n = \frac{2.5 \times 10^{-4}}{hf} = \frac{2.5 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

As 0.50 % of these photons emits photo electrons,

$$\begin{bmatrix} 100 : 0.5 \\ n : ? \end{bmatrix}$$

$\therefore$ Number of photo electrons emitted in 1 sec is,

$$N = \frac{0.50 \times n}{100} = \frac{0.5 \times 2.5 \times 10^{-4}}{100 \times 10.6 \times 1.6 \times 10^{-19}}$$

$$= 7.37 \times 10^{11}\ s^{-1}$$

The minimum energy of photo electon is = 0 J. Such photo electrons spend all the energy gained from the photon against the work function.

Maximum energy of photo electorn :

$E = hf - \phi_0 = 10.6\ eV - 5.2\ eV \quad (\because hf = 10.6\ eV \text{ and } \phi_0 = 5.2\ eV)$

$= 5.4\ eV$

Illustration 6 : Radius of a beam of radiation of wavelength 5000 Å is 10^{-3} m. Power of the beam is 10^{-3} W. This beam is normally incident on a metal of work function 1.9 eV. What will be the charge emitted by the metal per unit area in unit time ? Assume that each incident photon emits one electron.

$h = 6.625 \times 10^{-34}$ J s

Solution : Power of the beam of light = 10^{-3} W

$\therefore$ Amount of energy incident in unit time = 10^{-3} J

If the number of photons corresponding to this energy is n,

$$nhf = nh\frac{c}{\lambda} = 10^{-3} \Rightarrow n = \frac{10^{-3} \times \lambda}{hc}$$

$$\therefore n = \frac{10^{-3} \times 5000 \times 10^{-10}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} \quad (\because \lambda = 5000 \text{ Å} = 5000 \times 10^{-10}\ m)$$

These photons are incident on the surface of radius 10^{-3} m in one second.

$\therefore$ Number of photons incident per unit area in one second,

$$n_1 = \frac{10^{-3} \times 5000 \times 10^{-10}}{6.625 \times 10^{-34} \times 3 \times 10^{8} \times \pi \times (10^{-3})^2}$$

Each photon emits one electron and charge on electron being $e = 1.6 \times 10^{-19}$ C

$\therefore$ amount of charge emitted per unit area in unit time.

$$Q = n_1 e = \frac{10^{-3} \times 5000 \times 10^{-10} \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34} \times 3 \times 10^{8} \times 3.14 \times 10^{-6}} = 128.6 \text{ C}$$

Illustration 7 : Work function of some metals are Na : 1.92 eV, K : 2.2 eV, Cd : 4.1 eV, Ni : 5 eV. A laser beam from He-Cd of wavelength 3300 Å is incident on it. From which of the metals photo electrons will be emitted, if the distance of the source is initially 1 m from the metals. If it is brought to the distance of 10 cm will there be any change in emission ?

$h = 6.625 \times 10^{-34}$ J s. $c = 3 \times 10^{8}$ m s^{-1}, 1 eV = 1.6×10^{-19} J.

Solution : For photo-electric effect to be observed, energy of each photon should be at least equal to or more than work function of the metal.

$$\therefore hf = h\frac{c}{\lambda} \geq \text{work-function, } \phi_0$$

$$\text{Energy of Incident radiation} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10}} \text{ J} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10} \times 1.6 \times 10^{-19}}$$

$(\because 1\ eV = 1.6 \times 10^{-19}$ J)

Energy of Incident radiation = 3.76 eV

This result shows that the metal which has the work function 3.76 eV or less, may produce photoelectric effect. In the given list of metals Na and K may produce photoelectric effect, while in Cd or Ni this effect is not observed.

While the source is brought nearer, from 1 m to 10 cm, the intensity of incident light will of course increase, but its frequency will remain same. Hence, Na and K will emit more number of photo electrons and photo electric cuurent will increase, but still photo electric effect will not be seen in Cd and Ni.

Illustration 8 : U. V. light of wavelength 200 nm is incident on polished surface of Fe. Work function of the surface is 4.5 eV. Find, (1) stopping potential (2) maximum kinetic energy of photo-electrons (3) maximum speed of photo electrons.

$h = 6.625 \times 10^{-34}$ J s, $c = 3.00 \times 10^8$ m s^{-1}, 1 $eV = 1.6 \times 10^{-19}$ J.

$m = 9.11 \times 10^{-31}$ kg

Solution : $eV_0 = \frac{1}{2} mv^2_{max} = hf - \phi_0 = \frac{hc}{\lambda} - \phi_0$

First we find $\frac{hc}{\lambda}$, to calculate V_0.

$$\frac{hc}{\lambda} = \frac{6.625\times10^{-34}\times3\times10^8}{200\times10^{-9}} = 9.94 \times 10^{-19} \text{ J} = 6.21 \ eV$$

Now, $eV_0 = \frac{hc}{\lambda} - \phi_0 = 6.21 - 4.5$ ($\because \phi = 4.5 \ eV$) $= 1.71 \ eV$

$\therefore V_0 = 1.71$ V

Now,

$$\therefore \frac{1}{2}mv^2_{max} = eV_0 = 1.71 \ eV = (1.71)(1.6 \times 10^{-19}) \text{ J} = 2.74 \times 10^{-19} \text{ J}$$

$$\therefore v^2_{max} = \left(\frac{2.74\times10^{-19}\times2}{9.11\times10^{-31}}\right) = 6.0 \times 10^{11}$$

$$\therefore v_{max} = 7.75 \times 10^5 \text{ m s}^{-1}$$

Illustration 9 : A crystal of Cu emits $8.3 \times 10^{10} \ \frac{\text{photo-electrons}}{m^2 s}$. Atomic mass of Cu is 64 g mol^{-1} and its density is 8900 kg m^{-3}. Supposing that photo electrons are emitted from first five layers of atoms of Cu, will one electron be emitted per how many (average) atoms ? Consider the crystal to be a simple cubic lattice.

Solution : As the number of photo electrons are given as photo electrons/m^{2}s, consider the cube of crystal of length 1 m. Volume of such a crystal = $1 \times 1 \times 1 = 1$ m^3. Now density is 8900 kg m^{-3}. Hence, the mass of such crystal is 8900 kg. As the atomic mass is 64 g mol^{-1}, number of atoms in 64×10^{-3} kg of Cu will be same as Avogadro number.

64×10^{-3} kg : 6.02×10^{23}

$\therefore$ 8900 kg : number of atoms (?)

$\therefore$ Number of atoms in 8900 kg of Cu, N = $\dfrac{6.02\times10^{23}\times8900}{64\times10^{-3}}$ (1)

These atoms form simple cubic lattice.

If in one row there are n number of atoms, in one layer there may be n^2 number of atoms.

In 5 layers number of atoms = $5n^2$

Note that total number of atoms in the given cube is n^3

$\therefore$ N = n^3

$\therefore$. From equation (1),

$$n^3 = \frac{6.02\times10^{23}\times8900}{64\times10^{-3}}$$

$$\therefore n = \left(\frac{6.02\times10^{23}\times8900}{64\times10^{-3}}\right)^{\frac{1}{3}} = 4.37 \times 10^9$$

$\therefore$ $5n^2 = 5 \times (4.37 \times 10^9)^2 = 9.55 \times 10^{19}$

$\therefore$ 8.3×10^{10} photo electrons are emitted from 9.55×10^{19} atoms.

If 8.3×10^{10} photo electrons are emitted from $5n^2$ atoms, from how many atoms one electron is emitted ?

8.3×10^{10} : $5n^2$

1 : ? (number of atoms)

$$\therefore \frac{5n^2}{8.3\times10^{10}} = \frac{9.55\times10^{19}}{8.3\times10^{10}}$$

$\therefore$ Number of atoms emitting one photo-electrons = 1.15×10^9

Illustration 10 : Light of 4560 Å of 1 mW is incident on photo-sensitive surface of Cs (Cesium). If the quantum efficiency of the surface is 0.5 %, what is the amount of photo-electric current produced ?

Solution : Meaning of light of 1 mW is that 1 mJ = 10^{-3} J of energy is being incident on the surface in 1 s. This light is being incident in the form of photons of energy hf. If n photons are incident.

$nhf = 10^{-3}$ (1)

Out of n photons only 0.5% photons emit photo-electrons, as the quantum efficiency is 0.5%.

Now, 0.5% of n, is

$$\begin{bmatrix} 100 : 0.5 \\ n : ? \end{bmatrix}$$

$\therefore$ Number of photo-electrons = $\dfrac{n\times0.5}{100}$

Photo electric current is produced by these electrons is being emitted in 1 s.

Photoelectric current, I = Number of photo–electrons emitted in 1 s × charge of electron

$$\therefore \ I = \frac{n \times 0.5}{100} \times 1.6 \times 10^{-19} \text{ A} \qquad (2)$$

But from equation (1),

$$n = \frac{10^{-3}}{hf} = \frac{10^{-3}}{6.625 \times 10^{-34} \times \frac{c}{\lambda}} \qquad (\because f = \frac{c}{\lambda})$$

$$\therefore \ n = \frac{10^{-3} \times 4560 \times 10^{-10}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} = 2.303 \times 10^{15}$$

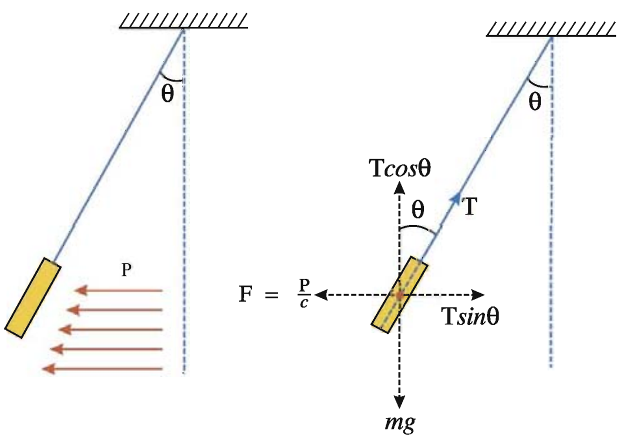
Replacing the value of n in equation (2),

$$I = \frac{2.303 \times 10^{15} \times 0.5 \times 1.6 \times 10^{-19}}{100}$$

$$\therefore \ I = 1.84 \times 10^{-6} \text{ A} = 1.84 \ \mu\text{A}$$

Illustration 11(a) : As shwon in the figure, light of energy P(joule) is incident on a small, flat strip of metal of mass m, suspended with the help of weightless string of length l in 1 s. All the energy incident on it is absorbed and the strip remains in equilibrium at an angle θ with respect to vertical. If the light is monochromatic, find angle θ.

Solution : When electromagnetic radiations are incident on a surface, force is produced due to pressure. Here, P joule of energy is incident in 1 s. If this radiation is made of photons and n photons are incident in 1 s,



$$nhf = \text{P} \qquad (1)$$

Now, momentum of each photon, $p = \frac{hf}{c}$ (2)

Replacing the value of hf from (1) in (2),

$$p = \frac{\text{P}}{nc}$$

$\therefore$ momentum of n photons $= np = \frac{\text{P}}{c}$

The strip gains this much momentum every second.

$\therefore$ Rate of change of momentum $= \frac{\text{P}}{c}$ = Force

$$\therefore \ \text{F} = \frac{\text{P}}{c} \qquad (3)$$

This force is shown in the Figure.

As the strip is in equilibrium, equating their vertical and horizontal components,

$$\left.\begin{aligned} \text{T}\cos\theta &= mg \\ \text{and T}\sin\theta &= \frac{\text{P}}{c} \end{aligned}\right\} \ \therefore \ \tan\theta = \frac{\text{P}}{cmg} \Rightarrow \theta = \tan^{-1}\left(\frac{\text{P}}{cmg}\right)$$

Illustration 11(b) : If the strip is slightly displaced from its state of equilibrium, find the period of it's simple harmonic oscillations.

Solution : Here, effective gravitational acceleration = $\vec{g}_e = \frac{\vec{P}}{mc} + \vec{g}$

$$\therefore \ |\vec{g}_e| = \sqrt{\left(\frac{P}{mc}\right)^2 + g^2}$$

$$\text{Now, } T = 2\pi \sqrt{\frac{l}{g_e}} = \sqrt{\frac{l}{\sqrt{\left(\frac{P}{mc}\right)^2 + g^2}}}$$

$$\therefore \ T = 2\pi \left[\frac{l}{\left\{\left(\frac{P}{mc}\right)^2 + g^2\right\}^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

7.5 Photocell

A Photocell (which is also known as electric eye) is a technological application of the photoelectric effect. In some photocells single layer of photosensitive material is used. A schematic diagram of a typical photocell is shown in the figure 7.5.

The wall of the photocell is made of glass or quartz. When the light (of suitable frequency) is incident on the photosensitive surface, a photocurrent of few micro ampere is normally obtained. When intensity of incident light is changed the photo electric current also changes. Using this property of photocell, control systems are operated and the intensity of light can be measured.

They are used in light meters, photographic camera, electric bell, burglar alarm, fire alarm. In astronomy, they are used to study the spectra of stars and their temperatures.

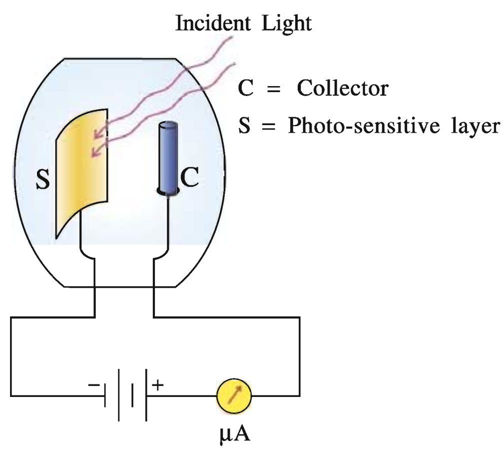


Figure 7.5 A Photocell

7.6 Matter Waves - Wave Nature of Particles

The photoelectric and compton effect have confirmed that light behaves as a collection of particles and not as a wave. At the same time, we also know that the phenomena of diffraction, interference and polarization can be understood only when light behaves as wave. This is a paradox of the existence of two quite different (the wave and the particle) nature of the same physical quantity (light). One possibility is to suppose that light propagates in the form of wave but assumes particle character at the instant of absorption or emission (i.e. during the interaction with matter). This explanation suggests that radiation shows dual nature; (continuous) wave-like extended and (discrete) quantized particle behaviour under the suitable conditions.

According to the theory of relativity, Lorentz transformation for a change of reference frame requires that relation like between E and f must necessarily hold for momentum (p) and wave-vector (k). Since for photon rest mass (m_0) is zero, its momentum is given by (see equation 7.4.4),

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \quad (\because\ c = f\lambda) \qquad (7.6.1)$$

$$\text{or } \lambda = \frac{h}{p} \qquad (7.6.2)$$

Based on this requirement, in 1924, Louis de Broglie arugued that if light (which consists of waves according to classical mechanics) can sometimes behave like particles. Then it should be possible for matter (which consists of particles according to classical picture) to exhibit wave-like behaviour under favourable circumstances. **"Nature should be symmetric with respect to radiation and particles."** The dual nature of radiation and particle must be a part of some general law of nature. That is, radiation and matter both show dual nature : particle and wave.

Thus, according to de Broglie, equation (7.6.2) is also true for material particles. For a particle with mass m and moving with a speed v (i.e., momentum, $p = mv$), when showing wave nature, corresponding wavelength can be found by using equation (7.6.2), as

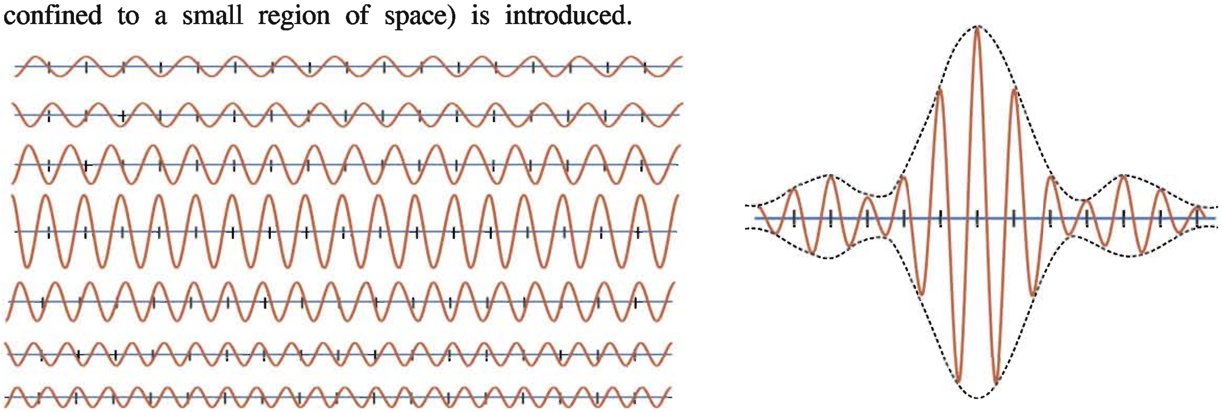
$$\lambda = \frac{h}{mv} \qquad (7.6.3)$$

This wavelength is known as de Broglie wavelength of the particle. We must remember that it is not that any kind of wave is attached to the matter particle. Under some circumstances, the behaviour of the particle can be explained by its wave nature.

Actually, the concept of matter particle as a wave was well supported by Erwin Schroedinger (1926) through his differential wave equation. He showed that this wave equation (for matter waves associated to particles) together with some physically-required conditions leads to quantized (discrete) nature of various physical quantities which supports the wave nature of particles. While the experimental evidences for matter as a wave were due to Davisson-Germer experiment, (which we will study in the next section), Kikuchi's diffraction experiment and Thomson's experiment showing associated de Broglie waves of electrons.

However, the most serious problem raised by the discovery of the wave nature of matter concerns the very basic definition of a 'particle'. Classically, particle means a point-like object endowed with a precise position and momentum. The de Broglie's hypothesis, which also supports wave-like (i.e. an extended in space) behaviour of matter, questions about how to measure accurately position and momentum of a material particle.

A pure harmonic wave extending in space obviously cannot represent point-like particle. This suggests that the wave activity of a wave representing a particle must be limited to (or nearby to) the space occupied by the particle. For this reason an idea of wave packet (i.e. a wave which is confined to a small region of space) is introduced.



(a) Harmonic Waves with Slightly Differing Wave Lengths.

(b) Amplitude Variation Due to Superposition of Harmonic Waves.

Figure 7.6 Construction of Wave Packat

We know that when many harmonic waves with slightly varying wave lengths are superposed (Hey, don't forget superposition principle), non-zero displacement of resulting wave is limited to small part of the space (See figure 7.6). In this sense, it would seem reasonable to suppose that the particle is within the region of the packet. Further, the probability of finding the particle is more in a region in which the displacement of the resulting wave is greater. If we use a single harmonic wave to represent a particle, the probability of finding a particle anywhere from $-\infty$ to $+\infty$ is equal. (This is because amplitude of a harmonic wave is finite and equal everywhere.) In other words, the position of the particle becomes totally uncertain. But, since the harmonic wave has unique wave length (λ), according to equation (7.6.3), its momentum is unique and certain.

If the concept of wave-packet (a group of superimposing waves of different wave lengths) is used to represent particle, position of the particle is more certain and is proportional to the size of the wave-packet. But as several waves of different wave lengths are used to represent a particle, its momentum is no longer unique and becomes uncertain.

Thus, the fundamental dual nature of radiation and particle introduces uncertainty in the simultaneous measurement of physical quantities.

Heisenberg's Uncertainty Principle : According to Heisenberg's uncertainty principle, if the uncertainty in the x-coordinate of the position of a particle is Δx and uncertainty in the x-component of its momentum is Δp (i.e. in one dimension) then

$$\therefore \ \Delta x \cdot \Delta p \geq \frac{h}{2\pi} \geq \hbar \text{ (Read as } h \text{ cut or } h \text{ cross).} \qquad (7.6.4)$$

Now, if $\Delta x \rightarrow 0$ then $\Delta p \rightarrow \infty$

and $\Delta p \rightarrow 0$ then $\Delta x \rightarrow \infty$

Similarly, one finds uncertainty principle associated in measuring energy of a particle and time as,

$$\therefore \ \Delta E \cdot \Delta t \geq \hbar \qquad (7.6.5)$$

Only for Information : We discussed about the probability of the particle to be at a definite point. In fact the wave functions representing a particle can be mathematically obtained in the form of solutions of typical differential equations (Schroedinger's equation). These wave functions may be real or complex according to the situation. According to Max Born, the probability of finding a particle at any point in the space in one dimension is proportional to the square of the magnitude of such a wave function ($|\psi|^2 = \psi^*\psi$). Hence, we have to deal with such probabilities while discussing about microscopic particles. This branch of physics is called **wave mechanics.**

You might have noted that the approach of physics based on quantum mechanics is not deterministic like classical physics.

So, for a microscopic particle like an electron, it is meaningless to question whether it is a particle or a wave. Actually it is neither a wave nor a particle. It is more fundamental physical reality whose behaviour can be understood with particle mechanics in some situation and with wave mechanics in the other. The mathematical studies developed in reference to the wave and particle nature are merely two disciplines to understand the nature.

Noted writer Margenau compares the question : "wave or particle ?" with the question "what is the colour of an egg of an elephant ?" This question is meaningful only if an egg of an elephant exists !

Illustration 12 : Find the certainty with which one can locate the position of (1) a bullet of mass 25 g and (2) an electron, both moving with a speed 500 m/s, accurate to 0.01 %. Also, draw inferences based on your results. Mass of an electron is 9.1×10^{-31} kg.

Solution : (1) Uncertainty in measurement of momentum of a bullet is 0.01% of its exact value. i.e., $\Delta p = 0.01\%$ of mv.

$$= \left(\frac{0.01}{100}\right) \times (25 \times 10^{-3}) \times (500)$$

$$= 1.25 \times 10^{-3} \text{ kg m s}^{-1}$$

Therefore, corresponding uncertainty in the determination of position is

$$\therefore \Delta x = \frac{\hbar}{\Delta p} \text{ (using equation 7.5.4)}$$

$$= \frac{6.625 \times 10^{-34}}{2 \times 3.14 \times (1.25) \times 10^{-3}} \times 10^{-3} \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

$$= 8.44 \times 10^{-32} \, m.$$

Conclusion : The value of Δx is too small compared to the dimension of the bullet, and can be neglected. That is, position of the bullet is determined accurately.

(2) Uncertainty in measurement of momentum of an electron is

$$\therefore \Delta p = \left(\frac{0.01}{100}\right) \times (9.1 \times 10^{-31}) \times (500) = 4.55 \times 10^{-32} \text{ kgms}^{-1}$$

Corresponding uncertainty in position is

$$\Delta x = \frac{6.625 \times 10^{-34}}{2 \times 3.14 \times 4.55 \times 10^{-32}} = 0.23 \times 10^{-2} \text{ m} = 2.3 \text{ mm}$$

Conclusion : Uncertainty in position for an electron (2.3 mm) is too large compared to the dimension of an electron, when it is assumed to be as a particle. Consequently, the concept of an electron as a tiny particle does not hold.

7.7 Davisson-Germer Experiment

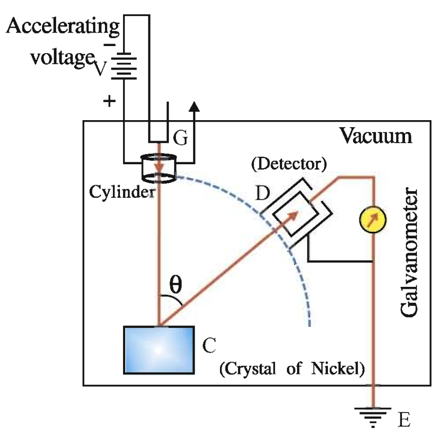


Figure 7.7 Arrangement for Davisson-Germer Experiment

Till 1927, De Broglie's hypothesis did not get any experimental confirmation. In 1927, two scientists named Davisson and Germer performed series of experiments at Bell laboratory to study scattering of electron by a piece of Nickle placed in vacuum.

The device used by them is shown in figure 7.7.

Here, G is the electron gun having tungsten filament coated with barium oxide. Filament is heated with L. T. (Low Tension = low p.d.). Hence, it emits electrons.

Now, these electrons can be accelerated under appropriate electric field produced by H. T. (High Tension). These electrons pass through a cylinder having a small hole and form a thin beam of electrons which is incident on a piece of Nickel and get scattered by it (in fact by its atoms). To detect the electrons scattered in different directions a detector D is arranged which can be moved on a circular scale as shown in figure 7.7. The output current from this detector passes through a galvanometer. The amount of current represents the number of electrons scattered in that direction.

According to classical physics, number of electrons scattered in different directions does not depend much on the angle of scattering. Also, this number hardly depends on the energy of incident electrons. Davisson and Germer tested these predictions of classical physics using the piece of Nickel as the scatterer.

During one of their experiments the bottle filled with liquefied air burst and the surface of the piece of Nickel was damaged. They heated the piece of Nickel to a high temperature and then cooled it to level its surface. Again when the experiment was repeated they found "something unusual". They found that the results of diffraction of electrons by Nickel are similar to the diffraction of X-rays by a crystal. This can happen only if electrons act as waves. This happened because when the piece of Nickel was heated and then cooled it was converted into a single crystal.

In this experiment the intensity of electron beam scattered at different angles of scattering, can be measured for the given accelerating voltage. Angle of scattering (θ) is the angle between the incident beam and scattered beam of electrons. The graphs of intensity $\rightarrow \theta$ for the observations taken by Davisson and Germer between 44 V to 68 V are shown qualitatively in figure 7.8.

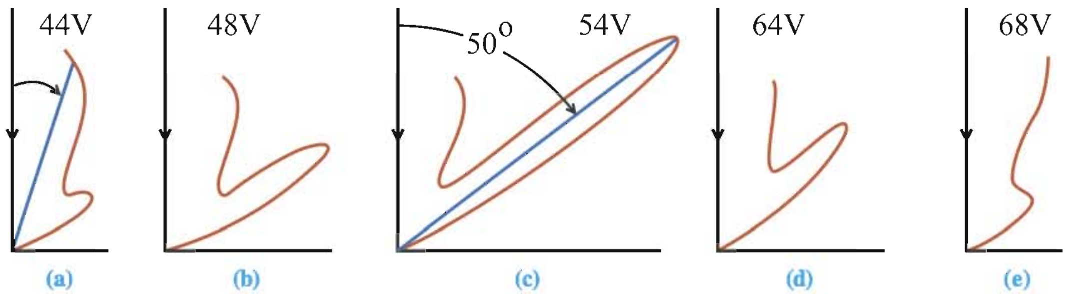


Figure 7.8 Results for Davisson Germer Experiment

The graphs indicate that the number of electrons scattered at a specific angle of scattering is maximum for the given accelerating voltage. See the graph of 54 V carefully. Here, the number of electrons scattered at an angle of 50^o is found to be maximum. These experimental results can be understood if the electrons are considered as the waves having de Broglie wavelength and if we accept that electrons are scattered just as X-rays by a crystal. The interatomic distance of Nickel is known. With this information and using the equation of scattering wavelength of electron can be obtained experimentally.

If the accelerating voltage is V and charge of an electron is e, energy of electron is

$$\frac{1}{2}mv^2 = e\text{V}$$

$$\therefore\ m^2v^2 = 2me\text{V}$$

$$\therefore\ mv = \sqrt{2me\text{V}}$$

But wavelength, $\lambda = \frac{h}{mv}$

$$\therefore\ \lambda = \frac{h}{\sqrt{2me\text{V}}} \qquad (7.7.1)$$

In above equation substituting V = 54 V, $h = 6.625 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C, we get $\lambda = 1.66 \times 10^{-10}$ m. The value of λ obtained in the experiment was 1.65×10^{-10} m. **Thus, accidentally it was proved that an electron behaves as wave also.**

For Information Only : The development of quantum physics is very interesting. This is the magnificent knowledge of mankind struggling to know the nature. Not only that but it is the confluence of rivers like science, mathematics and philosophy. Diving in it we realize how magnificent is the nature !

Illustration 13 : Suppose you are late in reaching the school, and you are going at the speed of 3.0 m s^{-1}. If your mass is 60 kg. assuming that you are a particle find your de Broglie wavelength $h = 6.625 \times 10^{-34}$ J S.

Solution : $p = mv = 60 \times 3.0 = 1.8 \times 10^2$ kg m s^{-1}

$$\text{Now, } \lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{1.8 \times 10^2} = 3.68 \times 10^{-36} \text{ m}$$

Note : This wavelength is even smaller than the radius of the nucleus (~ 10^{-15} m) by 10^{-21} times. If you want to make your wave properties "regular", your mass should be reduced to unimaginable level.

Illustration 14 : A proton falls freely under gravity of Earth. What will be its de Broglie wavelength after 10 s of its motion ? Neglect the forces other than gravitational force.

$g = 10$ m s^{-2}, $m_p = 1.67 \times 10^{-27}$ kg, $h = 6.625 \times 10^{-34}$ J s

Solution : From $v = v_0 + gt$,

$v = gt$

$\therefore$ momentum, $p = m_p v = m_p gt$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{m_p gt}$$

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{1.67 \times 10^{-27} \times 10 \times 10}$$

$$\therefore \lambda = 3.96 \times 10^{-9} \text{ m} = 39.6 \text{ Å}$$

Illustration 15 : An electron is at a distance of 10 m from a charge of 10 C. Its total energy is 15.6×10^{-10} J. Find its de Broglie wavelength at this point.

$h = 6.625 \times 10^{-34}$ J s; $m_e = 9.1 \times 10^{-31}$ kg; $k = 9 \times 10^9$ SI,

$e = 1.6 \times 10^{-19}$ C

Solution : Potential energy of an electron, $U = -k\frac{(q)(e)}{r}$

$$\therefore U = -\frac{9 \times 10^9 \times 10 \times 1.6 \times 10^{-19}}{10}$$

$\therefore U = -14.4 \times 10^{-10}$ J (1)

Now total energy E = Kinetic energy K + Potential energy U

$\therefore K = E - U$

$= 15.6 \times 10^{-10} + 14.4 \times 10^{-10}$

$\therefore$ K = 30 × 10^{-10} J

But, $K = \frac{p^2}{2m_e}$

$\therefore\ p = \sqrt{2Km_e}$

$$\lambda = \frac{h}{\sqrt{2Km_e}} = \frac{6.625\times10^{-34}}{\sqrt{2\times30\times10^{-10}\times9.1\times10^{-31}}}$$

$\therefore\ \lambda = 8.97 \times 10^{-15}$ m

Illustration 16 : Compare energy of a photon of X-rays having 1 Å, wavelength with the energy of an electron having same de Broglie wavelength. $h = 6.625 \times 10^{-34}$ J s; $c = 3 \times 10^8$ m s^{-1}; $m_e = 9.1 \times 10^{-31}$ kg

Solution : For photon,

Energy, $E_p = hf = \frac{hc}{\lambda}$ $\qquad\qquad \lambda = 1$ Å $= 10^{-10}$ m

$$\therefore\ E_p = \frac{6.625\times10^{-34}\times3\times10^8}{10^{-10}} = 19.87 \times 10^{-16} \text{ J}$$

For an electron;

Energy, $E_e = \frac{p^2}{2m}$

According to de Broglie relation, $p = \frac{h}{\lambda}$

$$\therefore\ E_e = \frac{h^2}{\lambda^2(2m)} = \frac{(6.625\times10^{-34})^2}{(10^{-10})^2\times2\times9.1\times10^{-31}} = 2.41 \times 10^{-17} \text{ J}$$

$$\therefore\ \frac{E_p}{E_e} = \frac{19.87\times10^{-16}}{2.41\times10^{-17}}$$

$$\therefore\ \frac{E_p}{E_e} = 82.4$$

Thus, energy of photon is 82.4 times the energy of electron having same wavelength.

Illustration 17 : Wavelength of an electron having energy E is $\lambda_0 = \frac{h}{\sqrt{2mE}}$, where m is the mass of an electron. Find the wavelength of the electron when it enters in X-direction in the region having potential V(x). If we imagine that due to the potential, electron enters from one medium to another, what is the refractive index of the medium ?

Solution : Energy of electron in the region having potential

E = (Kinetic energy)K + (Potential energy)U

$\therefore\ E = \frac{p^2}{2m} - eV(x)$

$$\therefore\ p = [2m(E + eV(x)]^{\frac{1}{2}}$$

$$\therefore\ \lambda = \frac{h}{p} = \frac{h}{[2m(E + eV(x)]^{\frac{1}{2}}}$$

Now, refractive index $= \dfrac{\lambda_0}{\lambda} = \dfrac{[2m(E + eV(x)]^{\frac{1}{2}}}{(2mE)^{\frac{1}{2}}}$ $\quad (\because\ \lambda_0 = \dfrac{h}{\sqrt{2mE}})$

$$\therefore\ \text{Refractive index} = \left[\frac{E + eV(x)}{E}\right]^{\frac{1}{2}}$$

Illustration 18 : Consider the radius of a nucleus to be 10^{-15} m. If an electron is assumed to be in such nucleus, what will be its Energy ?

Mass of electron = 9.1×10^{-31} kg; $h = 6.625 \times 10^{-34}$ J s

Solution : As the electron acts as a wave in this situation, the maximum uncertainty in its position.

$\therefore\ \Delta x = 2r = 2 \times 10^{-15}$ m $\qquad$ r = radius of the nucleus = 10^{-15} m

Now, according to Heisenberg's principle

$$\therefore\ \Delta x \cdot \Delta p \approx \frac{h}{2\pi}$$

$$\therefore\ \Delta p \approx \frac{h}{2\pi\Delta x} = \frac{6.625\times10^{-34}}{2\times3.14\times2\times10^{-15}} = 0.5274 \times 10^{-19}$$

Now, if this uncertainty is (approximately) taken as the momentum ($p \approx \Delta p$), energy of electron

$$E = \frac{p^2}{2m}$$

$$= \frac{(0.5274\times10^{-19})^2}{2\times9.1\times10^{-31}}\ \text{J} = \frac{(0.5274\times10^{-19})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}}\ \text{eV} = 9.55 \times 10^3\ \text{MeV}$$

Now, the binding energy of a nucleus is several MeV. As compared to it the energy of an electron in the nucleus is very large. Hence, electron can not reside in a nucleus.

Illustration 19 : Find the wave packet formed due to the superposition of two harmonic waves represented by $y_1 = A\ sin(\omega t - kx)$ and $y_2 = A\ sin[(\omega + d\omega)t - (k + dk)x]$

Solution : According to the principle of superposition,

$$y = y_1 + y_2$$

$$= A\ sin(\omega t - kx) + A\ sin[(\omega + d\omega)t - (k + dk)x]$$

Using the relation $sin\ A + sin\ B = 2\ sin\left(\frac{A+B}{2}\right) \cdot cos\left(\frac{A-B}{2}\right)$

$$y = 2A\ cos\left(\frac{xdk - td\omega}{2}\right) \cdot sin\left[(\omega t - kx) + \left(\frac{td\omega - xdk}{2}\right)\right]$$

As amplitude of the wave packet = $2A\ cos\left(\frac{xdk - td\omega}{2}\right)$, it depends both on the position and time.

SUMMARY

1. Difficulties by the classical theoretical explanation of certain experimental observations like, energy distribution in black-body radiation, stability of an electrically neutral atom and its spectra, specific heats of solids and diatomic molecules at low temperatures, etc., have forced scientists to think totally differently.
2. Planck with his revolutionary idea that energy of microscopic oscillating dipoles is quantized to *hf*. And total energy is always an integral multiple of the smallest quantum of energy (*hf*), the photon. Here, *h* is known as the Planck's constant. The photon possesses all the properties of a material particle.
3. Planck's hypothesis could solve black-body radiation problem successfully.
4. To bring an electron out of the metal, some minimum amount of energy must be supplied to an electron, which is known as work function of the metal. The work function depends on the type of metal, nature of its surface and its temperature.
5. Corresponding to work function minimum frequency required to eject photoelectron is known as the threshold frequency.
6. Dependence of photoelectric current on the intensity of incident light, value of maximum kinetic energy of an emitted photoelectron on frequency of incident light and not on its intensity, instantaneous (within 10^{-9} sec) emission of photoelectrons cannot be explained by the wave nature of light.
7. Assuming light as a particle, Einstein could solve the mystery of the photoelectric effect. His photoelectric equation $\frac{1}{2}mv^2_{max} = eV_0 = hf - \phi_0$ is in accordance with the energy conservation law.
8. Photoelectric effect and Compton effect have confirmed the dual nature of radiation.
9. On the symmetry argument, de Broglie had further proposed dual nature for material particles. Which was supported by experimental observations (e.g., Davisson-Germer experiment) as well as by theoretical calculations (e.g., Schroedinger wave equation).
10. This confirms the dual (particle and wave) nature for both radiation and matter particles.
11. The non-zero value of Planck's constant (*h*) alongwith Heisenberg's uncertainty principle measures the inadequacy of the classical mechanics.

EXERCISES

For the following statements choose the correct option from the given options :

1. Cathode rays ....
   (A) are the atoms moving towards the cathod.
   (B) are electromagnetic waves.
   (C) are negative ions travelling from cathode to anode.
   (D) are electrons emitted by cathode and travelling towards anode.
2. Which of the following statement is not true for a photon ?
   (A) Photon produces pressure (B) Photon has energy *hf*.
   (C) Photon has momentum $\frac{hf}{c}$ (D) Rest mass of photon is zero
3. The velocity of photon emitted in photo-electric effect depends on the properties of photosensitive surface and..........
   (A) frequency of incident light (B) state of polarization of incident light
   (C) time for which the light is incident (D) intensity of incident light
4. Photoelectric effect represents that
   (A) electron has a wave nature (B) light has a particle nature
   (C) (1) and (2) both (D) none of the above

5. De Broglie wavelength of a particle moving with velocity 2.25×10^8 m s^{-1} is same as the wavelength of photon. The ratio of kinetic energy of the particle to the energy of photon is..........
Velocity of light = 3×10^8 m s^{-1}

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{7}{8}$

6. Energy of photon is $E = hf$ and its momentum is $p = \frac{h}{\lambda}$, where λ is the wavelength of photon. With this assumption speed of light wave is ....

(A) $\frac{p}{E}$ (B) $\frac{E}{p}$ (C) Ep (D) $\left(\frac{E}{p}\right)^2$

7. Wavelength λ_A and λ_B are incident on two identical metal plates and photo electrons are emitted. If $\lambda_A = 2\lambda_B$, the maximum kinetic energy of photo electrons is ....

(A) $2K_A = K_B$ (B) $K_A < \frac{K_B}{2}$ (C) $K_A = 2K_B$ (D) $K_A > \frac{K_B}{2}$

8. Cathode rays travelling in the direction from east to west enter in an electric field directed from north to south. They will deflect in ....
(A) east (B) west (C) south (D) north

9. If photoelectric effect is not seen with the ultraviolet radiations in a given metal, photo electrons may be emitted with the ....
(A) infrared waves (B) radio waves (C) X-rays (D) visible light

10. Photons of energy 1 eV and 2.5 eV successively illuminate a metal whose work function is 0.5 eV, The ratio of maximum speed of emitted electron is ....
(A) 1 : 2 (B) 2 : 1 (C) 3 : 1 (D) 1 : 3

11. When frequencies f_1 and f_2 are incident on two identical photo sensitive surfaces, maximum velocities of photo electrons of mass m are v_1 and v_2, hence ....

(A) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$ (B) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(C) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$ (D) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

12. A proton and an α-particle are passed through same potential difference. If their initial velocity is zero, the ratio of their de Broglie's wavelength after getting accelerated is.\
(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) $2\sqrt{2}$: 1

13. Mass of photon in motion is ....

(A) $\frac{c}{hf}$ (B) $\frac{h}{\lambda}$ (C) hf (D) $\frac{hf}{c^2}$

14. Wavelength of an electron having energy 10 keV is .... Å.
(A) 0.12 (B) 1.2 (C) 12 (D) 120

15. If the momentum of an electron is required to be same as that of wave of 5200 Å wavelength, its velocity should be .... m s^{-1}.
(A) 10^3 (B) 1.2×10^3 (C) 1.4×10^3 (D) 2.8×10^3

16. The uncertainty in position of a particle is same as it's de Broglie wavelength, uncertainty in its momentum is ....

(A) $\frac{\hbar}{\lambda}$ (B) $\frac{2\hbar}{3\lambda}$ (C) $\frac{\lambda}{\hbar}$ (D) $\frac{3\lambda}{2\hbar}$

17. A proton and electron are lying in a box having unpenetrable walls, the ratio of uncertainty in their velocities are .... [m_e = mass of electron and m_p = mass of proton.]

(A) $\frac{m_e}{m_p}$ (B) $m_e \cdot m_p$ (C) $\sqrt{m_e \cdot m_p}$ (D) $\sqrt{\frac{m_e}{m_p}}$

18. When α-particles are accelerated under the p.d. of V volt, their de Broglie's wavelength is .... Å [Mass of α-particle is 6.4×10^{-27} kg and its charge is 3.2×10^{-19} C.]

(A) $\frac{0.287}{\sqrt{V}}$ (B) $\frac{12.27}{\sqrt{V}}$ (C) $\frac{0.103}{\sqrt{V}}$ (D) $\frac{1.22}{\sqrt{V}}$

19. De Broglie wavelength of a proton and α-particle is same, ... physical quantity should be same for both.

(A) velocity (B) energy (C) frequency (D) momentum

20. To reduce de Broglie wavelength of an electron from 10^{-10} m to 0.5×10^{-10} m, its energy should be ....

(A) increased to 4 times (B) doubled

(C) halved (D) decreased to fourth part

21. The de-Broglie wavelength of a proton and α–particle is same. The ratio of their velocities will be ....

[α–particle is the He-nucleus, having two protons and two neutrons. Thus, its mass $m_\alpha \approx 4m_p$; where m_p is the mass of the proton.]

(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

22. The de-Broglie wavelength associated with a particle with rest mass m_0 and moving with speed of light in vacuum is ....

(A) $\frac{h}{m_0 c}$ (B) 0 (C) ∞ (D) $\frac{m_0 c}{h}$

23. An image of sun is formed by convex lens of focal length 40 cm on the metal surface of a photoelectric cell, and a photoelectric current I is produced. If now another lens with half the focal length but with same diameter is used to focus the sun image, on the photoelectric cell, photoelectric current becomes ....

(A) $\frac{I}{4}$ (B) 2 I (C) I (D) $\frac{I}{2}$

24. In quantum mechanics, a particle ....

(A) can be regarded as a group of harmonic waves.

(B) can be regarded as a single wave of definite wave-length only

(C) can be regarded as only a pair of two harmonic waves

(D) is a point-like object with mass.

25. Which of the following physical quantity has the dimension of planck constant (h) ?

(A) Force (B) Angular momentum

(C) Energy (D) Power

ANSWERS

1. (D)	2. (A)	3. (A)	4. (B)	5. (B)	6. (B)
7. (B)	8. (D)	9. (C)	10. (A)	11. (A)	12. (D)
13. (D)	14. (A)	15. (C)	16. (A)	17. (A)	18. (C)
19. (D)	20. (A)	21. (D)	22. (B)	23. (C)	24. (A)
25. (B)					

Answer the following questions in brief :

1. What is photon ?
2. What is ultraviolet catastrophe ?
3. Write Planck's hypothesis to explain energy distribution for cavity radiation.
4. Write Planck's revolutionary idea to explain energy distribution for cavity radiation.
5. Define work function of metal.
6. On which factors work function of metal depends ?
7. What is thermionic emission ?
8. Define field emission.
9. Give definition of photoelectric emission.
10. What is threshold frequency ? On which factor does threshold frequency depend ?
11. What is stopping potential ?
12. Which physical quantity can be infered from the knowledge of shopping potential ?
13. On what factor does the stopping potential depend ?
14. Write de Broglie hypothesis.
15. Define wave packet.
16. State Heisenberg's Uncertainty principle.
17. Write the conclusion of Davisson-Germer's experiment.
18. If the threshold wave length of Na element is 6800 $\overset{o}{A}$, find its work function in eV.
19. Calculate the energy of photon in eV for a radiation of wavelength 5000 $\overset{o}{A}$?

Answer the following questions :

1. Write the characteristics of photoelectric emission.
2. How wave theory fails to explain the experimental results of photoelectric effect ?
3. Explain Einstein's explanation for photoelectric effect.
4. Write the properties of a photon.
5. Write a short note on photo cell.
6. Explain the experimental arrangement of Davisson-Germer experiment.
7. Explain the conclusions of Davisson-Germer experiment.
8. Calculate the maximum kinetic energy (eV) of a photo electron for a radiation of wave length 4000 $\overset{o}{A}$ incident on a surface of metal having work function 2 eV ?
9. A light beam of 6000 $\overset{o}{A}$ wavelength and 39.6 w/m^2 intensity is incident on a metal surface. If 1 % photon of the incident photon comitts the photo electron, calculate the number of photo electron emitted per second ?

Solve the following examples :

1. A small piece of Cs (work function = 1.9 eV) is placed 22 cm away from a large metal plate. The surface charge density on the metal plate is 1.21×10^{-9} C m^{-2}. Now, light of 460 nm wavelength is incident on the piece of Cs. Find the maximum and minimum energies of photo electrons on reaching the plate. Assume that no change occurs in electric field produced by the plate due to the piece of Cs.

   [**Ans. :** Minimum energy = 29.83 eV, Miximum energy = 30.63 eV)

2. Threshold wavelength of tungsten is 2.73×10^{-5} cm. Ultraviolet light of wavelength 1.80×10^{-5} cm is incident on it. Find, (1) threshold frequency, (2) work function (3) maximum kinetic energy (in joule and eV) (4) stopping potential and (5) maximum and minimum velocity of an electron.

   [**Ans. :** (1) $f_0 = 1.098 \times 10^{15}$ Hz $\approx 1.1 \times 10^{15}$ Hz, (2) ϕ = 4.54 eV, (3) $K_{max} = 3.76 \times 10^{-19}$ J = 2.35 eV, (4) V_0 = 2.35 V, (5) $v_{max} = 9.09 \times 10^5$ m s^{-1} $\approx 9.1 \times 10^5$ m s^{-1}, V_{min} = 0 m s^{-1})

3. Wavelength of light incident on a photo-sensitive surface is reduced from 3500 Å to 290 nm. Find the change in stopping potential $h = 6.625 \times 10^{-34}$ J s. [**Ans. :** 73.42×10^{-2} V]

4. An electric bulb of 100 W converts 3% of electrical energy into light energy. If the wavelength of light emitted is 6625 Å, find the number of photons emitted is 1 s. $h = 6.625 \times 10^{-34}$ J s. [**Ans. :** 10^{19}]

5. When a radiation of wavelength 3000 Å is incident on a metal, stopping potential is found to be 1.85 V and on making radiation of 4000 Å incident on it the stopping potential is found to be 0.82 V. Find (1) Planck's constant (2) Work function of the metal (3) Threshold wavelength of the metal.. [**Ans. :** (1) $h = 6.59 \times 10^{-34}$ J s (2) ϕ_0 = 2.268 eV (3) λ_0 = 5440 Å.]

6. Work function of Zn is 3.74 eV. If the sphere of Zn is illuminated by the X-rays of wavelength 12 Å, find the maximum potential produced on the sphere.

   $h = 6.25 \times 10^{-34}$ J s. [**Ans. :** 1032.2 V]

7. Find the energy of photon in each of the following :

   (1) Microwaves of wavelength 1.5 cm (2) Red light of wavelength 660 nm

   (3) Radiowaves of frequency 96 MHz (4) X–rays of wavelength 0.17 nm

   [**Ans. :** (1) 8.3×10^{-5} eV (2) 1.9 eV (3) 4×10^{-7} eV (4) 7.3 keV]

8. Human eye can experience minimum 19 photons per second. Light of 560 nm wavelength is required for it. What is the minimum power necessary to excite optic nerves ?

   [**Ans. :** 67.4×10^{-19} W]

9. Power produced by a star is 4×10^{28} W. If the average wavelength of the emitted radiations is considered to be 4500 Å, find the number of photons emitted in 1 s.

[**Ans. :** 9.054×10^{46} photons/s]

10. What should be the ratio of de Broglie wavelenghts of an atom of nitrogen gas at 300 K and 1000 K. Mass of nitrogen atom is 4.7×10^{-26} kg and it is at 1 atm pressure. Consider it as an ideal gas. [**Ans. :** 1.826]

11. Monochromatic light of wavelength 3000 Å is incident normally on a surface of area 4 cm^2. If the intensity of light is 150 $\frac{mW}{m^2}$, find the number of photons being incident on this surface in one second. [**Ans. :** 9.05×10^{13} s^{-1}]

12. A star which can be seen with naked eye from Earth has intensity 1.6×10^{-9} W m^{-2} on Earth. If the corresponding wavelength is 560 nm, and the diameter of the lens of human eye is 2.5×10^{-3} m, find the number of photons entering in our eye in 1 s.

[**Ans. :** 9×10^4 photons/s]

13. Find the velocity at which mass of a proton becomes 1.1 times its rest mass, $m_p = 1.6 \times 10^{-27}$ kg. Also, calculate corresponding temperature. For simplicity, consider a proton as non-interacting ideal-gas particle at 1 atm pressure.

[$c = 3 \times 10^8$ ms^{-1}, $k_B = 1.38 \times 10^{-23}$ SI] [**Ans. :** $v = 0.42$ C, 6.75×10^{11} K]

14. Output power of He-Ne LASER of low energy is 1.00 mW. Wavelength of the light is 632.8 nm. What will be the number of photons emitted per second from this LASER ? $h = 6.25 \times 10^{-34}$ J s. [**Ans. :** 3.18×10^{15} s^{-1}]

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